Formation Control of Nonholonomic Mobile Robots with Omnidirectional Visual Servoing and Motion Segmentation

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Motivation

- Formation control is ubiquitous
  - Safety in numbers
  - Decreased aerodynamic drag
  - Higher traffic throughput
  - Applications in defense, space exploration, etc
Previous Work

Formation control has a very rich literature (short list):

- **String stability for line formations [Swaroop et.al. TAC96]**
  - Formation can become unstable due to error propagation

- **Mesh stability of UAVs [Pant et.al. ACC01]**
  - Generalization of string stability to a planar mesh

- **Input-to-state stability [Tanner et.al ICRA02]**
  - Structure of interconnections and amount of information communicated affects ISS

- **Feasible formations [Tabuada et.al. ACC01]**
  - Differential geometric conditions on feasibility of formations under kinematic constraints of mobile robots

- **Vision-based formation control [Das et.al. TAC02]**
  - Leader position estimated by vision; Formation control in task space
Our Approach

- Distributed formation control (no explicit communication)
- Formation specified in image plane of each follower
- Multi-body motion segmentation to estimate leader position
- Followers employ tracking controller in the image plane
- Naturally incorporate collision avoidance by exploiting geometry of omni-directional images
Outline

- Omnidirectional vision
  - Central panoramic cameras, back-projection ray
  - Central panoramic optical flow equations
  - Multi-body motion segmentation

- Distributed formation control
  - Leader-follower dynamics in image plane
  - Feedback linearization control design
  - Collision avoidance using navigation functions

- Experimental results
  - Motion segmentation of robots in real sequence
  - Vision-based formation control simulations
Central Panoramic Camera

- Catadioptric camera is lens-mirror combination
- Central panoramic: single effective focal point
  - Parabolic mirror, orthographic lens
  - Hyperbolic camera, perspective lens
- Efficiently compute back-projection ray associated with each pixel in image
Central Panoramic Optical Flow

Optical flow induced by a planar camera motion with velocities $\Omega = (0, 0, \Omega_z)^T$ and $V = (V_x, V_y, 0)^T$

Central Panoramic Projection Model

$$
\begin{bmatrix}
  x \\
  y
\end{bmatrix} = \frac{1}{-Z + \xi \sqrt{X^2 + Y^2 + Z^2}}
\begin{bmatrix}
  X \\
  Y
\end{bmatrix}
$$

Central Panoramic Optical Flow

$$
\begin{bmatrix}
  \dot{x} \\
  \dot{y}
\end{bmatrix} = \begin{bmatrix}
  -y \\
  x
\end{bmatrix} \Omega_z + \frac{1}{\lambda} \begin{bmatrix}
  1 - \rho x^2 & -\rho xy \\
  -\rho xy & 1 - \rho y^2
\end{bmatrix}
\begin{bmatrix}
  V_x \\
  V_y
\end{bmatrix}
$$

$\rho = \frac{\xi}{1 + z}$
Central Panoramic Motion Segmentation

- Optical flows of pixels $i = 1, \ldots, n$ in frames $j = 1, \ldots, m$ live in a 5 dimensional subspace.
- Optical flows can be factorized into structure and motion:

$$\begin{bmatrix} \dot{x}_{ij} & \dot{y}_{ij} \end{bmatrix} = S_i M_j^T$$

\[
S_i = \begin{bmatrix}
x_i & -y_i & \frac{1-\rho_i x_i^2}{\lambda_i} & -\rho_i x_i y_i & \frac{1-\rho_i y_i^2}{\lambda_i}
\end{bmatrix} \in \mathbb{R}^{1 \times 5}
\]

\[
M_j = \begin{bmatrix}
0 & \Omega_{zj} & V_{xj} & V_{yj} & 0 \\
\Omega_{zj} & 0 & V_{xj} & V_{yj} & 0
\end{bmatrix} \in \mathbb{R}^{2 \times 5}.
\]

- Given $k$ independent motions,

$$W \triangleq \begin{bmatrix}
\dot{x}_{11} & \dot{y}_{11} & \cdots & \dot{x}_{1m} & \dot{y}_{1m} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\dot{x}_{n1} & \dot{y}_{n1} & \cdots & \dot{x}_{nm} & \dot{y}_{nm}
\end{bmatrix} = \begin{bmatrix}
S_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & S_k
\end{bmatrix} \begin{bmatrix}
M_1^T \\
M_2^T \\
\vdots \\
M_k^T
\end{bmatrix} = SM^T$$

- Number of independent motions is obtained as:

$$k = \frac{1}{5} \text{rank}(W)$$
Central Panoramic Motion Segmentation

Independent motions live in 5 dimensional subspaces of a higher-dimensional subspace. Motion segmentation can be solved using Generalized Principal Component Analysis.

- Project onto a subspace of dimension 6.
- Apply GPCA: fit and differentiate a polynomial.

\[ b_1 \sim D_{p_n}(\mathbf{z}_1) \]
\[ b_2 \sim D_{p_n}(\mathbf{z}_2) \]

\[ L = \{ \mathbf{z}_0 + tv \} \]

\[ p_n(x) = 0 \]

\[ b_1^T x = 0 \]
\[ b_2^T x = 0 \]
Image Leader-Follower Dyr

- Kinematic model \[
\begin{bmatrix}
\dot{X} \\
\dot{Y}
\end{bmatrix}
= \begin{bmatrix}
\cos \theta \\
\sin \theta
\end{bmatrix} v, \quad \dot{\theta} = \omega
\]

- Inputs \( v \in \mathbb{R}, \omega \in \mathbb{R} \)

- Leader position \((x, y)^T\) in follower’s camera

Central panoramic leader-follower dynamics

\[
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix}
= -\begin{bmatrix}
1 - \rho x^2 & -y \\
-\rho xy & -x
\end{bmatrix} \begin{bmatrix}
v_f \\
\omega_f
\end{bmatrix}
+ \begin{bmatrix}
1 - \rho x^2 & -\rho xy & -y \\
-\rho xy & 1 - \rho y^2 & -x
\end{bmatrix} \begin{bmatrix}
F_{\ell f} \\
\omega_{\ell}
\end{bmatrix}
\]

- Write as drift-free control system:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix}
= H(x, y)u_f + d_{\ell f}
\]

- Recover leader velocity using optical flow of background

\[
d_{\ell f} = \begin{bmatrix}
\dot{x}_\ell \\
\dot{y}_\ell
\end{bmatrix}
- H(x_\ell, y_\ell)H(x_w, y_w)^{-1} \begin{bmatrix}
\dot{x}_w \\
\dot{y}_w
\end{bmatrix}
\]
Omnidirectional Visual Servoing

- Controlling in Cartesian coordinates, leader trajectory intersects circle
- Controlling in polar coordinates, follower mostly rotates
- Trajectory passing through inner circle is a collision
Omnidirectional Visual Servoing

- Leader position \((\alpha, r)\), desired leader position \((\alpha_d, r_d)\)

Feedback Linearization Control Law in Polar Coordinates

\[
u_f = \begin{bmatrix}
\frac{\lambda}{(1-\rho r^2) \cos(\alpha)} \\
\frac{\sin(\alpha)}{r(1-\rho r^2) \cos(\alpha)} \\
0
\end{bmatrix}
\begin{bmatrix}
0 \\
1
\end{bmatrix}
\left[
\begin{bmatrix}
k_1(r - r_d) \\
k_2(\alpha - \alpha_d)
\end{bmatrix} + \ddot{d}_{\ell f}
\right]
\]

Degenerate configurations

- \(\cos(\alpha) = 0\) due to nonholonomy of mobile robot
- Robot can not move sideways instantaneously
- \(r = 1\) due to geometry of central panoramic cameras
- Corresponds to horizon points at infinity
Omnidirectional Visual Servoing

- Can avoid degenerate configurations with pseudo-feedback linearizing control law

\[
u_f = \begin{bmatrix}
\frac{\lambda \cos(\alpha)}{(1-pr^2)} & 0 \\
\frac{\sin(\alpha) \cos(\alpha)}{r(1-pr^2)} & 1
\end{bmatrix}
\left(
\begin{bmatrix}
k_1(r - r_d) \\
k_2(\alpha - \alpha_d)
\end{bmatrix}
+ \tilde{d}_l_f
\right)
\]

- However, the formation is only Input-to-State Stable (ISS)
- Can easily modify control law to achieve collision avoidance by using a Navigation Function
Experimental Results

- Multi-body motion segmentation
Wedge Formation

- Green follows red \( r_d = 1/\sqrt{2}, \quad \theta_d = \pi/6 \)
- Blue follows red \( r_d = 1/\sqrt{2}, \quad \theta_d = -\pi/6 \)
Wedge Formation

Distance to leader (in pixels)

Angle to leader (in degrees)
String Formation

- Green follows red \( r_d = \frac{1}{\sqrt{2}}, \quad \theta_d = 0 \)
- Blue follows green \( r_d = \frac{1}{\sqrt{2}}, \quad \theta_d = 0 \)
String Formation

Distance to leader (in pixels)

Angle to leader (in degrees)
Conclusions

- A framework for distributed formation control in the omni-directional image plane
- An algorithm for multi-body motion segmentation in omni-directional images

Future work
- Generalize formation control to UAV dynamics
- Hybrid theoretic formation switching control
- Implement on BEAR fleet of UGVs and UAVs