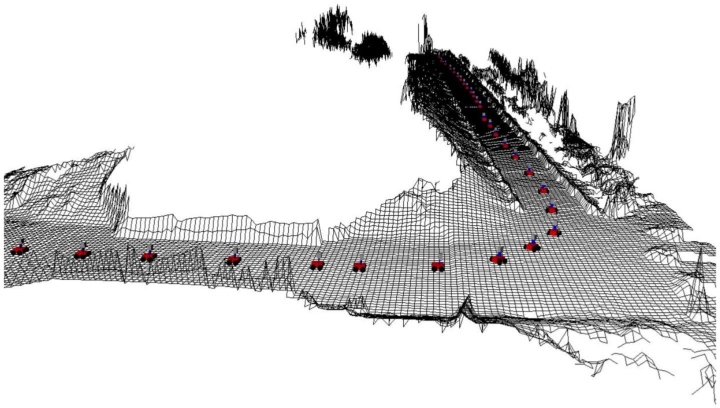
Introduction to Mobile Robotics

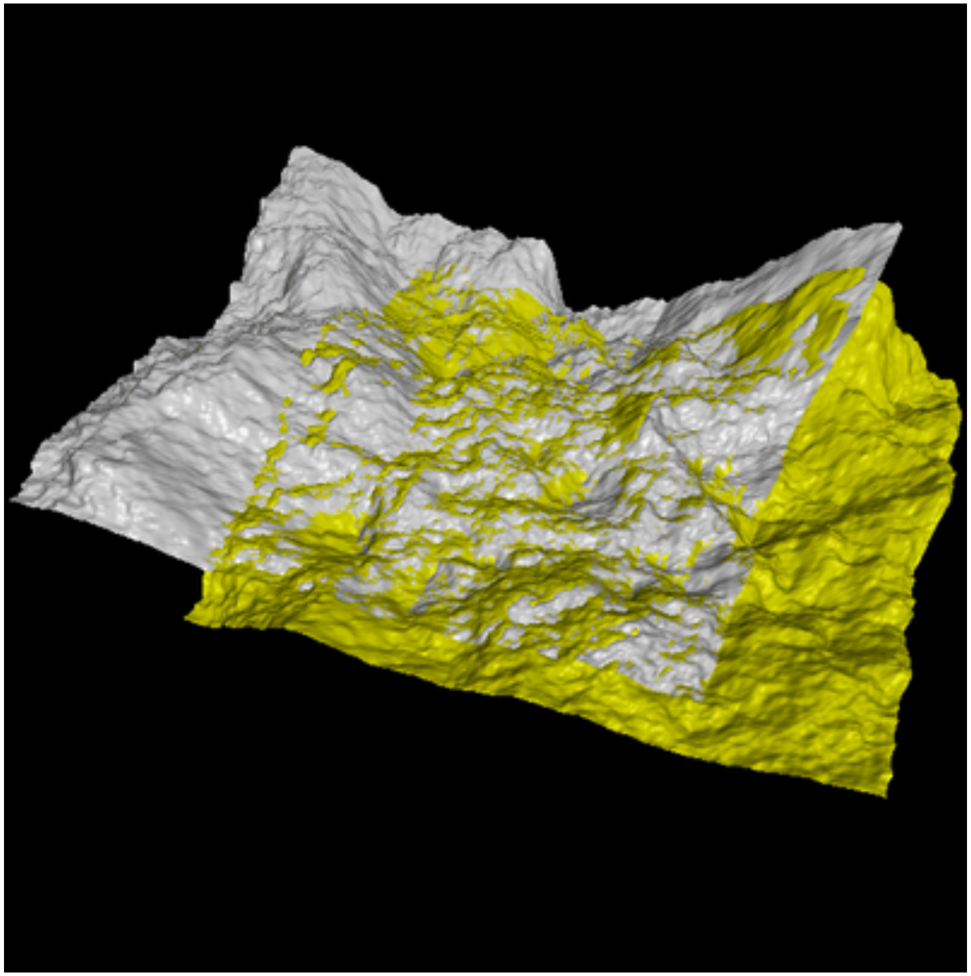
Iterative Closest Point Algorithm

Slides adopted from: Wolfram Burgard, Cyrill Stachniss,

Maren Bennewitz, Kai Arras and Probabilistic Robotics Book

Motivation





The Problem

- Given: two corresponding point sets:

$$X = \{x_1, ..., x_n\}$$

$$P = \{p_1, ..., p_n\}$$

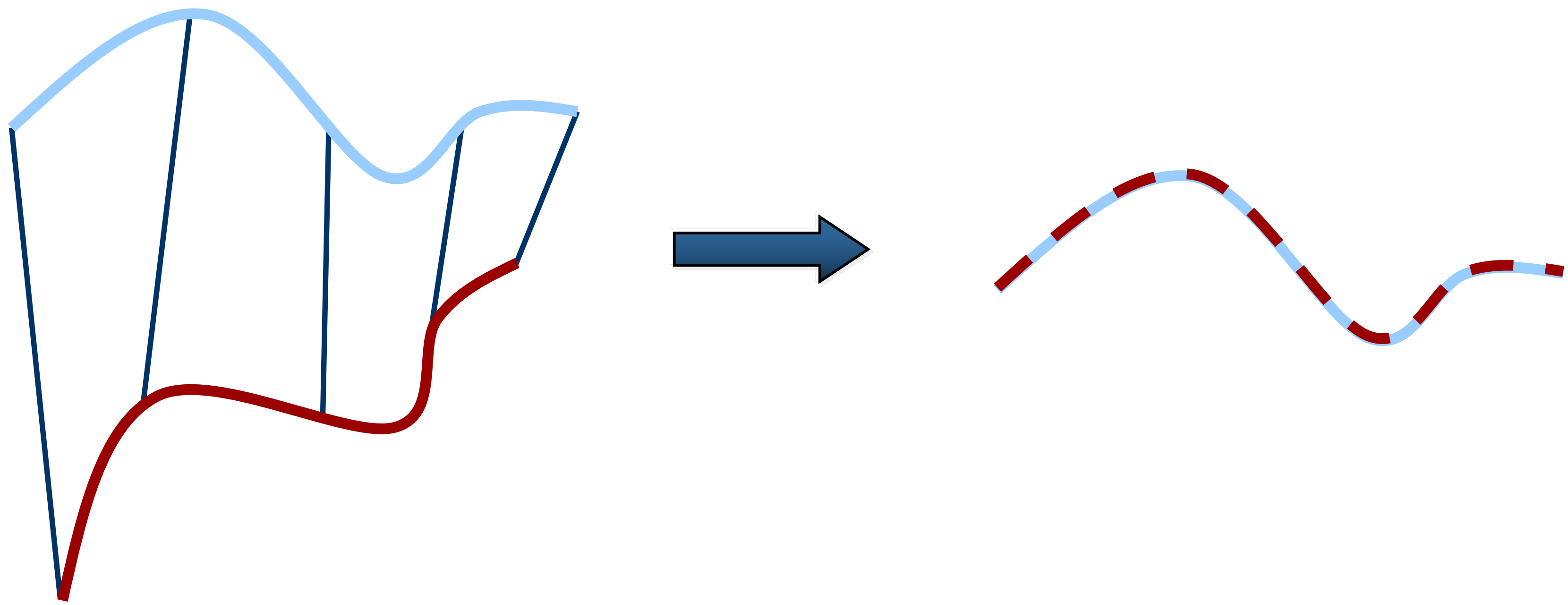
- Wanted: translation t and rotation R that minimizes the sum of the squared error:

$$E(R,t) = \frac{1}{N_p} \sum_{i=1}^{N_p} ||x_i - Rp_i - t||^2$$

Where x_i and p_i are corresponding points.

Key Idea

- If the correct correspondences are known, the correct relative rotation/translation can be calculated in closed form.



Center of Mass

$$\mu_x = \frac{1}{N_x}\sum_{i=1}^{N_x} x_i \quad \text{and} \quad \mu_p = \frac{1}{N_p}\sum_{i=1}^{N_p} p_i$$

are the centers of mass of the two point sets.

Idea:

- Subtract the corresponding center of mass from every point in the two point sets before calculating the transformation.
- The resulting point sets are:

$$X' = \{x_i - \mu_x\} = \{x'_i\}$$

and

$$P' = \{p_i - \mu_p\} = \{p'_i\}$$

SVD

Let $W = \sum_{i=1}^{N_p} x_i' p_i'^T$

denote the singular value decomposition (SVD) of W by:

$$W = U \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} V^T$$

where $U, V \in \mathbb{R}^{3\times 3}$ are unitary, and

$\sigma_1 \geq \sigma_2 \geq \sigma_3$ are the singular values of W.

SVD

Theorem (without proof):

If rank(W) = 3, the optimal solution of E(R,t) is unique and is given by:

$$R = UV^T$$

$$t = \mu_x - R\mu_p$$

Th

$$E(R,t) = \frac{1}{N_p}\sum_{i=1}^{N_p} ||x_i - Rp_i - t||^2$$

(R,t) is:

$$E(R,t) = \sum_{i=1} (||x_i'||^2 + ||y_i'||^2) - 2(\sigma_1 + \sigma_2 + \sigma_3)$$

Proof

(1)

$p=\{p_1,\dots p_n\}$ $\qquad$ $q=\{q_1,\dots q_n\}$

two sets of 3D points

$$E(R,t) = \arg\min_{R,t} \sum_{i=1}^{n} w_i \|Rp_i+t-q_i\|^2 \quad \text{for } w_i>0$$

here we assume $w_i=1$

1. Assume R is fixed and solve for translation

$$\frac{\partial E}{\partial t}=0 \qquad \sum_{i=1}^{n} 2(Rp_i+t-q_i) = 2t\sum_{i=1}^{n} 1 + 2R\sum_{i=1}^{n} p_i - 2\sum_{i=1}^{n} q_i$$

denote $\mu_p = \frac{\sum_{i=1}^{n} p_i}{n}$ $\qquad$ $\mu_q = \frac{\sum_{i=1}^{n} q_i}{n}$ mean of each point cloud

then $t = \mu_q - R\mu_p$

2. substitute t to original $E(R,t)$

$$\sum_{i=1}^{N} \|Rp_i+t-q_i\|^2 = \sum_{i=1}^{N} \|Rp_i+\mu_q - R\mu_p - q_i\|^2$$

$$= \sum_{i=1}^{N} \|R(p_i-\mu_p) - (q_i-\mu_q)\|^2$$

denote $p_i' = p_i-\mu_p$, $q_i' = q_i-\mu_q$

$$= \sum_{i=1}^{N} \|Rp_i' - q_i'\|^2$$

$$\|Rp_i'-q_i'\|^2 = p_i'^T p_i' - 2q_i'^T R p_i' + q_i'^T q_i'$$

we have to solve $\arg\min_R \left(-2\sum_{i=1}^{n} q_i'^T R p_i'\right)$

(2)

This can be written as

$$\underbrace{\begin{bmatrix} q_1'^T \\ q_i'^T \\ \vdots \\ q_n'^T\end{bmatrix}}_{Y^T} R \underbrace{\begin{bmatrix} p_1 & p_2 & \cdots & p_n\end{bmatrix}}_{X} = tr(Y^T R X)$$

Trace properties

$$tr(AB) = tr(BA)$$

$$tr(Y^T R X) = tr(R X Y^T)$$

consider covariance matrix $S = XY^T$ take SVD of S

$$S = U\Sigma V^T$$

then trace $tr(RXY^T) = tr(RU\Sigma V^T) = tr(\Sigma V^T R U)$

ortogonal matrices

=> trace is minimized if

$$V^T R U = I \Rightarrow R = VU^T$$

columns $m_i^T m_i = 1$ are ortogonal

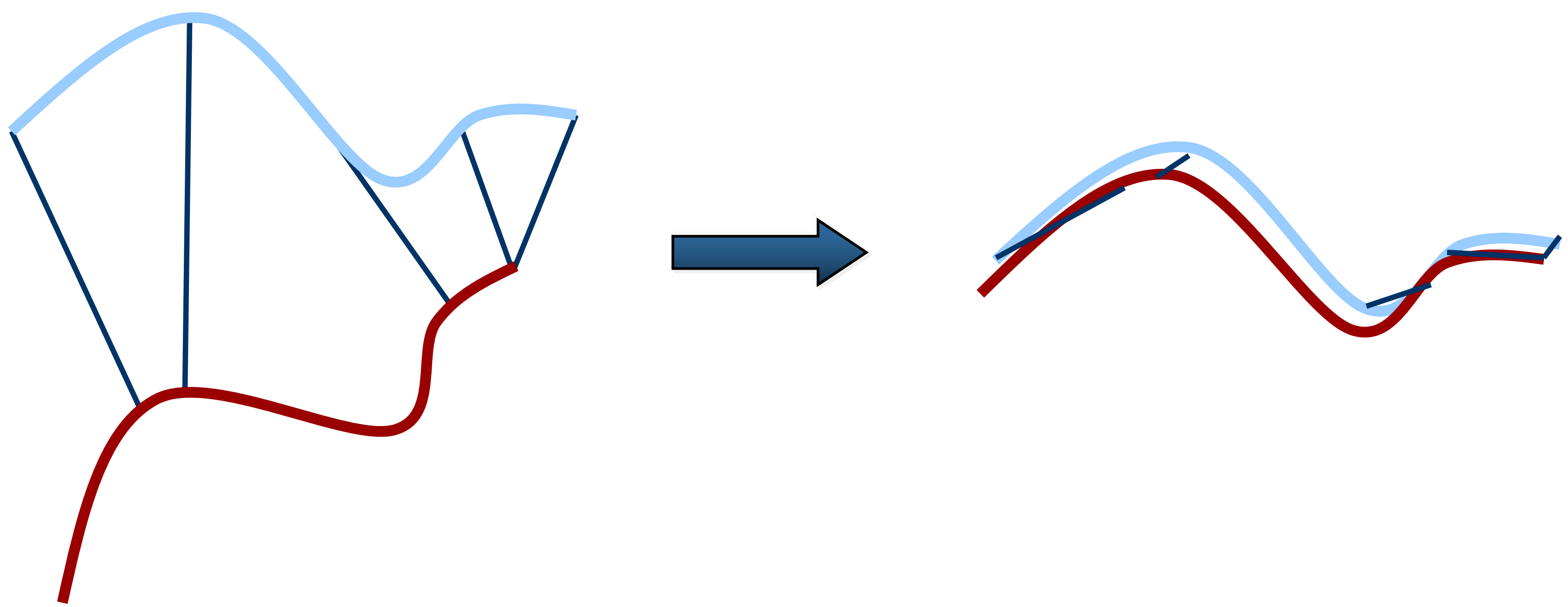
if $\det(VU^T) = -1$ then it contains reflection it is not an rotation matrix

→ we need to invert one row of a rotation matrix

$$\Rightarrow R = V \begin{bmatrix}1 & & \\ & 1 & \\ & & \det(VU^T)\end{bmatrix} U^T$$

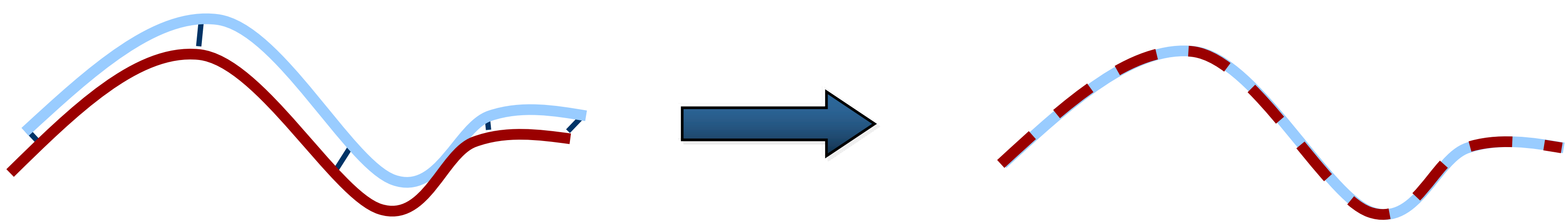
ICP with Unknown Data Association

- If correct correspondences are not known, it is generally impossible to determine the optimal relative rotation/translation in one step

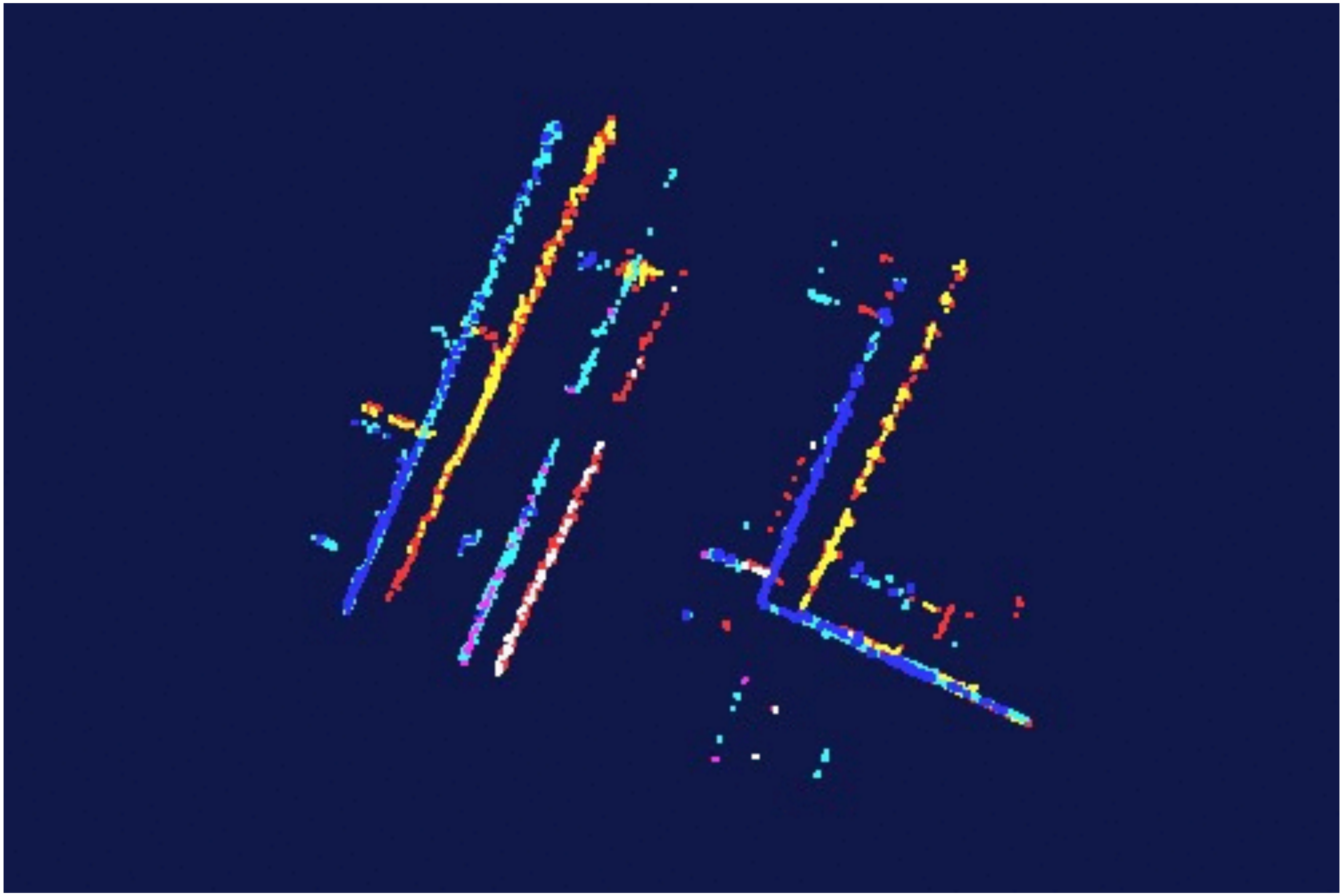


ICP-Algorithm

- Idea: iterate to find alignment
- Iterated Closest Points (ICP) [Besl & McKay 92]
- Converges if starting positions are "close enough"



Iteration-Example



ICP-Variants

- Variants on the following stages of ICP have been proposed:

  1. Point subsets (from one or both point sets)
  2. Weighting the correspondences
  3. Data association
  4. Rejecting certain (outlier) point pairs

Performance of Variants

- Various aspects of performance:
  - Speed
  - Stability (local minima)
  - Tolerance wrt. noise and/or outliers
  - Basin of convergence
    (maximum initial misalignment)
- Here: properties of these variants

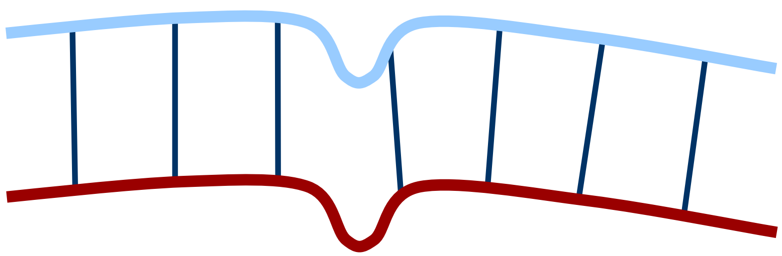
ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Data association
4. Rejecting certain (outlier) point pairs

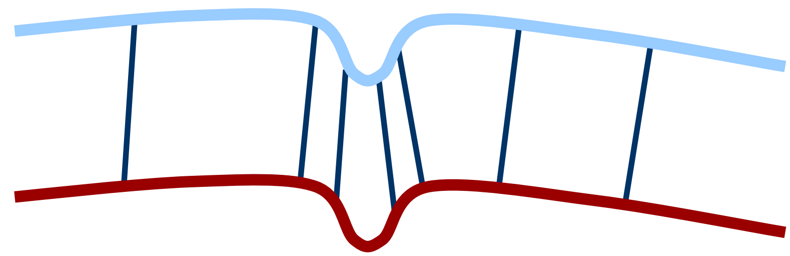
Selecting Source Points

- Use all points
- Uniform sub-sampling
- Random sampling
- Feature based Sampling
- Normal-space sampling
  - Ensure that samples have normals distributed as uniformly as possible

Normal-Space Sampling



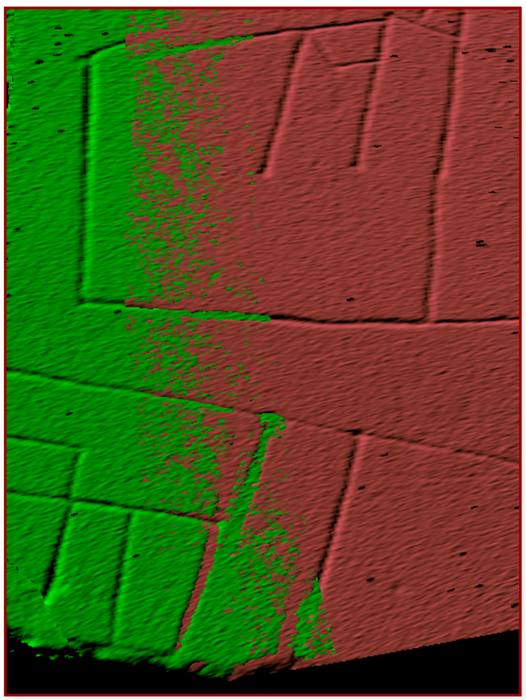
uniform sampling



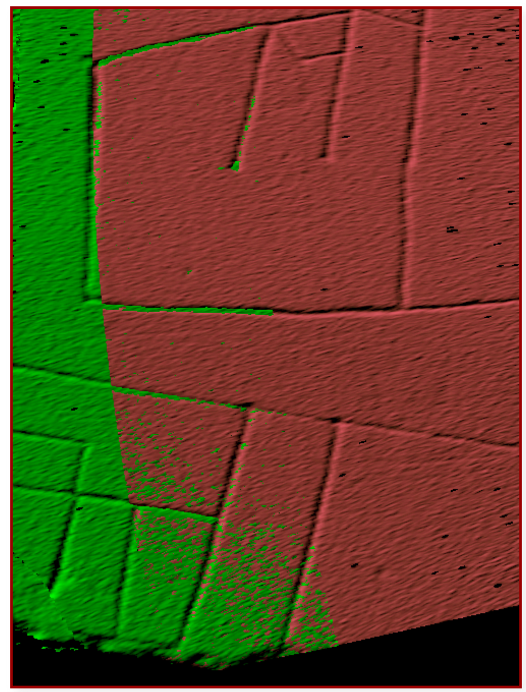
normal-space sampling

Comparison

- Normal-space sampling better for mostly-smooth areas with sparse features [Rusinkiewicz et al.]



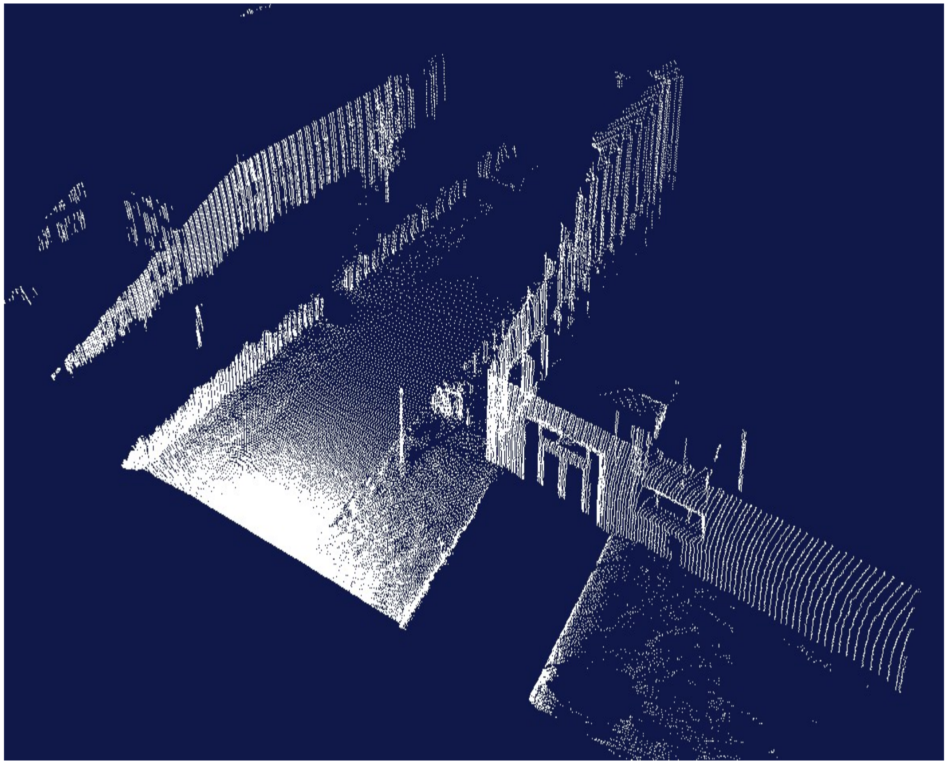
Random sampling



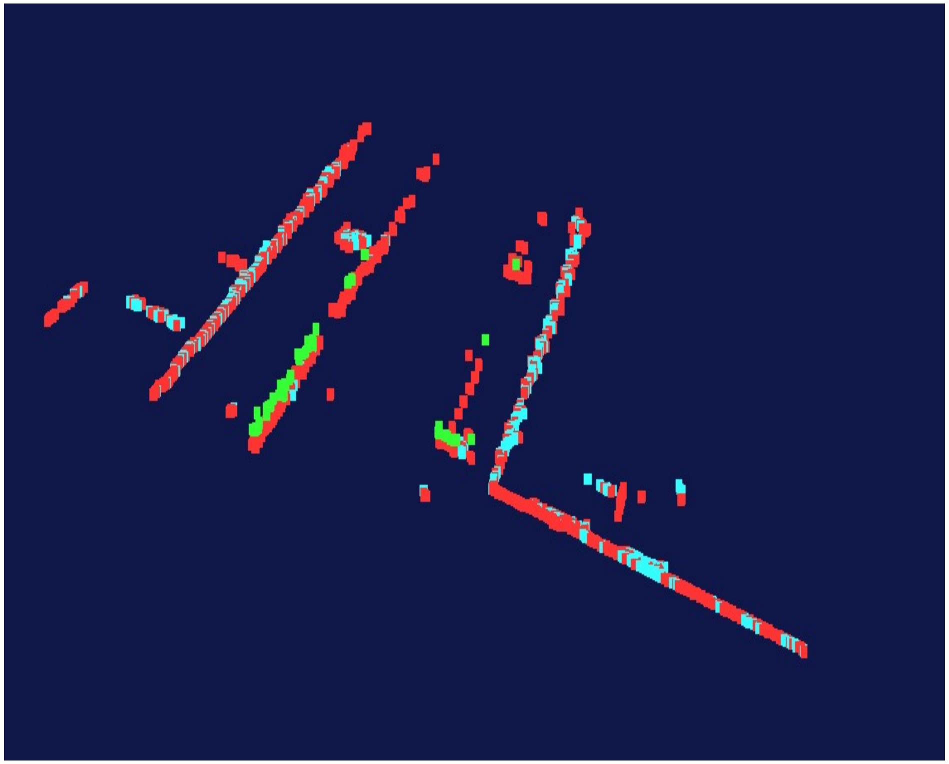
Normal-space sampling

Feature-Based Sampling

- try to find “important” points
- decrease the number of correspondences
- higher efficiency and higher accuracy
- requires preprocessing

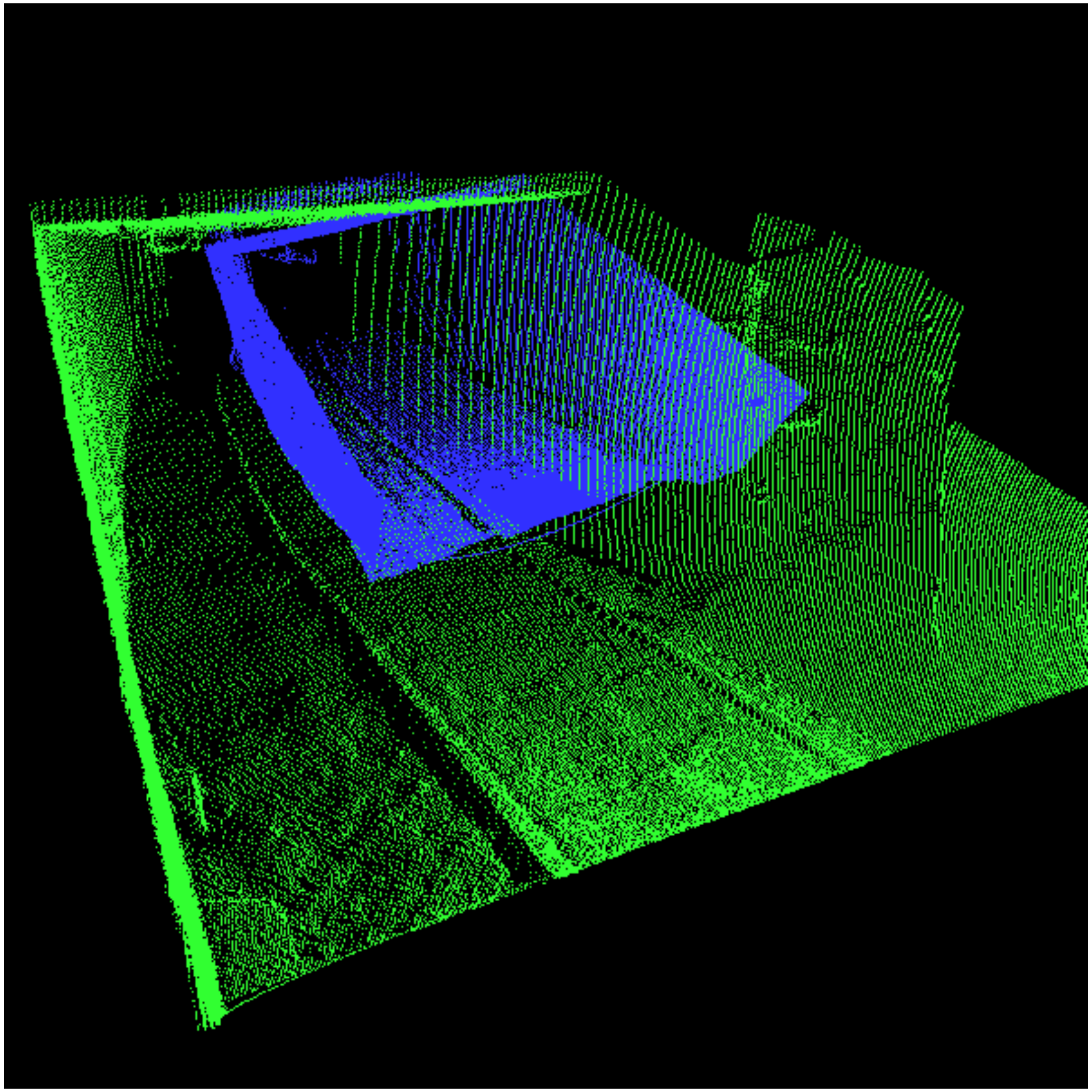


3D Scan (~200.000 Points)



Extracted Features (~5.000 Points)

Application



[Nuechter et al., 04]

ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Data association
4. Rejecting certain (outlier) point pairs

Selection vs. Weighting

- Could achieve same effect with weighting
- Hard to guarantee that enough samples of important features except at high sampling rates
- Weighting strategies turned out to be dependent on the data.
- Preprocessing / run-time cost tradeoff (how to find the correct weights?)

ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
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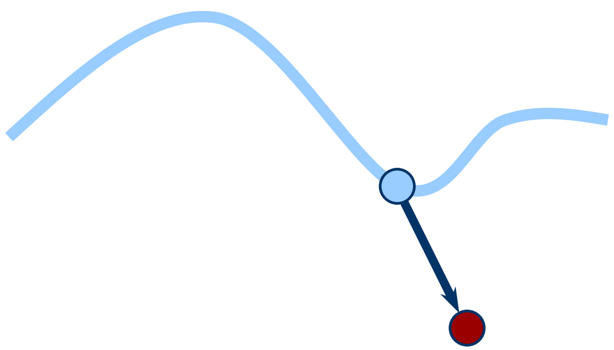
Data Association

- has greatest effect on convergence and speed
- Closest point
- Normal shooting
- Closest compatible point
- Projection
- Using kd-trees or oc-trees

Closest-Point Matching

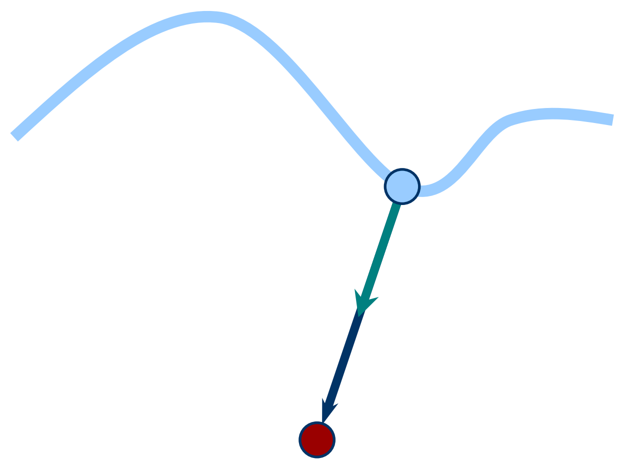
- Find closest point in other the point set



Closest-point matching generally stable, but slow and requires preprocessing

Normal Shooting

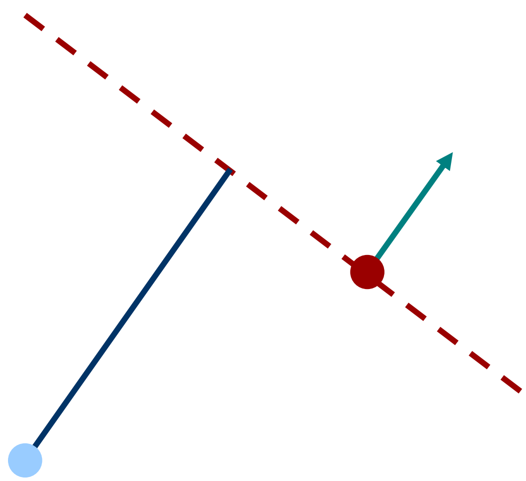
- Project along normal, intersect other point set

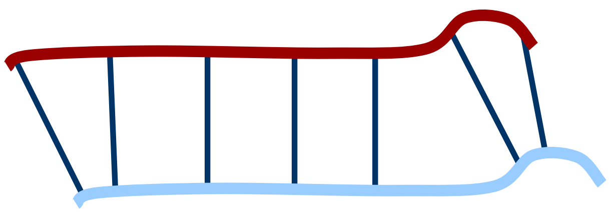


Slightly better than closest point for smooth structures, worse for noisy or complex structures

Point-to-Plane Error Metric

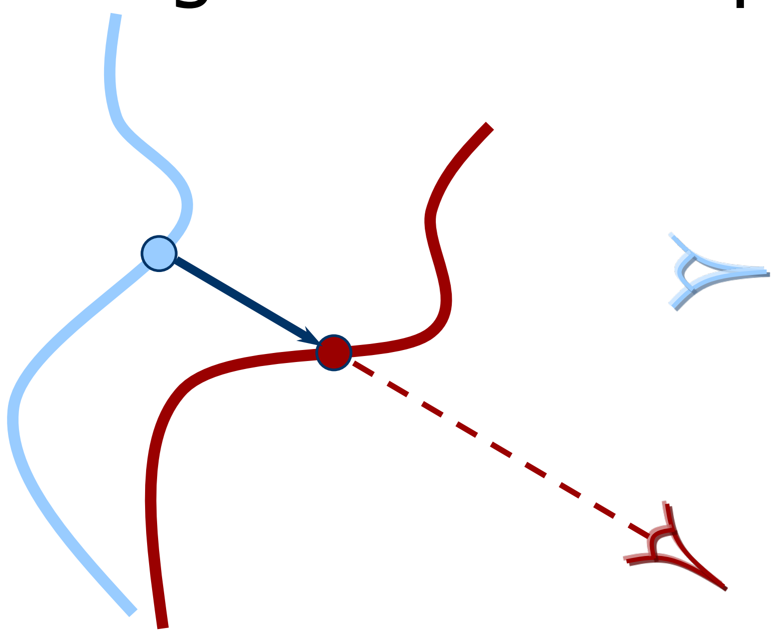
- Using point-to-plane distance instead of point-to-point lets flat regions slide along each other [Chen & Medioni 91]





Projection

- Finding the closest point is the most expensive stage of the ICP algorithm
- Idea: simplified nearest neighbor search
- For range images, one can project the points according to the view-point [Blais 95]



Projection-Based Matching

- Slightly worse alignments per iteration
- Each iteration is one to two orders of magnitude faster than closest-point
- Requires point-to-plane error metric

Closest Compatible Point

- Improves the previous two variants by considering the **compatibility** of the points
- Compatibility can be based on normals, colors, etc.
- In the limit, degenerates to feature matching

ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Nearest neighbor search
4. Rejecting certain (outlier) point pairs

Rejecting (outlier) point pairs

- sorting all correspondences with respect to there error and deleting the worst t%, Trimmed ICP (TrICP) [Chetverikov et al. 2002]
- t is to Estimate with respect to the Overlap

→ Problem: Knowledge about the overlap is necessary or has to be estimated

ICP-Summary

- ICP is a powerful algorithm for calculating the displacement between scans.
- The major problem is to determine the correct data associations.
- Given the correct data associations, the transformation can be computed efficiently using SVD.