CS 485: Autonomous Robotics

Sampling-Based Motion Planning

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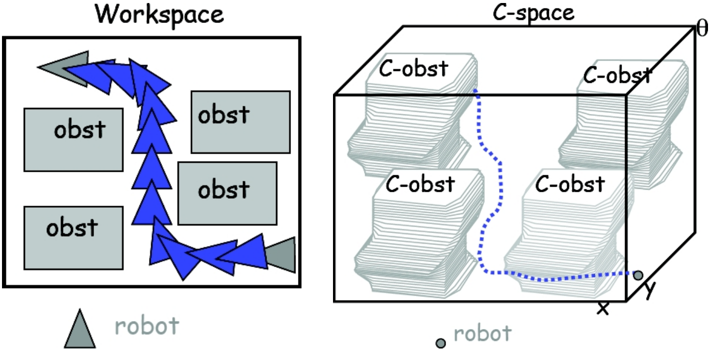
From Workspace to Configuration Space

- simple workspace obstacle transformed into complex configuration-space obstacle
- robot transformed into point in configuration space
- path transformed from swept volume to 1d curve



[fig from Jyh-Ming Lien]

Explicit Construction of Configuration Space/Roadmaps

- PSPACE-complete
- Exponential dependency on dimension
- No practical algorithms

Motion Planning for a Point Robot in 2D

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- Task: Compute collision-free path from initial to goal position

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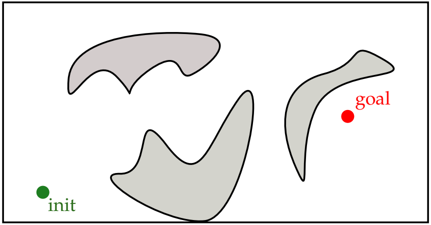

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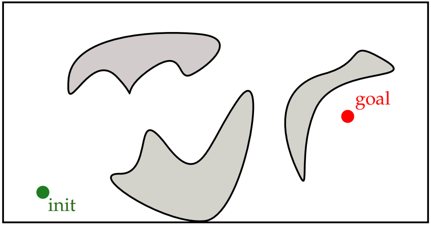

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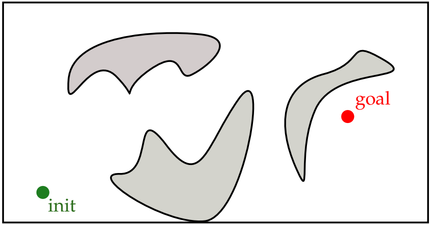

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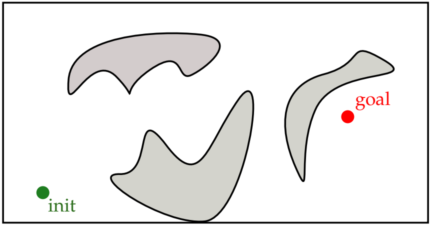

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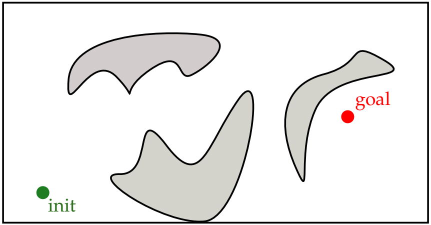

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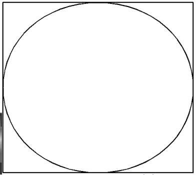

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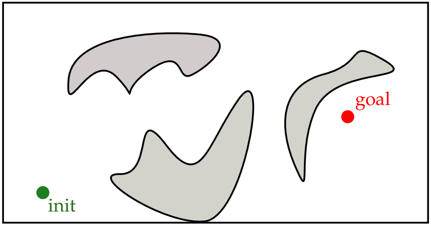

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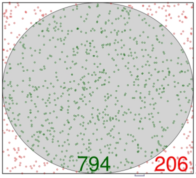

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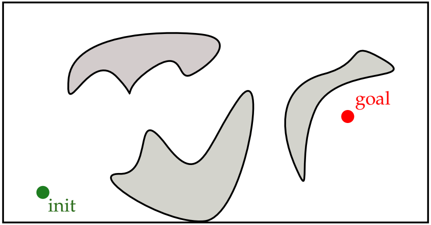

Monte-Carlo Idea:

- Define input space
- Generate inputs at random by *sampling* the input space
- Perform a deterministic computation using the input samples
- Aggregate the partial results into final result

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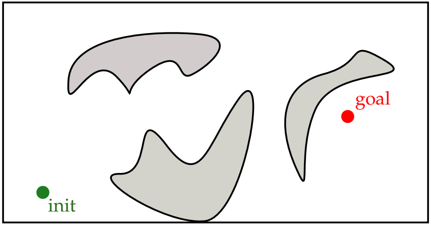

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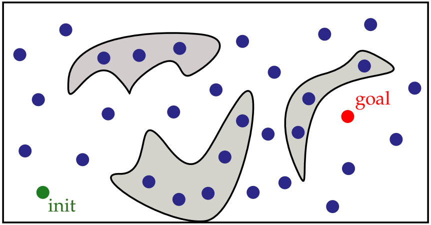

- Sample points

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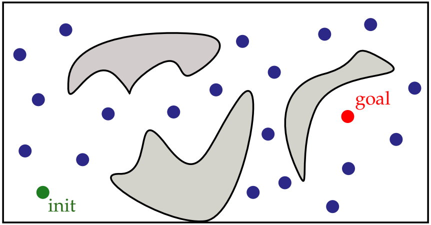

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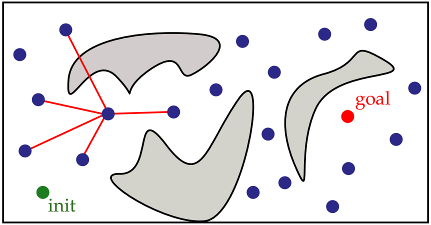

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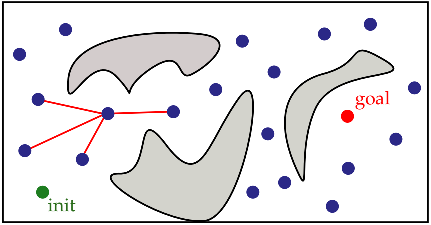

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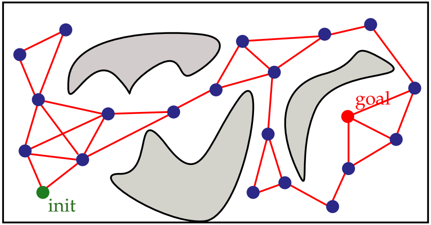

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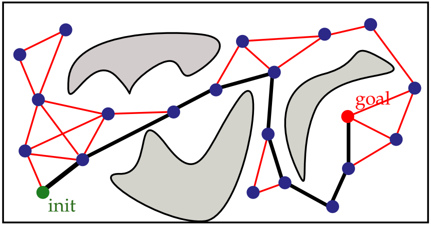

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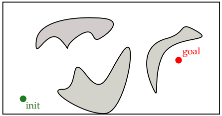
⇒ Collision-free path can be found by performing graph search on the roadmap

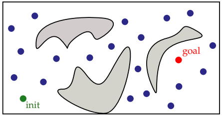
Probabilistic RoadMap (PRM) Method

[Kavraki, Švestka, Latombe, Overmars 1996]



0. Initialization

add q_{init} and q_{goal} to roadmap vertex set V



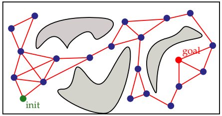
1. Sampling

repeat several times

$q \leftarrow \textsc{Sample}()$

if $\textsc{IsCollisionFree}(q) = \texttt{true}$

add q to roadmap vertex set V



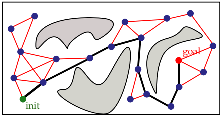
2. Connect Samples

for each pair of neighboring samples $(q_a, q_b) \in V \times V$

$\text{path} \leftarrow \textsc{GenerateLocalPath}(q_a, q_b)$

if $\textsc{IsCollisionFree}(\text{path}) = \texttt{true}$

add (q_a, q_b) to roadmap edge set E



3. Graph Search

search graph (V, E) for path from q_{init} to q_{goal}

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It offers *probabilistic completeness*

- When a solution exists, a probabilistically complete planner finds a solution with probability as time goes to infinity.
- When a solution does not exists, a probabilistically complete planner may not be able to determine that a solution does not exist.

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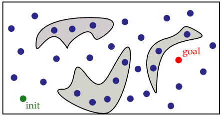

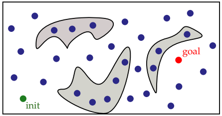

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- $x \leftarrow \textsc{rand}(\text{min}_x, \text{max}_x)$
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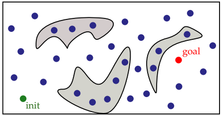
$\textsc{IsSampleCollisionFree}(q)$

- Point inside/outside polygon test

PRM Applied to 2D-point Robot



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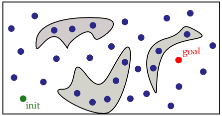

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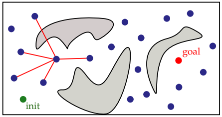

$\textsc{IsPathCollisionFree}(\text{path})$

- Segment-polygon intersection test

PRM Applied to 2D Rigid-Body Robot

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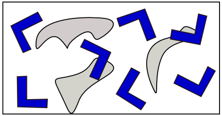

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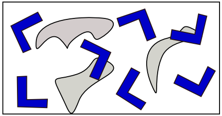

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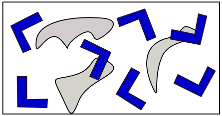

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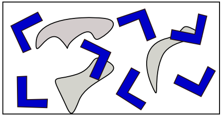

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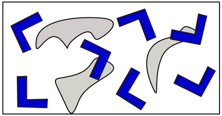

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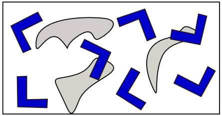

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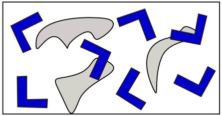

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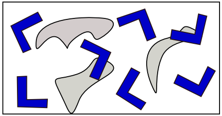

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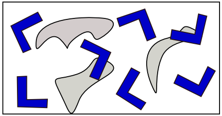

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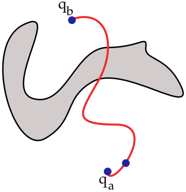

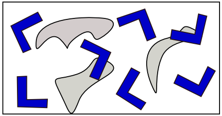

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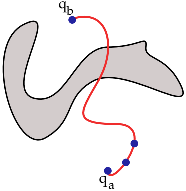

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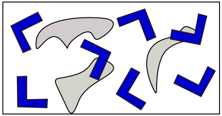

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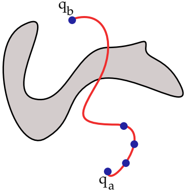
- Starts at q_a and ends at q_b: $\text{path}(0) = q_a, \text{path}(1) = q_b$
- Many possible ways of defining it, e.g., by linear interpolation

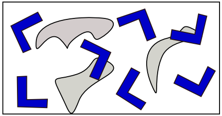
$$\text{path}(t) = (1 - t) * q_a + t * q_b$$

$\textsc{IsPathCollisionFree}(\text{path})$

- Incremental approach



PRM Applied to 2D Rigid-Body Robot



$q = (x, y, \theta) \leftarrow \textsc{Sample}()$

- $x \leftarrow \textsc{rand}(\min_x, \max_x)$; $y \leftarrow \textsc{rand}(\min_y, \max_y)$; $\theta \leftarrow \textsc{rand}(-\pi, \pi)$

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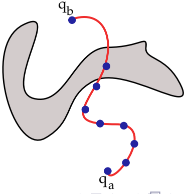

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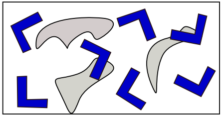

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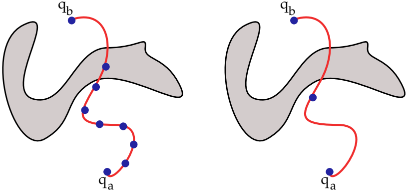

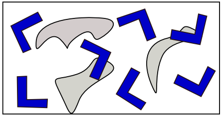

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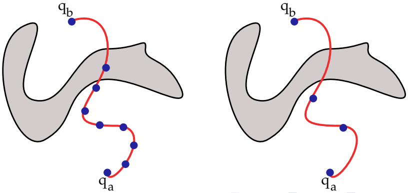
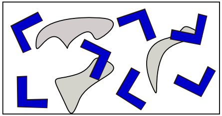


PRM Applied to 2D Rigid-Body Robot



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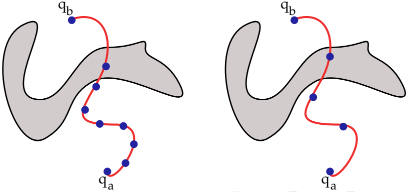

[piano] [manocha] [kcar] [tri] [buggy]

PRM Applied to Articulated Chain

$q = (\theta_1, \theta_2, \ldots, \theta_n) \leftarrow \textsc{Sample}()$

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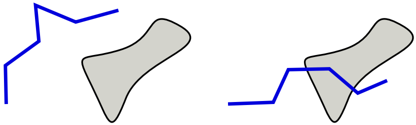

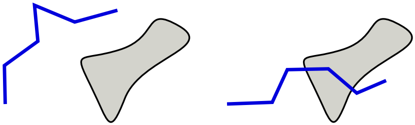

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- Place chain in configuration q (forward kinematics)
- Check for collision with obstacles

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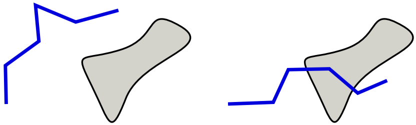

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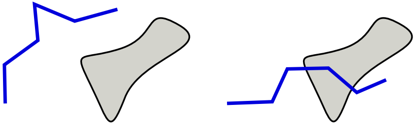

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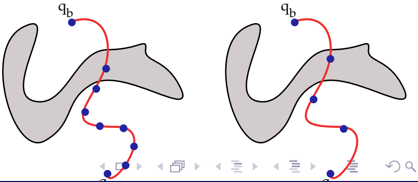

[everest] [skeleton] [knot] [manip]

Path Smoothing

- Solution paths produced by PRM planners tend to be long and non-smooth (due to sampling and edge connections)
- Post processing is commonly used to improve the quality of the paths
- A common practice is to repeatedly replace long paths by short paths

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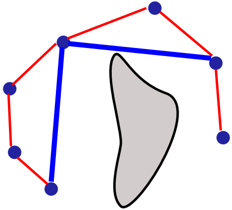
SmoothPath($q_1, q_2, \ldots, q_n$) – one version

```
for several times do
    select i and j uniformly at random from 1, 2, ..., n
    attempt to directly connect q_i to q_j
    if successful, remove the in-between nodes, i.e., q_{i+1}, ..., q_j
```



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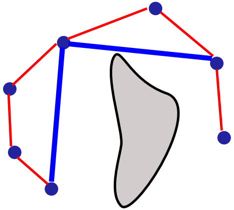
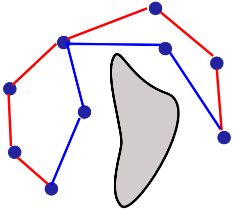
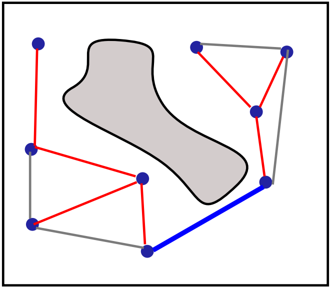


SmoothPath($q_1, q_2, \ldots, q_n$) – another version

```
for several times do
    select i and j uniformly at random from 1, 2, ..., n
    q ← generate collision-free sample
    attempt to connect q_i to q_j through q
    if successful, replace the in-between nodes q_{i+1}, ..., q_j by q
```



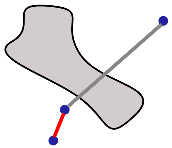
- Edge in cycle does not improve roadmap connectivity
- Edge is added to roadmap only if it connects two different roadmap components

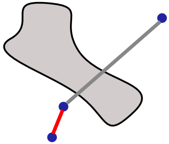


1: **if** SAMEROADMAPCOMPONENT(q_a, q_b) = `false` **then**
2: path ← GENERATEPATH(q_a, q_b)
3: **if** ISPATHCOLLISIONFREE(path) = `true` **then**
4: (q_a, q_b).path ← path
5: $E \leftarrow E \cup \{(q_a, q_b)\}$

- Disjoint-set data structure is used to speed up computation of SAMEROADMAPCOMPONENT(q_a, q_b)

Edges between neighboring nodes are more likely to be collision free than edges between far away nodes





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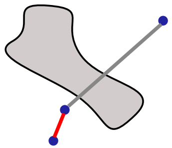

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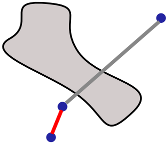

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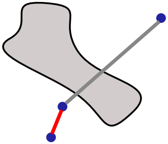

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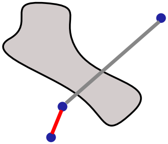

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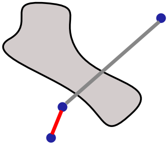

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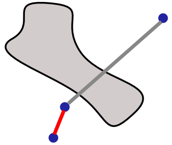

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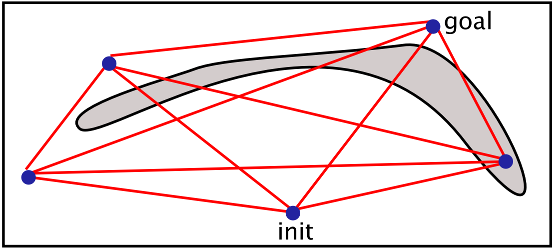
- Computational challenges of nearest neighbors in high-dimensional spaces
  - Efficiency deteriorates rapidly
  - Not much better than brute-force approach
- Alternative approach is to compute *approximate* nearest neighbors
  [Plaku, Kavraki: WAFR 2006, SDM 2007]
  - Minimal losses in accuracy of neighbors
  - No loss in accuracy of overall path planner
  - Significant computational gains

Perform collision checking only when necessary

[Bohlin, Kavraki: Handbook on Randomized Computing 2000]



LazyRoadmapConstruction

1: $V \leftarrow V \cup \{q_{\text{init}}, q_{\text{goal}}\}$; $E \leftarrow \emptyset$
2: **for** several times **do**
3: $q \leftarrow$ generate config uniformly at random; $q.\text{checked} \leftarrow$ `false`; $V \leftarrow V \cup \{q\}$
4: **for** each pair $(q_a, q_b) \in V \times V$ **do**
5: $(q_a, q_b).\text{res} \leftarrow 1.0$; $(q_a, q_b).\text{path} \leftarrow$ GeneratePath(q_a, q_b); $E \leftarrow E \cup \{(q_a, q_b)\}$

Lazy PRM

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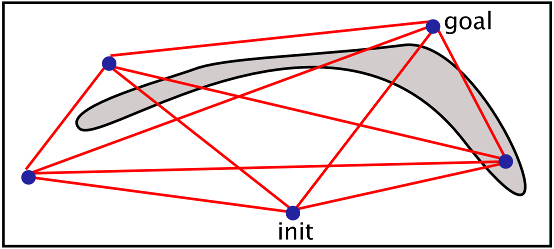
[Bohlin, Kavraki: Handbook on Randomized Computing 2000]

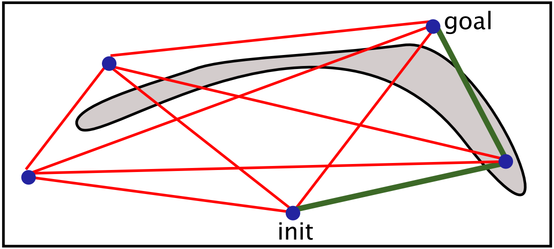


LAZYROADMAPCOLLISIONCHECKING

1: **for** several times **do**
2: $[q_1, q_2, \ldots, q_n] \leftarrow$ search $G = (V, E)$ for sequence of edges connecting q_{init} to q_{goal}
3: **for** $i = 1, 2, \ldots, n$ **do**
4: **if** $q_i.\text{checked} = \texttt{false}$ **and** ISCONFIGCOLLISIONFREE(q_i) $= \texttt{false}$ **then**
5: remove q_i from roadmap; **goto** line 2
6: **else**
7: $q_i.\text{checked} \leftarrow \texttt{true}$
8: **while** no edge collisions are found **and** minimum resolution not reached **do**
9: **for** $i = 1, 2, \ldots, n-1$ **do**
10: $(q_i, q_{i+1}).\text{res} \leftarrow (q_i, q_{i+1}).\text{res}/2$; check $(q_i, q_{i+1}).\text{path}$ at resolution $(q_i, q_{i+1}).\text{res}$
11: **if** collision found in $(q_i, q_{i+1}).\text{path}$ **then**
12: remove (q_i, q_{i+1}) from roadmap; **goto** line 2
13: **return** $(q_1, q_2).\text{path} \circ \cdots \circ (q_{n-1}, q_n).\text{path}$

Perform collision checking only when necessary

[Bohlin, Kavraki: Handbook on Randomized Computing 2000]



LAZYROADMAPCOLLISIONCHECKING

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2:      [q_1, q_2, ..., q_n] ← search G = (V, E) for sequence of edges connecting q_init to q_goal
3:      for i = 1, 2, ..., n do
4:          if q_i.checked = false and ISCONFIGCOLLISIONFREE(q_i) = false then
5:              remove q_i from roadmap; goto line 2
6:          else
7:              q_i.checked ← true
8:      while no edge collisions are found and minimum resolution not reached do
9:          for i = 1, 2, ..., n − 1 do
10:             (q_i, q_{i+1}).res ← (q_i, q_{i+1}).res/2; check (q_i, q_{i+1}).path at resolution (q_i, q_{i+1}).res
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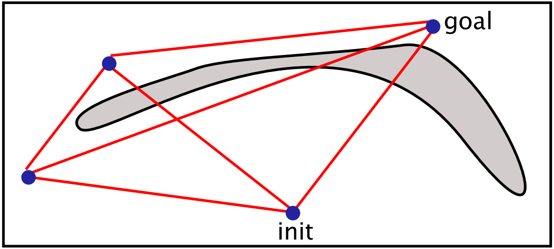
13:     return (q_1, q_2).path ∘ ··· ∘ (q_{n−1}, q_n).path
```

Perform collision checking only when necessary

[Bohlin, Kavraki: Handbook on Randomized Computing 2000]



LAZYROADMAPCOLLISIONCHECKING

```
1:  for several times do
2:     [q_1, q_2, ..., q_n] ← search G = (V, E) for sequence of edges connecting q_init to q_goal
3:     for i = 1, 2, ..., n do
4:        if q_i.checked = false and IsConfigCollisionFree(q_i) = false then
5:           remove q_i from roadmap; goto line 2
6:        else
7:           q_i.checked ← true
8:     while no edge collisions are found and minimum resolution not reached do
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Lazy PRM

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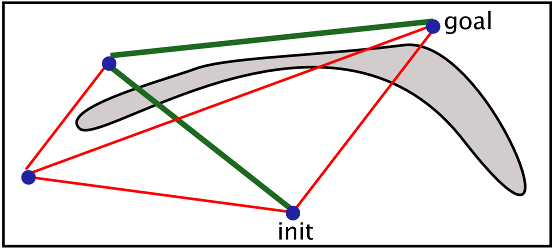
[Bohlin, Kavraki: Handbook on Randomized Computing 2000]



LAZYROADMAPCOLLISIONCHECKING

```
1:  for several times do
2:     [q_1, q_2, ..., q_n] ← search G = (V, E) for sequence of edges connecting q_init to q_goal
3:     for i = 1, 2, ..., n do
4:        if q_i.checked = false and IsConfigCollisionFree(q_i) = false then
5:           remove q_i from roadmap; goto line 2
6:        else
7:           q_i.checked ← true
8:     while no edge collisions are found and minimum resolution not reached do
9:        for i = 1, 2, ..., n − 1 do
10:          (q_i, q_{i+1}).res ← (q_i, q_{i+1}).res/2; check (q_i, q_{i+1}).path at resolution (q_i, q_{i+1}).res
11:          if collision found in (q_i, q_{i+1}).path then
12:             remove (q_i, q_{i+1}) from roadmap; goto line 2
13:    return (q_1, q_2).path ∘ ··· ∘ (q_{n−1}, q_n).path
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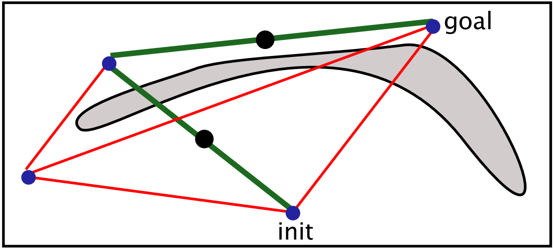

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Lazy PRM

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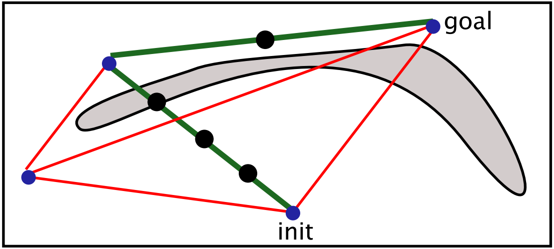

LazyRoadmapCollisionChecking

1: **for** several times **do**
2: $[q_1, q_2, \ldots, q_n] \leftarrow$ search $G = (V, E)$ for sequence of edges connecting q_{init} to q_{goal}
3: **for** $i = 1, 2, \ldots, n$ **do**
4: **if** $q_i.\mathrm{checked} = \texttt{false}$ **and** IsConfigCollisionFree$(q_i) = \texttt{false}$ **then**
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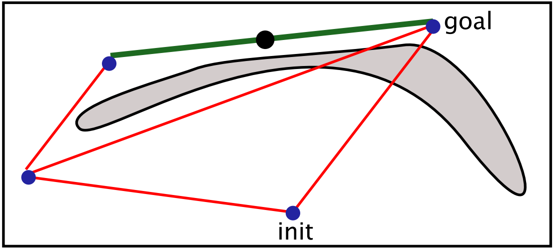

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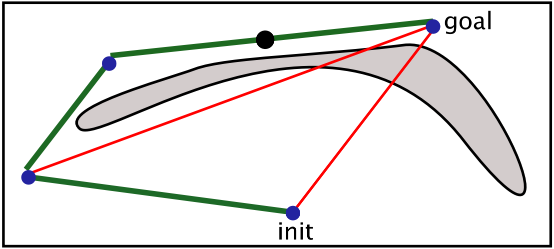

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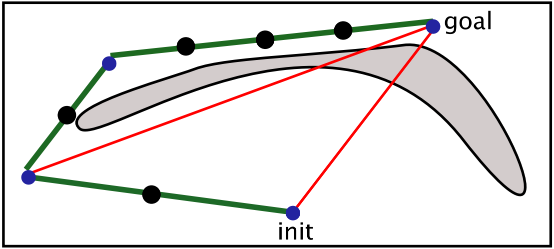

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1: **for** several times **do**
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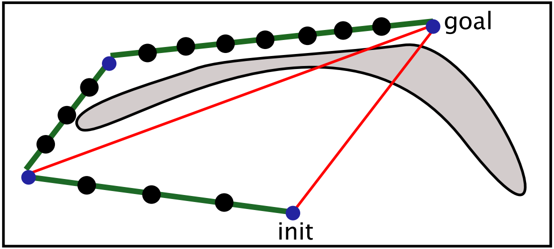

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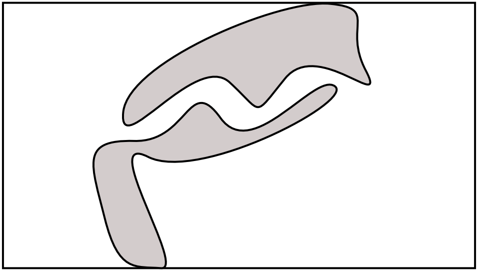


- Probability of generating samples via uniform sampling in a narrow passage is low due to the small volume of the narrow passage
- Generating samples inside a narrow passage may be critical to the success of the path planner
- Objective is then to design sampling strategies that can increase the probability of generating samples inside narrow passages

Gaussian Sampling in PRM

Objective: Increase Sampling Inside/Near Narrow Passages
Approach: Sample from a Gaussian distribution biased near the obstacles

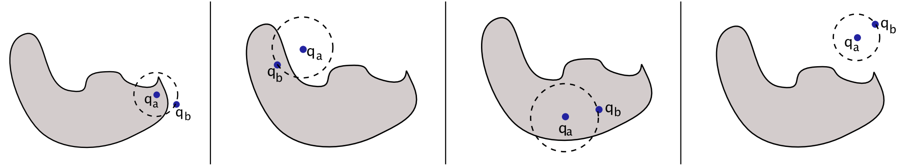
GenerateCollisionFreeConfig [Boor, Overmars, van Der Stappen: ICRA 1999]

$q_a \leftarrow$ generate config uniformly at random
$r \leftarrow$ generate distance from Gaussian distribution
$q_b \leftarrow$ generate config uniformly at random at distance r from q_a

$\mathrm{ok}_a \leftarrow$ IsConfigCollisionFree(q_a)
$\mathrm{ok}_b \leftarrow$ IsConfigCollisionFree(q_b)

if $\mathrm{ok}_a =$ `true` **and** $\mathrm{ok}_b =$ `false` **then return** q_a
if $\mathrm{ok}_a =$ `false` **and** $\mathrm{ok}_b =$ `true` **then return** q_b

return `null`

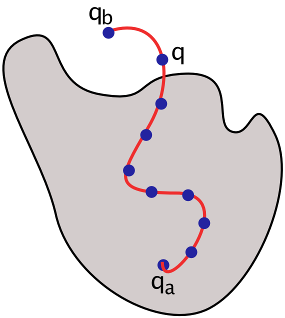


Obstacle-based Sampling in PRM

Objective: Increase Sampling Inside/Near Narrow Passages
Approach: Move samples in collision outside obstacle boundary

GENERATECOLLISIONFREECONFIG [Amato, Bayazit, Dale, Jones, Vallejo: WAFR 1998]

```
q_a ← generate config uniformly at random
if IsConfigCollisionFree(q_a) = true then
    return q_a
else
    q_b ← generate config uniformly at random
    path ← GeneratePath(q_a, q_b)
    for t = δ to |path| by δ do
        if IsConfigCollisionFree(path(t)) then
            return path(t)
return null
```

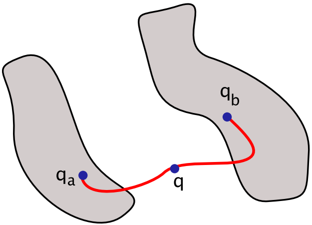


Objective: Increase Sampling Inside/Near Narrow Passages
Approach: Create "bridge" between samples in collision

[Hsu, Jiang, Reif, Sun: ICRA 2003]

GENERATECOLLISIONFREECONFIG

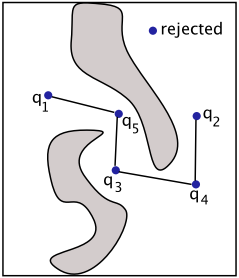
```
q_a ← generate config uniformly at random
q_b ← generate config uniformly at random
ok_a ← IsConfigCollisionFree(q_a)
ok_b ← IsConfigCollisionFree(q_b)
if ok_a = false and ok_b = false then
    path ← GeneratePath(q_a, q_b)
    q ← path(0.5|path|)
    if IsConfigCollisionFree(q) then
        return q
return null
```



Objective: Capture connectivity of configuration space with few samples
Approach: Generate samples that create new components or join existing components

GENERATECOLLISIONFREECONFIG [Nisseoux, Simeon, Laumond: Advanced Robotics J 2000]

```
q ← generate config uniformly at random
if IsConfigCollisionFree(q) = true then
    if q belongs to a new roadmap component then
        return q
    if q connects two roadmap components then
        return q
return null
```



- q_1: creates new roadmap component
- q_2: creates new roadmap component
- q_3: creates new roadmap component
- q_4: connects two roadmap components
- q_5: connects two roadmap components

Importance Sampling

Objective: Increase Sampling Inside/Near Narrow Passages
Approach: Improve roadmap connectivity

- Construct roadmap using given sampling strategy
- Identify roadmap nodes that lie in regions that are hard to connect
- Sample more in these regions

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- Weight $w(q)$ indicates difficulty of region around q

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  - $w(q) =$ number of times connections from/to q have failed
  - combination of different strategies

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  - combination of different strategies

- Select sample with probability $\frac{w(q)}{\sum_{q' \in V} w(q')}$
- Generate more samples around q
- Connect new samples to neighboring roadmap nodes

Combine Different Sampling Strategies

- Each sampling strategy has its strengths and weakness
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- A sampler S_i is then selected with probability

$$\frac{w_i}{\sum_j w_j}$$

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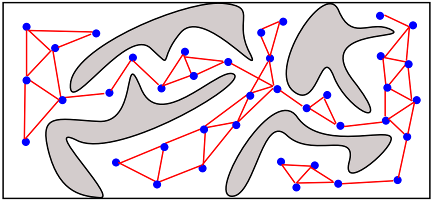
- Sampler weight is updated based on quality of performance

- Balance between being "smart and slow" and "dumb and fast"

- PRM-based planners aim to construct a roadmap that captures the whole connectivity of the configuration space



Motivation: Tree-based Motion Planning

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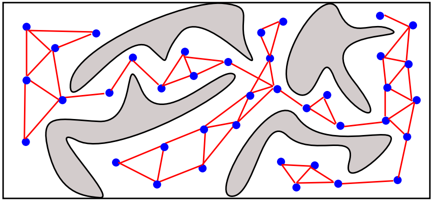

- Good when the objective is to solve *multiple* queries

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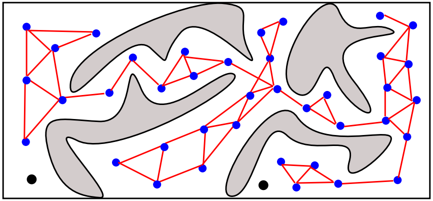

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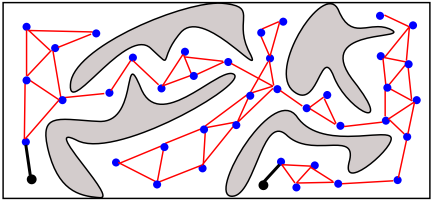

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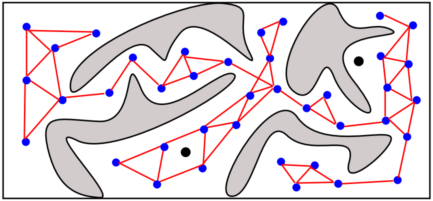

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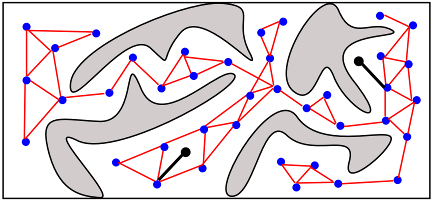

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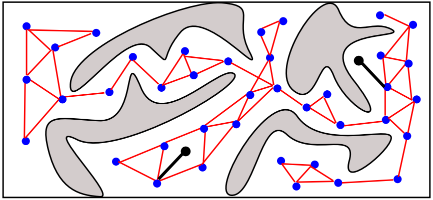
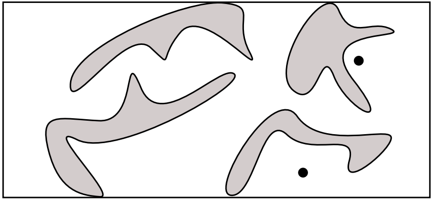
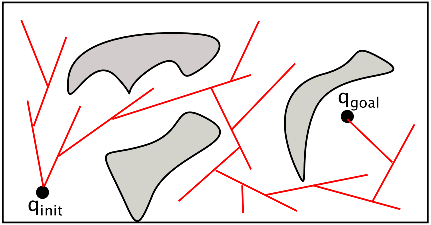


- Good when the objective is to solve *multiple* queries
- Maybe a bit too much when the objective is to solve a *single* query



Grow a tree in the free configuration space from q_{init} toward q_{goal}



$\textsc{TreeSearchFramework}(q_{\text{init}}, q_{\text{goal}})$

1: $\mathcal{T} \leftarrow \textsc{RootTree}(q_{\text{init}})$
2: **while** q_{goal} has not been reached **do**
3: $\quad q \leftarrow \textsc{SelectConfigFromTree}(\mathcal{T})$
4: $\quad \textsc{AddTreeBranchFromConfig}(\mathcal{T}, q)$

Critical Issues

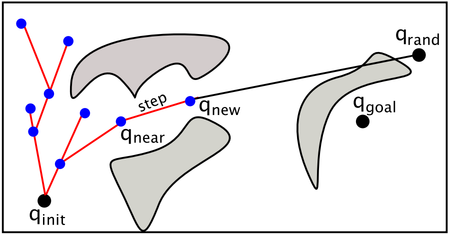
- How should a configuration be selected from the tree?
- How should a new branch be added to the tree from the selected configuration?

Rapidly-exploring Random Tree (RRT)

Pull the tree toward random samples in the configuration space

- RRT relies on nearest neighbors and distance metric $\rho : Q \times Q \leftarrow \mathbb{R}^{\geq 0}$
- RRT adds Voronoi bias to tree growth

[LaValle, Kuffner: 1999]



$\mathrm{RRT}(q_{\mathrm{init}}, q_{\mathrm{goal}})$

$\triangleright$ *initialize tree*
$\mathcal{T} \leftarrow$ create tree rooted at q_{init}
while solution not found **do**
$\triangleright$ *select configuration from tree*
  $q_{\mathrm{rand}} \leftarrow$ generate a random sample
  $q_{\mathrm{near}} \leftarrow$ nearest configuration in $\mathcal{T}$ to q_{rand} according to distance ρ
$\triangleright$ *add new branch to tree from selected configuration*
  path $\leftarrow$ generate path (not necessarily collision free) from q_{near} to q_{rand}
  if IsSubpathCollisionFree(path, 0, step) **then**
    $q_{\mathrm{new}} \leftarrow \mathrm{path}(\mathtt{step})$
    add configuration q_{new} and edge $(q_{\mathrm{near}}, q_{\mathrm{new}})$ to $\mathcal{T}$
$\triangleright$ *check if a solution is found*
    if $\rho(q_{\mathrm{new}}, q_{\mathrm{goal}}) \approx 0$ **then**
      return solution path from root to q_{new}

Aspects for Improvement

Suggested Improvements in the Literature

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- BasicRRT does not take advantage of q_{goal}
- Tree is pulled towards random directions based on the uniform sampling of Q
- In particular, tree growth is not directed towards q_{goal}

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- BasicRRT does not take advantage of q_{goal}
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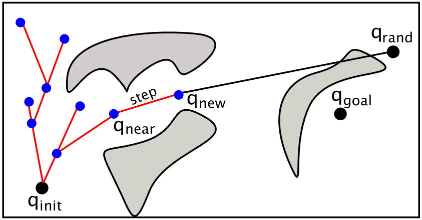
- In particular, tree growth is not directed towards q_{goal}

Suggested Improvements in the Literature

- Introduce goal-bias to tree growth (known as GoalBiasRRT)
  - q_{rand} is selected as q_{goal} with probability p
  - q_{rand} is selected based on uniform sampling of Q with probability $1 - p$
  - Probability p is commonly set to ≈ 0.05

Aspects for Improvement

- BasicRRT takes only one small step when adding a new tree branch



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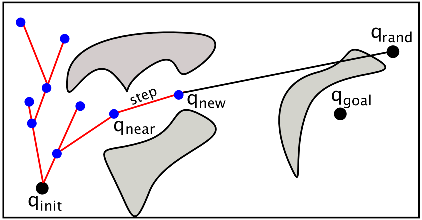

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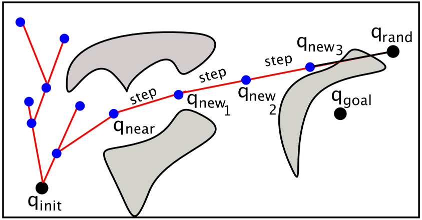

Suggested Improvements in the Literature

- Take several steps until q_{rand} is reached or a collision is found (CONNECTRRT)
- Add all the intermediate nodes to the tree

Expansive-Space Tree (EST)

Push the tree frontier in the free configuration space

[Hsu, Rock, Motwani, Latombe: 1999]

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- select q in $\mathcal{T}$ with probability $w(q)/\sum_{q' \in \mathcal{T}} w(q')$

AddTreeBranchFromConfig$(\mathcal{T}, q)$

- $q_{\rm near} \leftarrow$ sample a collision-free configuration near q
- $\rm path \leftarrow$ generate path from q to $q_{\rm near}$
- if $\rm path$ is collision-free, then add $q_{\rm near}$ and $(q, q_{\rm near})$ to $\mathcal{T}$

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[play movie]

Observations in High-Dimensional Problems

- Tree generally grows rapidly for the first few thousand iterations
- Tree growth afterwards slows down quite significantly
- Large number of configurations increases computational cost
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Possible improvements?

Bi-directional Trees

Grow two trees, rooted at q_{init} and q_{goal}, towards each other

- Bi-directional trees improve computational efficiency compared to a single tree
- Growth slows down significantly later than when using a single tree
- Fewer configurations in each tree, which imposes less of a computational burden
- Each tree explores a different part of the configuration space

BiTree($q_{\text{init}}, q_{\text{goal}}$)

```
T_init ← create tree rooted at q_init
T_goal ← create tree rooted at q_goal
while solution not found do
    add new branch to T_init
    add new branch to T_goal
    attempt to connect neighboring configurations
    from the two trees
    if successful, return path from q_init to q_goal
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1: $\mathcal{T}_{\text{init}} \leftarrow$ create tree rooted at q_{init}
2: $\mathcal{T}_{\text{goal}} \leftarrow$ create tree rooted at q_{goal}
3: **while** solution not found **do**
4: add new branch to $\mathcal{T}_{\text{init}}$
5: add new branch to $\mathcal{T}_{\text{goal}}$
6: attempt to connect neighboring configurations from the two trees
7: if successful, return path from q_{init} to q_{goal}

- Different tree planners can be used to grow each of the trees
- E.g., RRT can be used for one tree and EST can be used for the other

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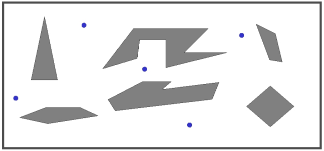
Sampling-based Roadmap of Trees (SRT) [Plaku, Bekris, Chen, Ladd, Kavraki: Trans on Robotics 2005]

- Hierarchical planner
- Top level performs global sampling (PRM-based)
- Bottom level performs local sampling (tree-based, e.g., RRT, EST)
- Combines advantages of global and local sampling

CreateTreesInRoadmap

```
V ← ∅; E ← ∅
while |V| < n_trees do
    T ← create tree rooted at a collision-free configuration
    use tree planner to grow T for some time
    add T to roadmap vertices V
```



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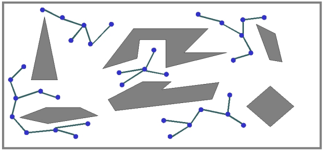

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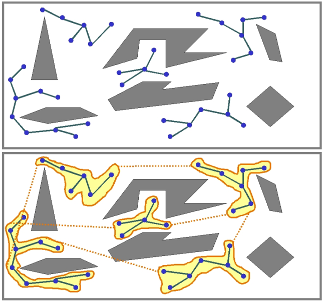

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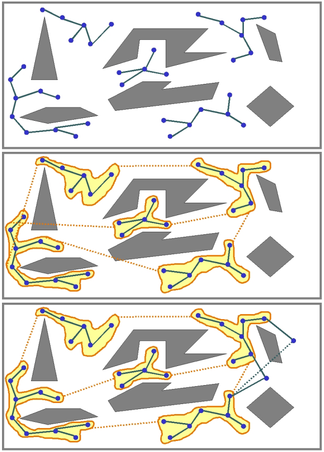

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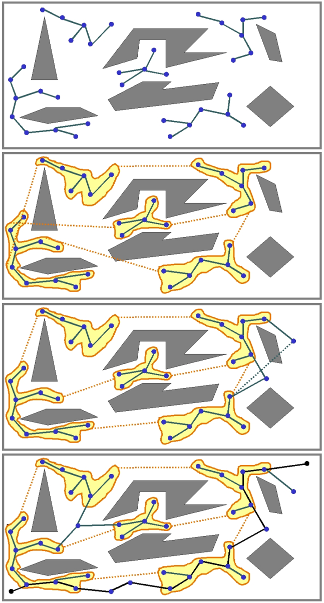

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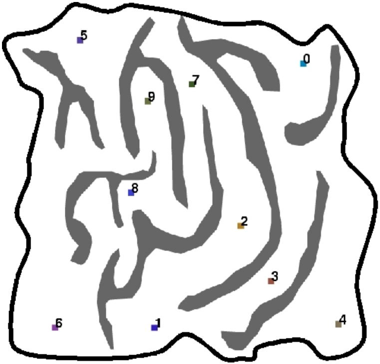

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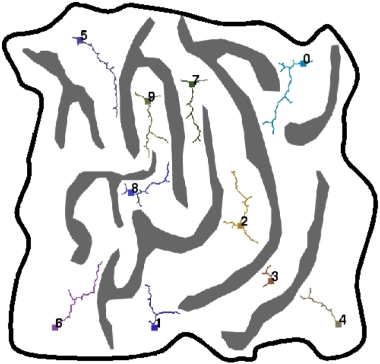

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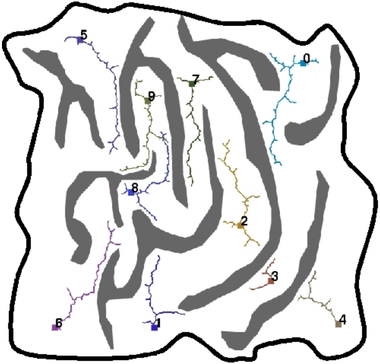

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SolveQuery(q_{init}, q_{goal})

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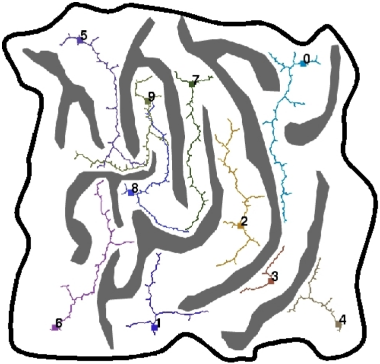

CREATETREESINROADMAP
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```

SELECTWHICHTREESTOCONNECT
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CONNECTTREESINROADMAP
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for each (T_1, T_2) ∈ E_pairs do
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```

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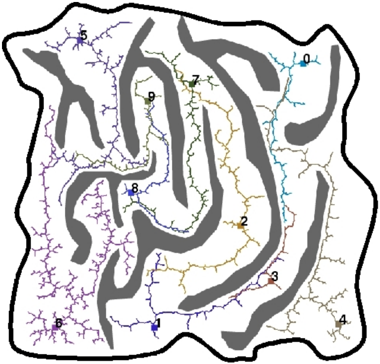

Advantages

- Explores small subset of possibilities by sampling
- Computationally efficient
- Solves high-dimensional problems (with hundreds of DOFs)
- Easy to implement
- Applications in many different areas

Disadvantages

- Does not guarantee completeness (a complete planner always finds a solution if there exists one, or reports that no solution exists)

Is it then just a heuristic approach? No. It's more than that

It offers *probabilistic completeness*

- When a solution exists, a probabilistically complete planner finds a solution with probability as time goes to infinity.
- When a solution does not exists, a probabilistically complete planner may not be able to determine that a solution does not exist.

Components

- Free configuration space Q_{free}: arbitrary open subset of $[0,1]^d$
- Local connector: connects $a, b \in Q_{\mathrm{free}}$ via a straight-line path and succeeds if path lies entirely in Q_{free}
- Collection of roadmap samples from Q_{free}

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Let $a, b \in Q_{\mathrm{free}}$ such that there exists a path γ between a and b lying in Q_{free}. Then the probability that PRM correctly answers the query (a, b) after generating n collision-free configurations is given by

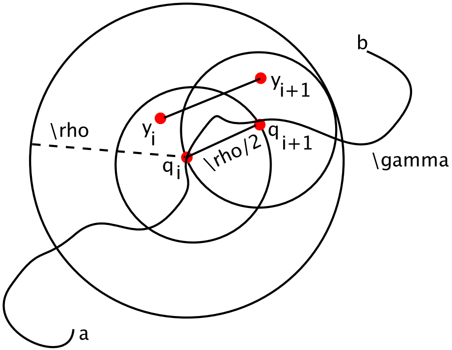
$$\Pr[(a, b)\mathrm{SUCCESS}] \geq 1 - \left\lceil \frac{2L}{\rho} \right\rceil e^{-\sigma \rho^d n},$$

where

- L is the length of the path γ
- $\rho = \mathrm{clr}(\gamma)$ is the clearance of path γ from obstacles
- $\sigma = \frac{\mu(B_1(\cdot))}{2^d \mu(Q_{\mathrm{free}})}$
- $\mu(B_1(\cdot))$ is the volume of the unit ball in $\mathbb{R}^d$
- $\mu(Q_{\mathrm{free}})$ is the volume of Q_{free}

Basic Idea

- Reduce path to a set of open balls in Q_{free}
- Calculate probability of generating samples in those balls
- Connect samples in different balls via straight-line paths to compute solution path



Proof Outline: Probabilistic Completeness of PRM (cont.)

- Note that clearance $\rho = \mathrm{clr}(\gamma) > 0$
- Let $m = \left\lceil \frac{2L}{\rho} \right\rceil$. Then, γ can be covered with m balls $B_{\rho/2}(q_i)$ where $a = q_1, \ldots, q_m = b$
- Let $y_i \in B_{\rho/2}(q_i)$ and $y_{i+1} \in B_{\rho/2}(q_{i+1})$.
  Then, the straight-line segment $\overline{y_i y_{i+1}} \in Q_{\mathrm{free}}$, since $y_i, y_{i+1} \in B_\rho(q_i)$
- $I_i \stackrel{def}{=}$ indicator variable that there exists $y \in V$ s.t. $y \in B_{\rho/2}(q_i)$
- $\Pr[(a,b)\mathrm{FAILURE}] = \Pr\left[\bigvee_{i=1}^m I_i = 0\right] = \sum_{i=1}^m \Pr[I_i = 0]$
  - Note that $\Pr[I_i = 0] = \left(1 - \frac{\mu(B_{\rho/2}(q_i))}{\mu(Q_{\mathrm{free}})}\right)^n$
    i.e., probability that none of the n PRM samples falls in $B_{\rho/2}(q_i)$
  - I_i's are independent because of uniform samling in PRM

Therefore, $\Pr[(a,b)\mathrm{FAILURE}] = m\left(1 - \frac{\mu(B_{\rho/2}(\cdot))}{\mu(Q_{\mathrm{free}})}\right)^n$

- $\frac{\mu(B_{\rho/2}(\cdot))}{\mu(Q_{\mathrm{free}})} = \frac{\left(\frac{\rho}{2}\right)^d \mu(B_1(\cdot))}{\mu(Q_{\mathrm{free}})} = \sigma\rho^d$

Therefore, $\Pr[(a,b)\mathrm{FAILURE}] = m\left(1 - \sigma\rho^d\right)^n \leq m e^{-\sigma\rho^d n} = \left\lceil \frac{2L}{\rho} \right\rceil e^{-\sigma\rho^d n}$ □

since $(1 - x) \leq e^{-x} \quad \forall x \geq 0$