CS 485 – Autonomous Robotics

Manipulation Planning

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What is Manipulation Planning?

[movie: industrial]

[movie: L-shape]

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- Body: articulated chain (what are configuration parameters)?
- Tool/grasper/end-effector (what are configuration parameters)?

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How is manipulation planning a motion planning problem?

- What moves where?
- Workspace?
- Configuration space?

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How is manipulation planning a motion planning problem?

- What moves where?
- Workspace?
- Configuration space?
  - Need to keep track of ? and ? moving in workspace?

Problem Formulation

Given:

- a description of the obstacles
- a description of the robot manipulator
- a description of the object to be manipulated
- a description of the initial and desired placements for the object

Objective:

- compute a sequence of motions where the robot manipulator grasps the object in its *initial placement* and places it in its *desired placement* while *avoding collisions*

- How to grasp the object?
- Is the grasp stable?
- Does the solution require re-grasping?
- When should the robot manipulator release the object and re-grasp it in a different configuration?

Two Representative Approaches

PRM-based: Nielsen and Kavraki, IROS 2000.

- Expands roadmap/graph to manipulation graph.
- Assumes stable robot grasps and object placements pre-computed and provided ahead of time.

RRT-based: Berenson et al., ICRA 2009.

- Approaches it as an inverse kinematics problem.
- Enriches any provided object placements with more and computes new robot grasps.

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Focus: efficient construction of manipulation graph.

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Focus: efficient construction of manipulation graph.

- Observation on whether motion of robot is with object grasped or not.

Observations

- Solution path consists of a sequence of transfer and transit paths
- Transfer path: subpath where object is stably grasped and moved by robot
- Transit path: subpath where object is left in a stable position while robot changes grasp

Each node is a triple $(q_{\mathrm{obj}}, g, q_{\mathrm{rob}})$, where:

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- $q_{\rm obj}$ specifies a stable placement (position and orientation) of the object
  - Provided or pre-computed before construction of graph
- g specifies a position and orientation of the robot tool relative to the placement of the object at which the tool is able to grasp the object
  - Provided before construction of graph
- $q_{\rm rob}$ is the configuration of the robot for which the robot tool is able to grasp the object placed at $q_{\rm obj}$ using the grasp g
  - Focus of this approach

Manipulation Graph: Edges

Transfer edge: Robot moves with object grasped by tool. What is changing?

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An edge $\left((q^i_{\mathrm{obj}}, g, q^i_{\mathrm{rob}}), (q^j_{\mathrm{obj}}, g, q^j_{\mathrm{rob}})\right)$ indicates a tranfer (local) path where the object is grasped according to g and the robot moves with the object from configuration $(q^i_{\mathrm{obj}}, q^i_{\mathrm{rob}})$ to $(q^j_{\mathrm{obj}}, q^j_{\mathrm{rob}})$

Transit edge: Robot moves to reposition its end effector/tool for object on ground.
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An edge $\left((q_{\mathrm{obj}}, g^i, q^i_{\mathrm{rob}}), (q_{\mathrm{obj}}, g^j, q^j_{\mathrm{rob}})\right)$ indicates a transit (local) path where the object is left at a stable placement q_{obj} while the robot changes grasp from $(g^i, q^i_{\mathrm{rob}})$ to $(g^j, q^j_{\mathrm{rob}})$

PRM Approach

- Node Generation:

  for $i = 1, \ldots, N$ **do** sample a node $(q^i_{\mathrm{obj}}, g^i, q^i_{\mathrm{rob}})$

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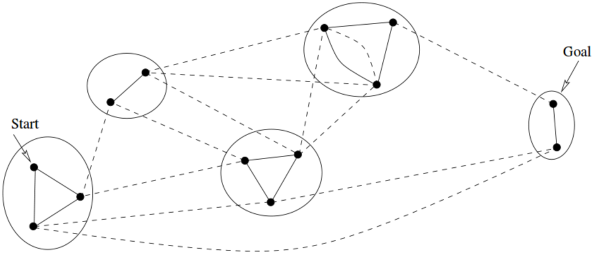
  How is local path generated for transfer or transit edge?



Solid lines represent transit paths, and dotted lines represent transfer paths.

Challenges:

- Each edge generation gives rise to a path-planning problem
- Must verify edge validity before adding it to manipulation graph
- Too many edge verifications (since graph could have large number of nodes)

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FuzzyPRM Idea

- Probabilistic edges instead of deterministic edges
- Use a probabilistic path planner to compute edge connections
- Probability associated with an edge e depends on the time spent by probabilistic path planner on e
- From the people that gave you the Lazy PRM...

[Nielsen, Kavraki: IROS 2000]

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3: **if** $g^i = g^j$ **then** add e as a transfer edge and set $prob(e) \leftarrow 0.9999$

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A two-level Fuzzy PRM for Manipulation Planning

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4:         **if** $prob(e) \neq 1$ **then**
5:             run low-level fuzzy PRM on e for a short period of time

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Low-Level Fuzzy PRM

if mode = "CONSTRUCTION" **then**
    add a new sample q to graph G_e
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if mode = “QUERY” **then**
  $\phi \leftarrow$ compute most probable path in G_e
  repeat
    $(q', q'') \leftarrow$ edge in ϕ with lowest probability

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    $\sigma \leftarrow$ compute most probable path in the
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if mode = "QUERY" then
    $\phi \leftarrow$ compute most probable path in $G_e$
    repeat
        $(q', q'') \leftarrow$ edge in $\phi$ with lowest
            probability
        if $prob(q', q'') \neq 1$ then
            run subdivision collision checking to
                validate $(q', q'')$ at resolution
                $\ell(q', q'')$
            increment $\ell(q', q'')$
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Query Stage

```
while no solution found do
    σ ← compute most probable path in the
    manipulation graph
    for each edge e ∈ σ do
        if prob(e) ≠ 1 then
            run low-level fuzzy PRM on e for a
            short period of time
            if success then
                prob(e) ← 1
            else
                prob(e) ← 1 − time(e)/total_time
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Low-Level Fuzzy PRM

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    prob(q, q′) ← P*(l)
if mode = "QUERY" then
    φ ← compute most probable path in G_e
    repeat
        (q′, q″) ← edge in φ with lowest
        probability
        if prob(q′, q″) ≠ 1 then
            run subdivision collision checking to
            validate (q′, q″) at resolution
            ℓ(q′, q″)
            increment ℓ(q′, q″)
            if collision then
                remove (q′, q″) from G_e and
                return failure
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    $(q', q'') \leftarrow$ edge in ϕ with lowest probability
    if $prob(q', q'') \neq 1$ **then**
      run subdivision collision checking to validate (q', q'') at resolution $\ell(q', q'')$
      increment $\ell(q', q'')$
      if collision **then**
        remove (q', q'') from G_e and `return failure`
      else
        update $prob(q', q'')$ based on collision resolution $\ell(q', q'')$

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for each pair of nodes $e = ((q^i_{\mathrm{obj}}, g^i, q^i_{\mathrm{rob}}), (q^j_{\mathrm{obj}}, g^j, q^j_{\mathrm{rob}})$ **do**
    if $g^i = g^j$ **then** add e as a transfer edge and set $prob(e) \leftarrow 0.9999$
    if $q^i_{\mathrm{obj}} = q^j_{\mathrm{obj}}$ **then** add e as a transit edge and set $prob(e) \leftarrow 0.9999$

Query Stage

while no solution found **do**
    $\sigma \leftarrow$ compute most probable path in the manipulation graph
    for each edge $e \in \sigma$ **do**
        if $prob(e) \neq 1$ **then**
            run low-level fuzzy PRM on e for a short period of time
            if success **then**
                $prob(e) \leftarrow 1$
            else
                $prob(e) \leftarrow 1 - \frac{time(e)}{total\_time}$

Low-Level Fuzzy PRM

if mode = "CONSTRUCTION" **then**
    add a new sample q to graph G_e
    add an edge(q, q') to all previous samples
    $prob(q, q') \leftarrow P^*(l)$
if mode = "QUERY" **then**
    $\phi \leftarrow$ compute most probable path in G_e
    repeat
        $(q', q'') \leftarrow$ edge in ϕ with lowest probability
        if $prob(q', q'') \neq 1$ **then**
            run subdivision collision checking to validate (q', q'') at resolution $\ell(q', q'')$
            increment $\ell(q', q'')$
            if collision **then**
                remove (q', q'') from G_e and return `failure`
            else
                update $prob(q', q'')$ based on collision resolution $\ell(q', q'')$
    until all edges in ϕ have prob 1
    return `success`

[Berenson, Srinivasa, Ferguson, Collet, Kuffner: ICRA 2009]

- Manipulation planners often require specification of a set of stable grasp configurations

Manipulation Planning with Workspace Goal Regions

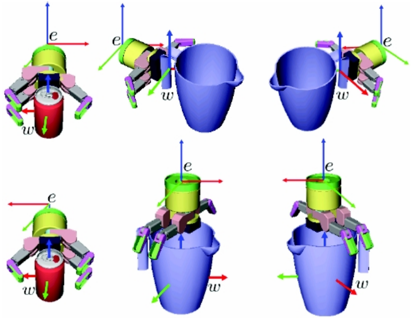
[Berenson, Srinivasa, Ferguson, Collet, Kuffner: ICRA 2009]

- Manipulation planners often require specification of a set of stable grasp configurations
- This forces the planner to use only these configurations as goals
- If the chosen goal configurations are unreachable, the planner will fail
- Even when reachable, it may take the planner a long time to find solutions to these goal configurations

[Berenson, Srinivasa, Ferguson, Collet, Kuffner: ICRA 2009]

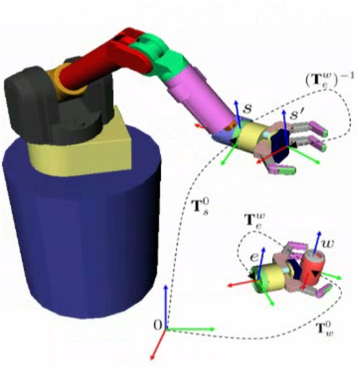
Proposed Approach

- Introduce concept of Workspace Goal Regions (WGRs)
- WGR allows the specification of continuous regions in the six-dimensional workspace of end-effector poses as goals for the planner



- Two WGRs describe grasping a soda can
- Bounds allow rotation around z axis of w

- Reference frame w attached at object specifying pre-computed *grasp pose*
- Workspace bounds B^w specifying flexibility around target grasp w: $[(x_{\min} x_{\max}), (y_{\min}, y_{\max}), (z_{\min}, z_{\max}), (\psi_{\min}, \psi_{\max}), (\theta_{\min}, \theta_{\max}), (\phi_{\min}, \phi_{\max})]$



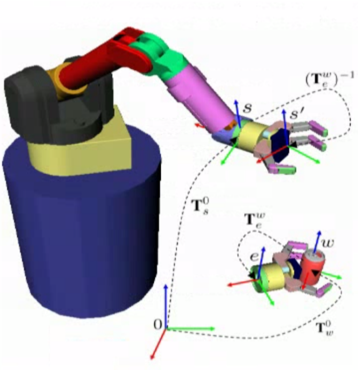
- To allow offset for end-effector, transform T_e^w specifies end-effector pose relative to the (w) reference frame of the desired grasp
- Simple operations can be done: $T_w^0 T_e^w$ now specifies a target pose of end effector in world coordinate frame
- One can sample alternative pose for end effector from B^w, and then convert to world coordinate frame to provide an end-effector goal pose to IK solver

Why Use WGRs for Manipulation Planning

- Sampling from B^w (in the provided range for each of the 6 coordinates that specify the pose of target, pre-specified grasp) gives alternative grasper pose in (w/object's) coordinate frame.
- Sample can be converted into new, sampled goal pose for end-effector.
- IK can be used to steer manipulator towards sampled goal end-effector pose.
- All encapsulated in an IK bi-directional RRT (IKBiRRT) so as to deal with the usual get-stuck (subptimal) behavior of gradient-descent type methods for IK.
- A distance measure can be specified to give a sense of how far or near two end-effector configurations are for RRT.

Some Sampling Target End-Effector Pose from WGR

- $d^w_{\mathrm{sample}} \leftarrow$ *sample* a random value between each of the bounds defined by B^w with uniform probability
- convert d^w_{sample} into a transformation matrix T^w_{sample}, which specifies the *sampled grasper pose* relative to the coordinate frame w of the target grasp.
- convert the *sampled grasper pose* into a *sampled pose for the end-effector*, still in the coordinate frame of w (target grasp pose)



$$T^w_{\mathrm{sample}} \cdot T^w_e$$

- convert the sampled end-effector pose in world coordinates

$$T^0_{\mathrm{sample}'} = T^0_w T^w_{\mathrm{sample}} T^w_e$$

- $T^0_{\mathrm{sample}'}$ is passed to an IK solver to generate solution(a)s, which are checked for collisions. Only collision-free solutions are added to the RRT.

- use FK to get end-effector pose at current q_s configuration: T_s^0 is pose of end-effector in world coordinates.
- get pose of grasp, if object held there, in world coordinates

$$T_{s'}^0 = T_s^0 (T_e^w)^{-1}$$

- convert it from world to coordinates of w

$$T_{s'}^w = (T_w^0)^{-1} T_{s'}^0$$

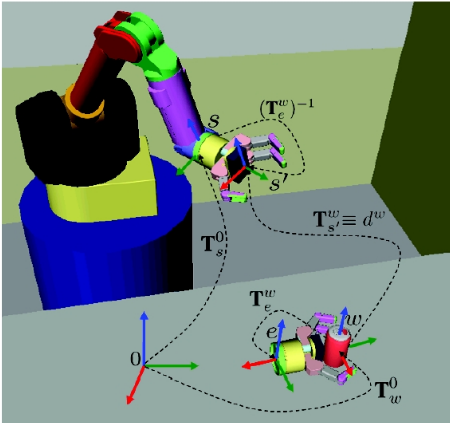
- convert $T_{s'}^w$ into a 6×1 displacement vector from origin of w frame

$$d^w = \begin{bmatrix} t_{s'}^w \\ \mathrm{arctan2}(R_{s'_{32}}^w, R_{s'_{33}}^w) \\ -\mathrm{arcsin}(R_{s'_{31}}^w) \\ \mathrm{arctan2}(R_{s'_{21}}^w, R_{s'_{11}}^w) \end{bmatrix}$$

- take into account bounds B^w to get 6×1 displacement vector Δx from d^w

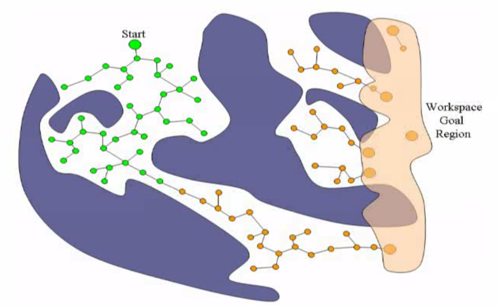
$$\Delta x_i = \begin{cases} d_i^w - B_{i,1}^w & \text{if } d_i^w < B_{i,1}^w \\ d_i^w - B_{i,2}^w & \text{if } d_i^w > B_{i,2}^w \\ 0 & \text{otherwise} \end{cases}$$

Distance to WGRs: $d(q_s, WGR)$



$$d(q_s, WGR) = ||\Delta x||$$

- Grows one tree from start and one tree from goal configuration.
- At each iteration chooses between one of two modes: exploration through standard BiRRT and sampling from the set of WGRs W. The probability of choosing the mode is controlled by the parameter P_{sample}.
- Goal configurations sampled from a WGR are injected into the backwards tree that grows from goal.
- Termination when both trees meet at some configuration.



Inverse Kinematics Bi-Directional RRT (IKBiRRT)

T_a.INIT(q_s); T_b.INIT(NULL)
while TIMEREMAINING() **do**
    $T_{\text{goal}} \leftarrow$ GETBACKWARDTREE(T_a, T_b)
    if $T_{\text{goal}}.size() = 0$ or $rand(0, 1) < P_{\text{sample}}$ **then**
        ADDIKSOLUTIONS(T_{goal})
    else
        $q_{\text{rand}} \leftarrow$ RANDCONFIG()

        $q^a_{\text{near}} \leftarrow$ NEARESTNEIGHBOR(T_a, q_{rand})
        $q^a_{\text{reached}} \leftarrow$ EXTENDTREE(T_a, q^a_{near}, q_{rand})

        $q^b_{\text{near}} \leftarrow$ NEARESTNEIGHBOR(T_b, q_{rand})
        $q^b_{\text{reached}} \leftarrow$ EXTENDTREE(T_b, q^b_{near}, q_{rand})

        if $q^a_{\text{reached}} = q^b_{\text{reached}}$ **then**
            return EXTRACTPATH(T_a, q^a_{reached}, T_b, q^b_{reached})
        else
            SWAP(T_a, T_b)

[movie]