1. (5) Give an example of connected, weighted undirected graph and a start vertex such that neither DFS search tree nor BFS search tree is a Minimum Weight Spanning tree, regardless how the adjacency lists are ordered.

2. (10) 25.1-5 (26.1.4 old) All pairs shortest paths computes \( L^{n-1} = W^{n-1} = L^0, W^{n-1} \), where the entry \( l_{ij}^{n-1} \) is the 'product' of \( i \)th row with \( j \)th column and is the shortest path distance between \( i \) and \( j \). Note that the row \( i \) of \( L^{n-1} \) is the solution to the single source shortest path for source \( i \). This row in matrix product \( A = BC \) is the \( i \)th row of \( A \) multiplied by the entire \( B \).

3. (10) 25.2-6 (26.2-5 old) In order to detect negative weight cycle we can just run Floyd Warshall algorithm one extra iteration to see if any value changes. If there are no negative weight cycles no change should occur.

4. (15) Transitive Closure 25-1 (old 26-1)
   a) At the beginning when there are no edges \( T[i, j] = 1 \) if \( i = j \) and 0 otherwise. The effect of adding an edge \((u, v)\) is to create a path via new edge of from every vertex that could reach \( u \) and to every vertex that could already be reached from \( v \). For each entry \( T_{ij} \) you have to check if \( T_{iu} == 1 \) and \( T_{vj} == 1 \) then \( T_{ij} := 1 \). This takes \( (V^2) \) because it is a double loop.

   b) Consider an example of a straight line graph \( v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_n \) and suppose that you try to insert and edge \( v_n \rightarrow v_1 \). After the edge is inserted, we have a cycle so all vertices are reachable from each other, hence all \( n^2 \) entries is 1. Since before inserting that edge only \( n(n+1)/2 \) entries were one, the remaining \( n^2 - n(n+1)/2 \) entries have to be changed, so any algorithm for updating transitive closure would take \( \Theta(V^2) \), hence \( \Omega(V^2) \).

   Using the above simple algorithm it would take \( \Theta(V^4) \) to update transitive closure, since we can have \( V^2 \) edges and inserting one edge takes \( \Theta(V^2) \). If we want to insert edge \((u, v)\), the notice that if \( T[i, v] = 1 \) then inserting that edge if not going to make any new vertices reachable from \( i \). Hence we do not have to go through the entire matrix.

   ```
   for i := 1 to |V|
   do if T[i,u] == 1 and T[i,v] == 0
       then for j:= 1 to |V|
           do if T[v,j] == 1
               then T[i,j] := 1;
   ```

   Show that the total running time over \( n \) insertions is \( O(V^3) \).

5. (10) 32.4-5 (old 34.4-5) Attach two copies if \( T \) next to each other and then search whether \( T' \) is a substring of \( T \).