CS 485: Autonomous Robotics

Bug Algorithms

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Outline

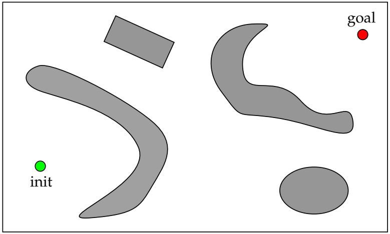
1. General Properties of Bug Path-Planning Algorithms

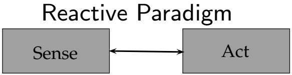
2. Bug Algorithms with Tactile (Contact) Sensors
   - `Bug0`
   - `Bug1`
   - `Bug2`

3. Bug Algorithms with Range Sensors
   - `TangentBug`

4. Summary

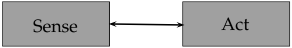
Problem: Compute a collision-free path from an initial to a goal position





- No global model of the world, i.e., obstacles are unknown
- Only local information acquired through sensing
- Inspired by insects

Reactive Paradigm



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Properties

- Complete algorithms, i.e., find solution if it exists, report no when there is no solution
- Theoretical lower and upper bounds on path length; optimal paths in certain cases

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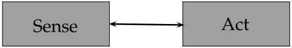

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Environment

- Two-dimensional scene filled with unknown obstacles
- Each obstacle is a simple closed curve of finite length and non-zero thickness
- A straight line crosses an obstacle finitely many times
- Obstacles do not touch each other
- Locally finite number of obstacles, i.e., any disc of finite radius intersects a finite set of obstacles
- Initial and goal positions are known

Bug Path-Planning Algorithms

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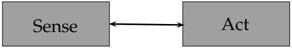

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Point Robot, Simple Motions

- Move straight toward `goal`
- Move along obstacle boundary
- Stop

Bug Path-Planning Algorithms

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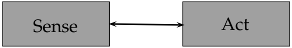

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Simple Sensing

- `Bug1`, `Bug2` assume essentially tactile (contact) sensing
- `TangentBug`, `VisBug`, `DistBug` deal with finite distance sensing
- `I-Bug` uses only signal strength emanating from goal

Tactile Sensor

- Provides current position
- Detects when a contact with an obstacle occurs

Bug with Tactile (Contact) Sensor

Tactile Sensor

- Provides current position
- Detects when a contact with an obstacle occurs

`Bug0`, `Bug1`, `Bug2` Algorithms – General Idea

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - follow obstacle boundary
  - at some point, leave the obstacle and head again toward `goal`

Tactile Sensor

- Provides current position
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Bug0, Bug1, Bug2 Algorithms – General Idea

repeat until goal is reached

- head toward goal
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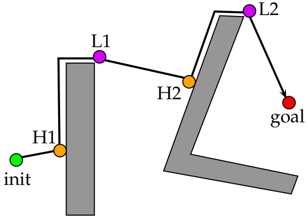
Path consists of a sequence of hit (H_i) and leave (L_i) points
Algorithms differ on how leave points are computed

repeat until goal is reached

- head toward goal
- if sensor reports contact with an obstacle then
  - follow obstacle boundary
    until can head toward goal again



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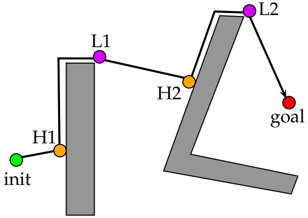

Is `Bug0` a complete algorithm?

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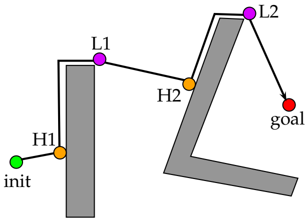

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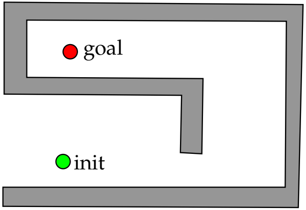

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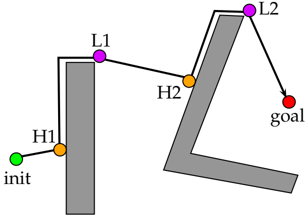

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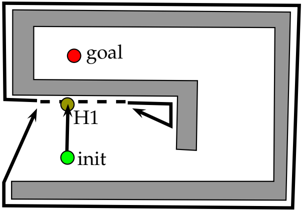

Bug0 fails to find a solution even though a solution exists

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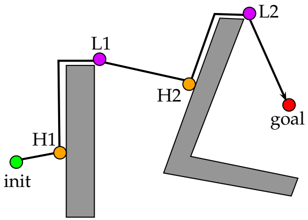

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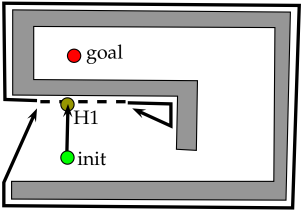

Bug0 fails to find a solution even though a solution exists
Bug0 has no memory

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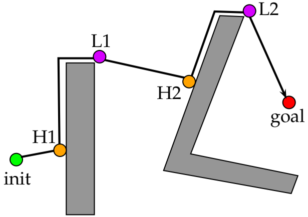

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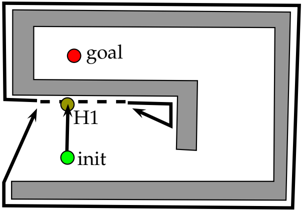

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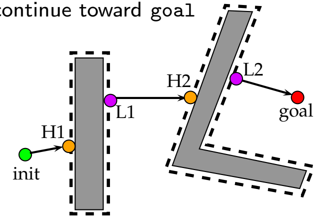
can we obtain a complete algorithm if Bug has some memory?

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle then
  - circumnavigate the obstacle and *remember* how close you get to the goal
  - return to that closest point (by wall following) and continue toward `goal`



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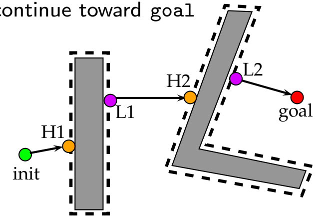

Bug1 Pseudocode

1: $L_0 \leftarrow$ `init`; $i \leftarrow 1$
2: **loop**

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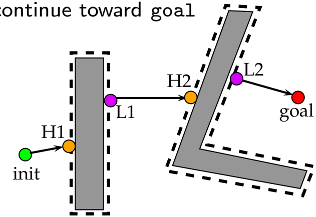

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loop

  repeat move on a straight line from L_{i-1} to `goal`

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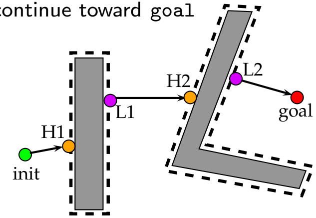

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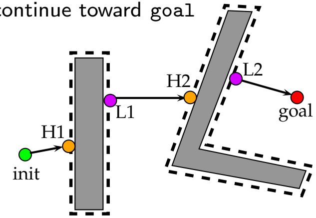

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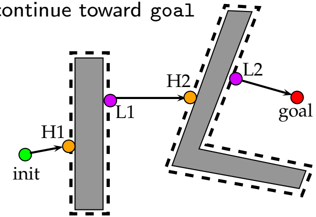

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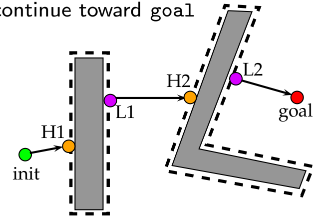

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    follow boundary from H_i to L_i along shortest route

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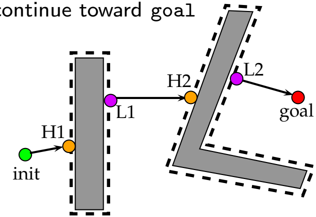

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  follow boundary from H_i to L_i along shortest route
  if move on straight line from L_i toward goal moves into obstacle **then** exit with failure
  else $i \leftarrow i + 1$

Lemma 1: When the bug leaves a leave point of an obstacle to continue its way toward `goal`, the bug never returns to this obstacle again

Proof Sketch:

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Thus,

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Thus, since $d(L_i, \mathtt{goal})$ is the shortest distance from the i-th obstacle to $\mathtt{goal}$ and since each each new hit point is closer than the last leave point, then bug cannot encounter the i-th obstacle again

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Lemma 2: Bug meets only a finite number of obstacles

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Thus, since $d(L_i, \texttt{goal})$ is the shortest distance from the i-th obstacle to goal and since each each new hit point is closer than the last leave point, then bug cannot encounter the i-th obstacle again

Lemma 2: Bug meets only a finite number of obstacles

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Bug1 Properties

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Corollary: `Bug1` *algorithm always terminates in finite time*

Proof Sketch: Follows immediately from Lemma 1 and Lemma 2

Theorem: `Bug1` is a complete path-planning algorithm, i.e., in finite time, `Bug1`

- *finds a path to `goal` when a path exists or*
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   - Thus, bug must have encountered this other intersection point (which is supposedly closer to the goal) when circumnavigating obstacle boundary, which contradicts definition of leave point

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Bug1 Lower and Upper Bounds on Path Length

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Therefore, upper bound

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What is n?

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- see proof of Lemma 2, distances from $H_1, L_1, H_2, L_2, \ldots$ to goal become smaller and smaller and are never more than $d(\texttt{init}, \texttt{goal})$. So, bug never encounters obstacles outside this disk

Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

call the line from `init` to `goal` the *m*-line

repeat until `goal` is reached

- head toward `goal`
- if sensor reports contact with an obstacle $\mathcal{O}_i$ then
  - follow $\mathcal{O}_i$ until *m*-line encountered again at a closer point to `goal`
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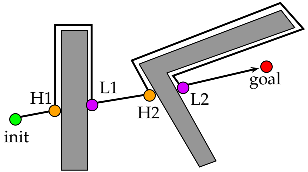

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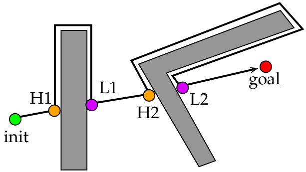

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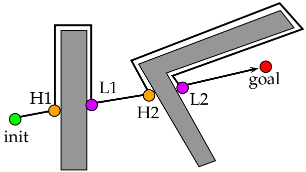

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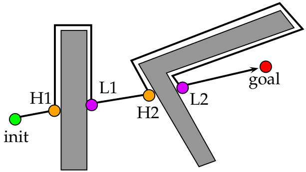

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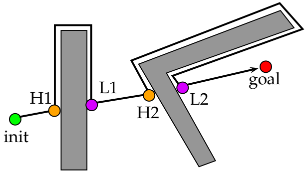
    (a) `goal` is reached or
    (b) H_i is re-encountered
    (c) *m*-line is re-encountered at Q s.t
        $Q \neq H_i$
        $d(Q, \texttt{goal}) < d(H_i, \texttt{goal})$, and
        line $(Q, \texttt{goal})$ does not cross $\mathcal{O}_i$ at Q



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- if sensor reports contact with an obstacle $\mathcal{O}_i$ then
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Bug2 Pseudocode

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  (a) `goal` is reached or
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  (c) *m*-line is re-encountered at Q s.t
    $Q \neq H_i$
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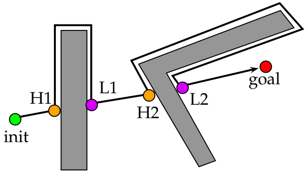

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Vladimir J. Lumelsky and Alexander A. Stepanov: Algorithmica (1987) 2:403–430

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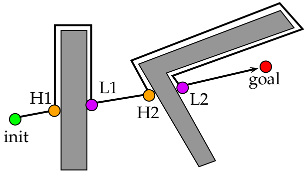

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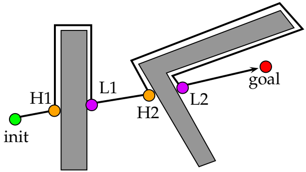
  else $L_i \leftarrow Q$; $i \leftarrow i + 1$



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Theorem: $\mathit{Bug2}$ is a complete path-planning algorithm. Moreover, the length of a path generated by $\mathit{Bug2}$ never exceeds the limit

$$d(init, goal) + \sum_i \frac{n_i p_i}{2},$$

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Proof Sketch for Lemma 3: Similar to for Bug1.
Proof Sketch for Lemma 4: (take-home exercise)

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Proof Sketch for Lemma 3: Similar to for `Bug1`.
Proof Sketch for Lemma 4: (take-home exercise)

Useful ideas:

- m-line intersects $\mathcal{O}_i$ n_i times
- At most n_i leave points from $\mathcal{O}_i$ (Why?)
- Half of them not valid (Why?)
- Distance traversed to reach each valid point is what?

Bug1 vs Bug2

Bug1 vs Bug2

Bug1 is an exhaustive search algorithm
– looks at all choices before commiting
Bug1 has a more stable performance

Bug2 is a greedy search algorithm
– takes first choice that looks better
Bug2 often outperforms Bug1, but not always

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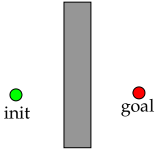

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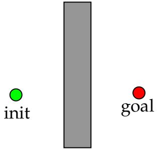

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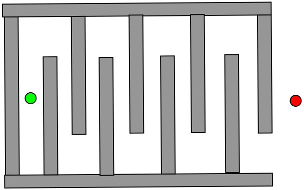

Bug1 beats Bug2 in this scene

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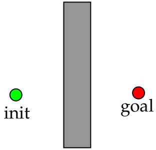

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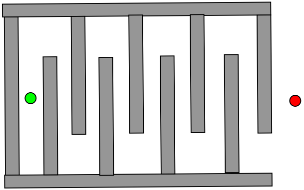

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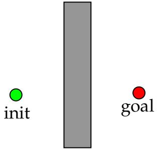

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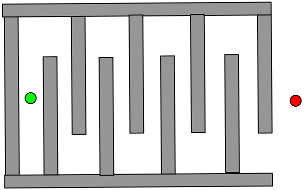

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what happens if Bug2 decides at random whether to turn counterclock-wise or clockwise each time it has follow an obstacle boundary?

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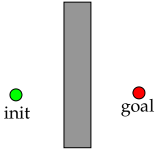

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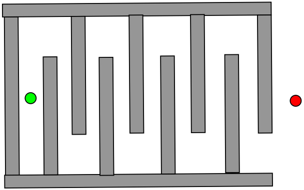

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what happens if Bug2 decides at random whether to turn counterclock-wise or clockwise each time it has follow an obstacle boundary?

can you draw a scene then where Bug1 beats Bug2 no matter how Bug2 decides to turn each time it has follow an obstacle boundary?

Bug with Range Sensor

Raw Distance Function $\rho : \mathbb{R}^2 \times [0, 2\pi) \rightarrow \mathbb{R}$

$$\rho(x, \theta) = \min_{\alpha \in [0, \infty)} \alpha \text{ such that the point } x + \alpha \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \in \bigcup_i \texttt{Boundary}(O_i)$$

- $\rho(x, \theta)$ is the distance to the closest obstacle along the ray emanating from point $x \in \mathbb{R}^2$ at an angle $\theta \in [0, 2\pi)$

Saturated Raw Distance Function $\rho_R : \mathbb{R}^2 \times [0, 2\pi) \rightarrow \mathbb{R}$ with Sensing Range $R \in \mathbb{R}^{\geq 0}$

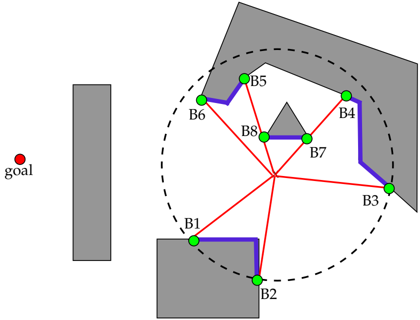
$$\rho_R(x, \theta) = \begin{cases} \rho(x, \theta), & \text{if } \rho(x, \theta) < R \\ \infty, & \text{otherwise} \end{cases}$$

- ρ_R has same value as ρ when obstacle is within sensing range R
- ρ_R has ∞ value when obstacles are outside the sensing range R

Ishay Kamon, Elon Romon, and Ehud Rivlin: IJRR (1998) 17:934–953

`TangentBug` relies on range sensor ρ_R to compute endpoints of finite continuous segments on obstacle boundaries

These segments constitute its local model of the world



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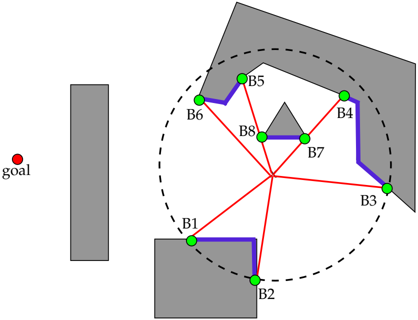

- `TangentBug` currently thinks it has unobstructed way to goal

TangentBug Algorithm – Idea

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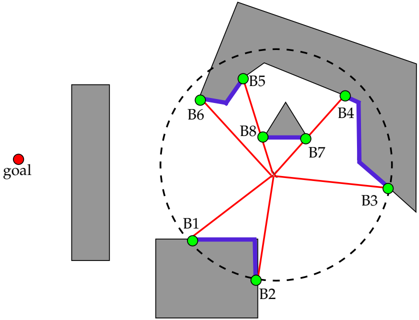

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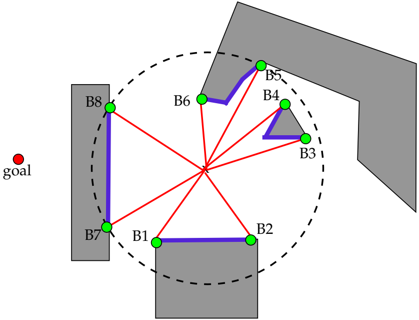

- TangentBug now sees that it can't go straight to the goal. What can it do?

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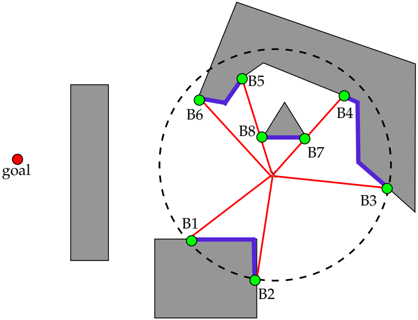

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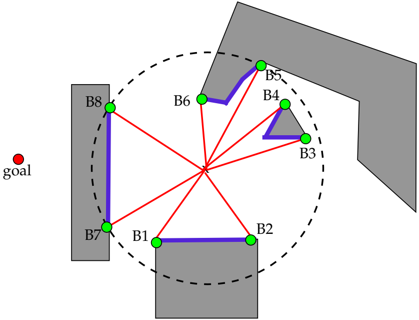

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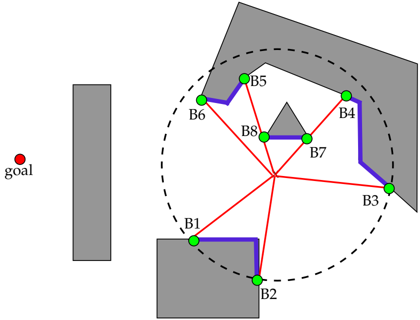

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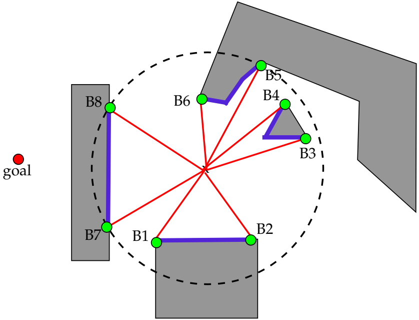

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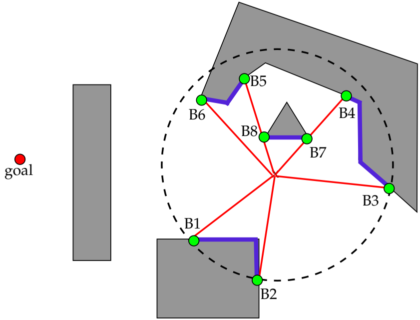

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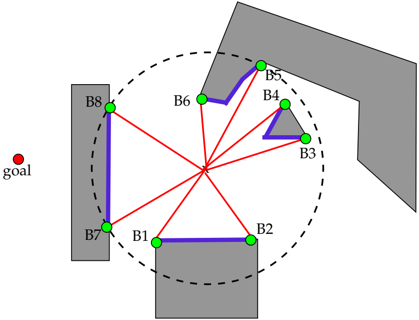

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- Choose the point B_i that minimizes heuristic distance $d(x, B_i) + d(B_i, \texttt{goal})$
- What if this distance starts increasing? Then, start following some boundary

- A motion-to-goal behavior as long as way is clear or there is a visible obstacle boundary point that decreases heuristic distance
- A boundary following behavior invoked when heuristic distance increases
- A value $d_{followed}$ which is the shortest distance between the sensed boundary and goal
- A value d_{reach} which is the shortest distance between blocking obstacle and goal (or distance to goal if no blocking obstacle visible)
- Terminate boundary following behavior when $d_{reach} < d_{followed}$

TangentBug Algorithm – Pseudocode

repeat until `goal` is reached

1. repeat
   - take sensor-range reading and compute continuous range segments
   - move toward point $n \in \{\texttt{goal}, B_1, B_2, \ldots\}$ that minimizes $h(x, n) = d(x, n) + d(n, \texttt{goal})$

   until
   - goal is reached, or
   - value of $h(x, n)$ begins to increase
2. follow boundary continuing in same direction as before repeating
   - update discontinuity points $\{B_1, B_2, \ldots\}$, d_{reach}, $d_{followed}$

   until
   - goal is reached, or
   - a complete cycle is performed (goal is unreachable)
   - $d_{reach} < d_{followed}$

Completeness proof similar to other bug-algorithm proofs, although the definition of hit and leave points is trickier

TangentBug Algorithm – Some Implementation Details

Basic problem: compute tangent to curve forming boundary of obstacle at any point, and drive the robot in that direction

- Let $D(x) = \min_c d(x, c), c \in \bigcup \texttt{Boundary}(O_i)$
- Let $G(x) = D(x) - W$, where W is some safe following distance
- Note that $\nabla G(x)$ points radially away from the object
- Define $T(x) = (\nabla G(x))$ the tangent direction
  - in a real sensor, this is just the tangent to the array element with lowest reading
- We could just move in the direction $T(x)$
  - open-loop control

Summary

- Bug0 is incomplete
- Bug1 is complete, safe, and reliable
- Bug2 is complete, better in some cases than Bug1, but worse in others
- TangentBug is complete, supports range sensors

Reactive paradigm with minimal global information

Point Robot, Simple Motions

- Move straight toward goal
- Move along obstacle boundary
- Stop