

# Introduction to Software Testing Logic Coverage (Ch. 8.1.4 & 8.1.5)

**Software Testing & Maintenance**

SWE 437

<http://go.gmu.edu/swe437>

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(Dr. B for short)

# Active Clauses – Determination

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Clause  $c_j$  **determines** the value of its predicate when the other clauses have certain values.

If  $c_j$  is changed, the value of the predicate changes

$c_j$  is called the *major clause*

Other clauses are *minor clauses*

This is called *making the clause active*.

# Determining Predicates

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$$P = A \vee B$$

if  **$B = true$** ,  $p$  is always true.

so if  **$B = false$** ,  $A$  determines  $p$ .

if  **$A = false$** ,  $B$  determines  $p$ .

$$P = A \wedge B$$

if  **$B = false$** ,  $p$  is always false.

so if  **$B = true$** ,  $A$  determines  $p$ .

if  **$A = true$** ,  $B$  determines  $p$ .

**Goal** : Find tests for each clause when the clause determines the value of the predicate.

# Infeasibility & Subsumption (8.1.4)

Consider the predicate:

$$(a > b \wedge b > c)$$

Realize the *abstract* test *tt* into a *concrete* test by finding values for *a*, *b*, and *c* that create the truth assignments *tt*

$$a=9, b=7, c=5$$

Now consider the predicate:

$$(a > b \wedge b > c) \vee c > a$$

Realize the *abstract* test *ttt* into a *concrete* test by finding values for *a*, *b*, and *c* that create the truth assignments *ttt*

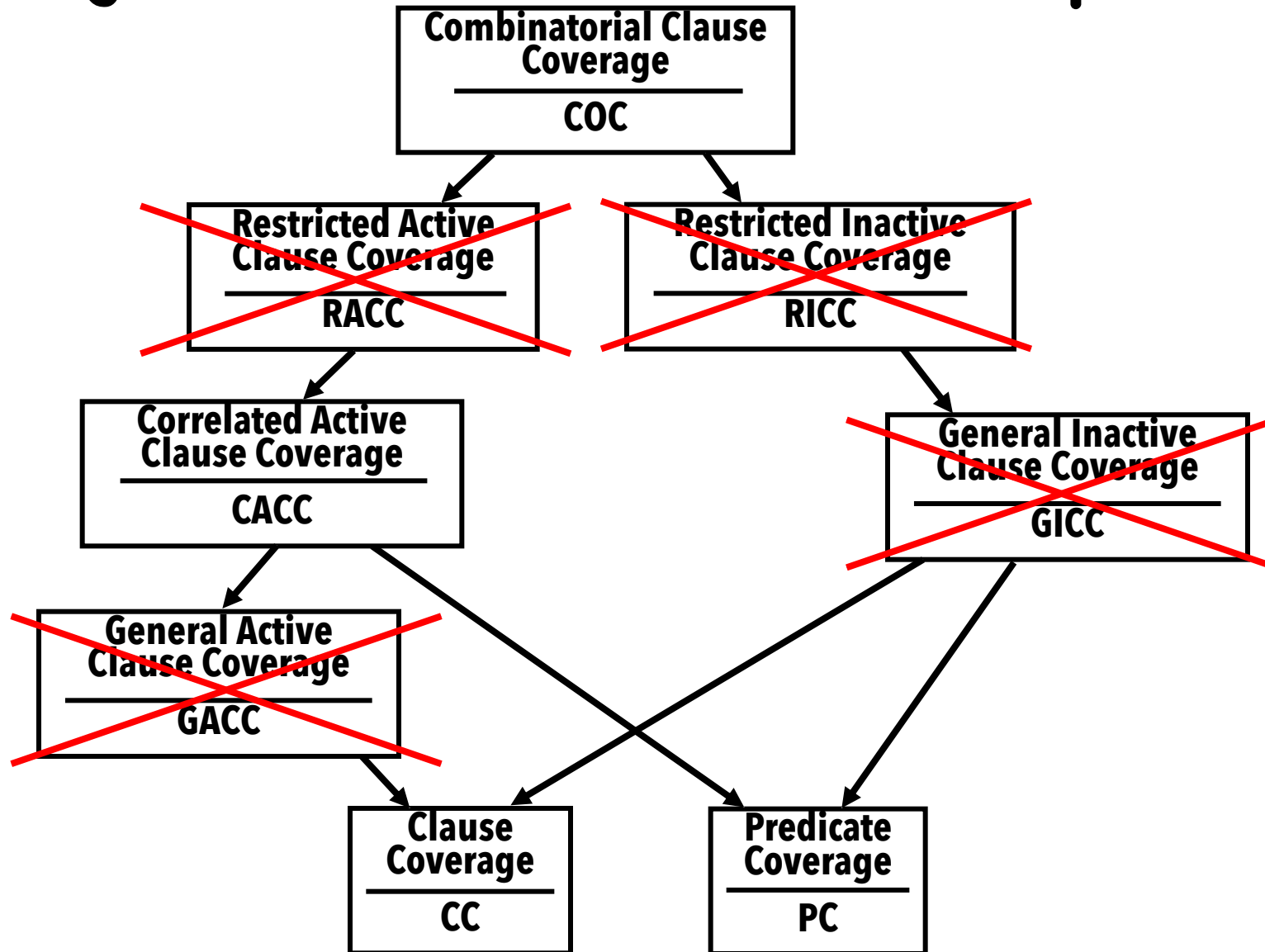
**Impossible!**

**Infeasible test requirements are recognized and ignored.**

Recognizing infeasible test requirements is generally **undecidable**.

(thus usually done by hand)

# Logic Criteria Subsumption



# Determination Techniques

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## 1. Informal **by inspection**

This is what we've been doing

Fast, but mistake-prone and does not scale—for experts

## 2. **Tabular** method

Very simple by hand

Few mistakes, slower, scales well to 5 or 6 clauses





## 3. **Definitional** method

More mathematical

Scales arbitrarily

# Tabular Method

Find pairs of rows in the truth table.

	a	b	$P=a \wedge b$	$p_a$	$p_b$
1	T	T	T		
2	T	F	F		
3	F	T	F		
4	F	F	F		

For  $p_a$ , find a **pair** of rows where

- **b is the same** in both
- **a is different**
- **P is different**

For  $p_b$ , find a **pair** of rows where

- **a is the same** in both
- **b is different**
- **P is different**



# Tabular Method

Find pairs of rows in the truth table.

	a	b	$P=a \wedge b$	$p_a$	$p_b$
1	T	T	T		
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For  $p_a$ , find a **pair** of rows where

- **b is the same** in both
- **a is different**
- **P is different**

For  $p_b$ , find a **pair** of rows where

- **a is the same** in both
- **b is different**
- **P is different**

Now do the same for "or"

	a	b	$P=a \vee b$	$p_a$	$p_b$
1	T	T	T		
2	T	F	T		
3	F	T	T		
4	F	F	F		



# In-class Exercise

## Tabular method

Use the tabular method to solve for  $P_a$ ,  $P_b$ , and  $P_c$ .  
Give solutions as pairs of rows.

	<b>a</b>	<b>b</b>	<b>c</b>	<b><math>a \wedge (b \vee c)</math></b>	<b><math>p_a</math></b>	<b><math>p_b</math></b>	<b><math>p_c</math></b>
<b>1</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>			
<b>2</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>T</b>			
<b>3</b>	<b>T</b>	<b>F</b>	<b>T</b>	<b>T</b>			
<b>4</b>	<b>T</b>	<b>F</b>	<b>F</b>	<b>F</b>			
<b>5</b>	<b>F</b>	<b>T</b>	<b>T</b>	<b>F</b>			
<b>6</b>	<b>F</b>	<b>T</b>	<b>F</b>	<b>F</b>			
<b>7</b>	<b>F</b>	<b>F</b>	<b>T</b>	<b>F</b>			
<b>8</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>			

# In-class Exercise

## Tabular method

*b* & *c* are the same, *a* differs, and *p* differs ... thus TTT and FTT cause *a* to determine the value of *p*

Likewise, for clause *c*, only one pair, TFT and TFF, cause *c* to determine the value of *p*

Again, *b* & *c* are the same, so TTF and FTF cause *a* to determine the value of *p*

Finally, this third pair, TFT and FFT, also cause *a* to determine the value of *p*

For clause *b*, only one pair, TTF and TFF cause *b* to determine the value of *p*

	<b>a</b>	<b>b</b>	<b>c</b>	<b><math>a \wedge (b \vee c)</math></b>	<b><math>p_a</math></b>	<b><math>p_b</math></b>	<b><math>p_c</math></b>
<b>1</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>			
<b>2</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>T</b>			
<b>3</b>	<b>T</b>	<b>F</b>	<b>T</b>	<b>T</b>			
<b>4</b>	<b>T</b>	<b>F</b>	<b>F</b>	<b>F</b>			
<b>5</b>	<b>F</b>	<b>T</b>	<b>T</b>	<b>F</b>			
<b>6</b>	<b>F</b>	<b>T</b>	<b>F</b>	<b>F</b>			
<b>7</b>	<b>F</b>	<b>F</b>	<b>T</b>	<b>F</b>			
<b>8</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>			

Three separate pairs of rows can cause *a* to determine the predicate.

Only one pair each for *b* and *c*.

# Definitional Method

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Scales better (more clauses), requires more math

Definitional approach:

$p_{c=true}$  is predicate  $p$  with every occurrence of  $c$  replaced by *true*

$p_{c=false}$  is predicate  $p$  with every occurrence of  $c$  replaced by *false*

To find values for the minor clauses, connect  $p_{c=true}$  and  $p_{c=false}$  with exclusive *OR*

$$p_c = p_{c=true} \oplus p_{c=false}$$

After solving,  $p_c$  describes exactly the values needed for  $c$  to determine  $p$

# Definitional Method Examples

$$\underline{p = a \vee b}$$

$$\begin{aligned} p_a &= p_{a=\text{true}} \oplus p_{a=\text{false}} \\ &= (\text{true} \vee b) \text{ XOR } (\text{false} \vee b) \\ &= \text{true XOR } b \\ &= !b \end{aligned}$$

$$\underline{p = a \wedge b}$$

*Use the definitional approach to solve for  $P_a$*

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*Use the definitional approach to solve for  $P_a$*

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*Use the definitional approach to solve for  $P_a$*

$$\underline{p = a \vee (b \wedge c)}$$

$$\begin{aligned} p_a &= p_{a=\text{true}} \oplus p_{a=\text{false}} \\ &= (\text{true} \vee (b \wedge c)) \oplus (\text{false} \vee (b \wedge c)) \\ &= \text{true} \oplus (b \wedge c) \\ &= !(b \wedge c) \\ &= !b \vee !c \end{aligned}$$

*Use the definitional approach to solve for  $P_a$*

*"NOT  $b \vee$  NOT  $c$ "* means either  $b$  or  $c$  must be false

# XOR Identity Rules

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Exclusive-OR (*xor*,  $\oplus$ ) means both cannot be true

That is,  $A \text{ xor } B$  means

*"A or B is true, but not both"*

$$\begin{aligned} p &= A \oplus A \wedge b \\ &= A \wedge \neg b \end{aligned}$$

$$\begin{aligned} p &= A \oplus A \vee b \\ &= \neg A \wedge b \end{aligned}$$

with fewer symbols ...

$$\begin{aligned} p &= A \text{ xor } (A \text{ and } b) \\ &= A \text{ and } !b \end{aligned}$$

$$\begin{aligned} p &= A \text{ xor } (A \text{ or } b) \\ &= !A \text{ and } b \end{aligned}$$



# Repeated Variables

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Definitional method yields the same tests no matter how the predicate is expressed

$$(a \vee b) \wedge (c \vee b) \equiv (a \wedge c) \vee b$$

$$(a \wedge b) \vee (b \wedge c) \vee (a \wedge c)$$

Only has 8 possible tests, not 64

Always use the simplest form of the predicate

and ignore contradictory truth table assignments

# A More Subtle Example

$$\underline{p = (a \wedge b) \vee (a \wedge ! b)}$$

$$\begin{aligned} p_a &= p_{a=\text{true}} \oplus p_{a=\text{false}} \\ &= ((\text{true} \wedge b) \vee (\text{true} \wedge ! b)) \oplus ((\text{false} \wedge b) \vee (\text{false} \wedge ! b)) \\ &= (b \vee ! b) \oplus \text{false} \\ &= \text{true} \oplus \text{false} \\ &= \text{true} \end{aligned}$$

$$\underline{p = (a \wedge b) \vee (a \wedge \neg b)}$$

$$\begin{aligned} p_b &= p_{b=\text{true}} \oplus p_{b=\text{false}} \\ &= ((a \wedge \text{true}) \vee (a \wedge ! \text{true})) \oplus ((a \wedge \text{false}) \vee (a \wedge ! \text{false})) \\ &= (a \vee \text{false}) \oplus (\text{false} \vee a) \\ &= a \oplus a \\ &= \text{false} \end{aligned}$$

***a*** always determines the value of this predicate  
***b*** never determines the value - ***b*** is **irrelevant** !

# Logic Coverage Summary

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Predicates are often **very simple**—in practice, most have less than 3 clauses

In fact, most predicates only have one clause !

With only clause, PC is enough

With 2 or 3 clauses, CoC is practical

Advantages of ACC and ICC criteria significant for large predicates

CoC is impractical for predicates with many clauses

**Control** software often has many complicated predicates, with lots of clauses

# In-class Exercise

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## Definitional method



$$P = (a \mid b) \& (a \mid c) \& d$$

Use the definitional method to solve for  $P_a$

First step:  $((T \mid b) \& (T \mid c) \& d) \text{ xor } ((F \mid b) \& (F \mid c) \& d)$

# In-class Exercise

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## Definitional method

$$P = (a \mid b) \& (a \mid c) \& d$$

Use the definitional method to solve for  $P_a$

First step:  $((T \mid b) \& (T \mid c) \& d) \text{ xor } ((F \mid b) \& (F \mid c) \& d)$

$$\begin{aligned} P_a &= ((T \mid b) \& (T \mid c) \& d) \text{ xor } ((F \mid b) \& (F \mid c) \& d) \\ &= (T \& T \& d) \text{ xor } (b \& c \& d) \\ &= d \text{ xor } (b \& c \& d) \end{aligned}$$

$$\begin{aligned} &\text{Using the identity: } A \text{ xor } (A \& b) == A \text{ and } !b \\ &= d \& !(b \& c) \\ &= d \& (!b \mid !c) \end{aligned}$$