

# Network Reliability and Cascading Blackouts

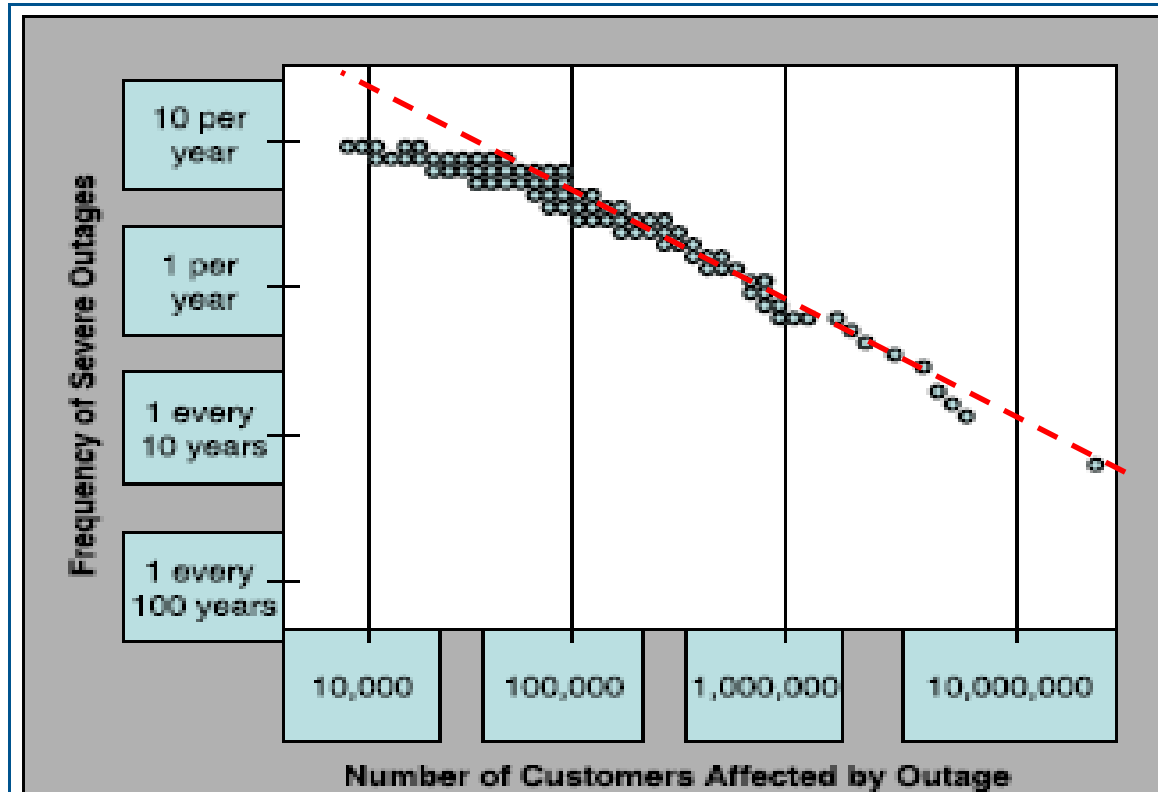
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# Rare Events in Power Grids

Individual Outages in North America, 1984-1997



**Power-tailed  
Distribution**

$$\Pr\{X > x\} \approx cx^{-a}$$

Source: U.S.-Canada Power System Outage Task Force. 2004. Final Report on the August 14, 2003 Blackout in the United States and Canada: Causes and Recommendations.

Adapted from John Doyle. 1999. Complexity and Robustness.

# Related Research

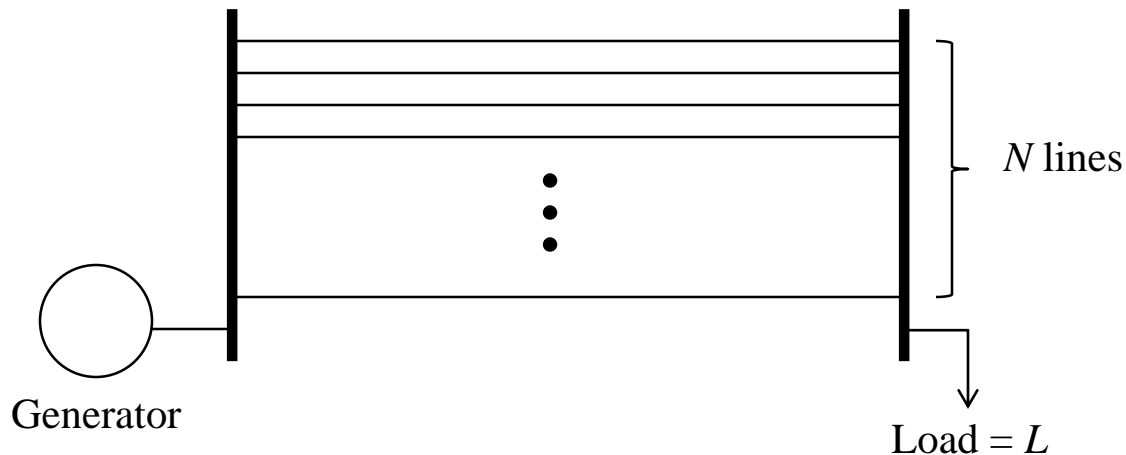
- Stochastic methods
  - Much work on rare-event stochastic reliability of static networks (assume line failures are independent)
- Cascading outages, detailed modeling methods
  - Cluster-based – identify network clusters and critical cut-sets
  - Enumeration of likely cascading paths
  - Monte-Carlo sampling – examination of random subset of cascading scenarios
    - Limited use of rare-event simulation methods

• Botev, Z., P. L'Ecuyer, G. Rubino, R. Simard, B. Tuffin. 2012. Static reliability estimation via generalized splitting. To appear in *INFORMS Journal on Computing*.

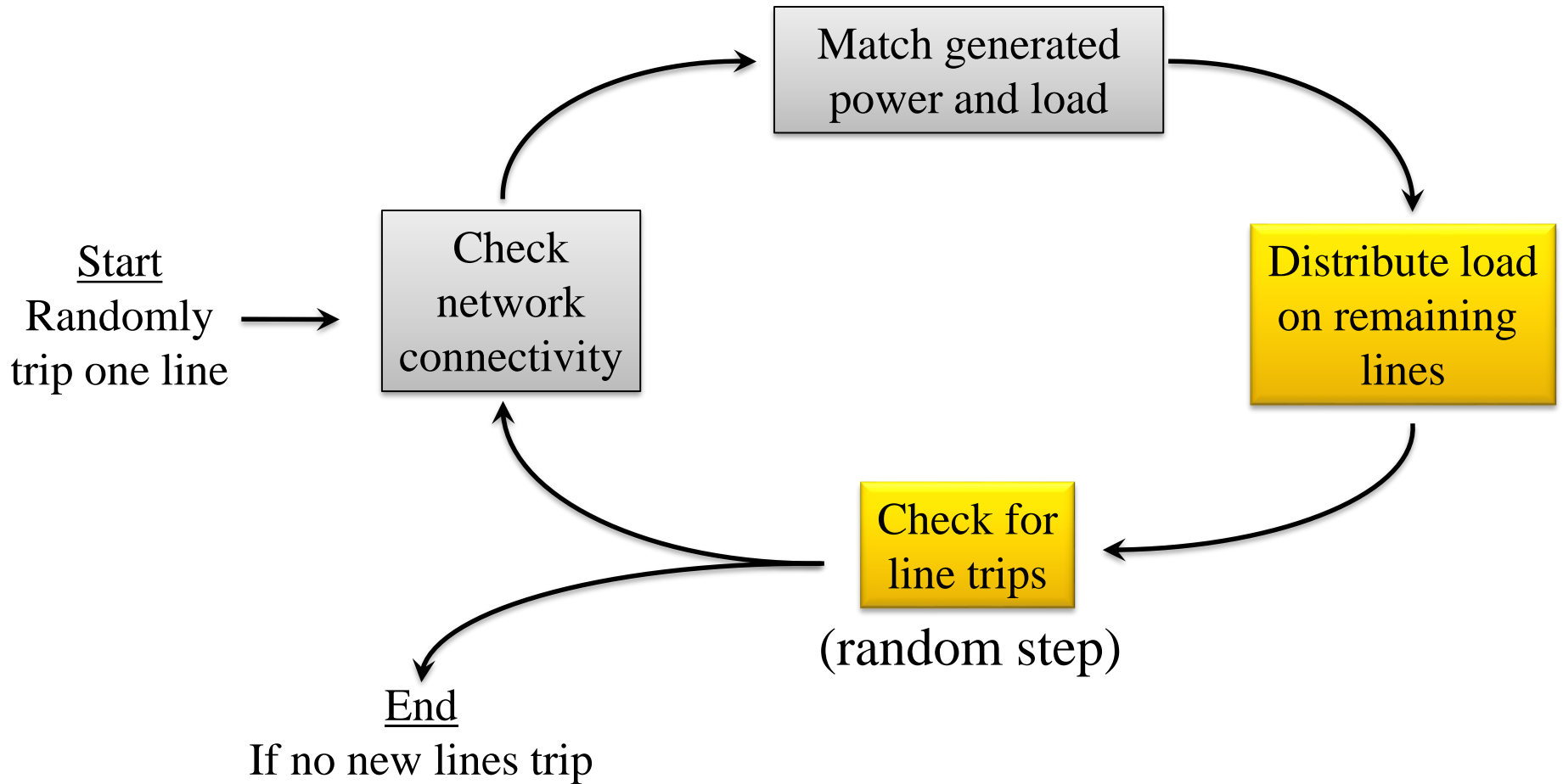
• Risk assessment of cascading outages: Part I – overview of methodologies. PES Task Force on Understanding, Prediction, Mitigation and Restoration of Cascading Failures, PES Annual Meeting, Detroit, MI, 2011.

# Example: Simple Cascading Model

- When a line fails, its load is equally distributed among the remaining lines
- Equivalent to  $N$  identical parallel lines connecting two buses



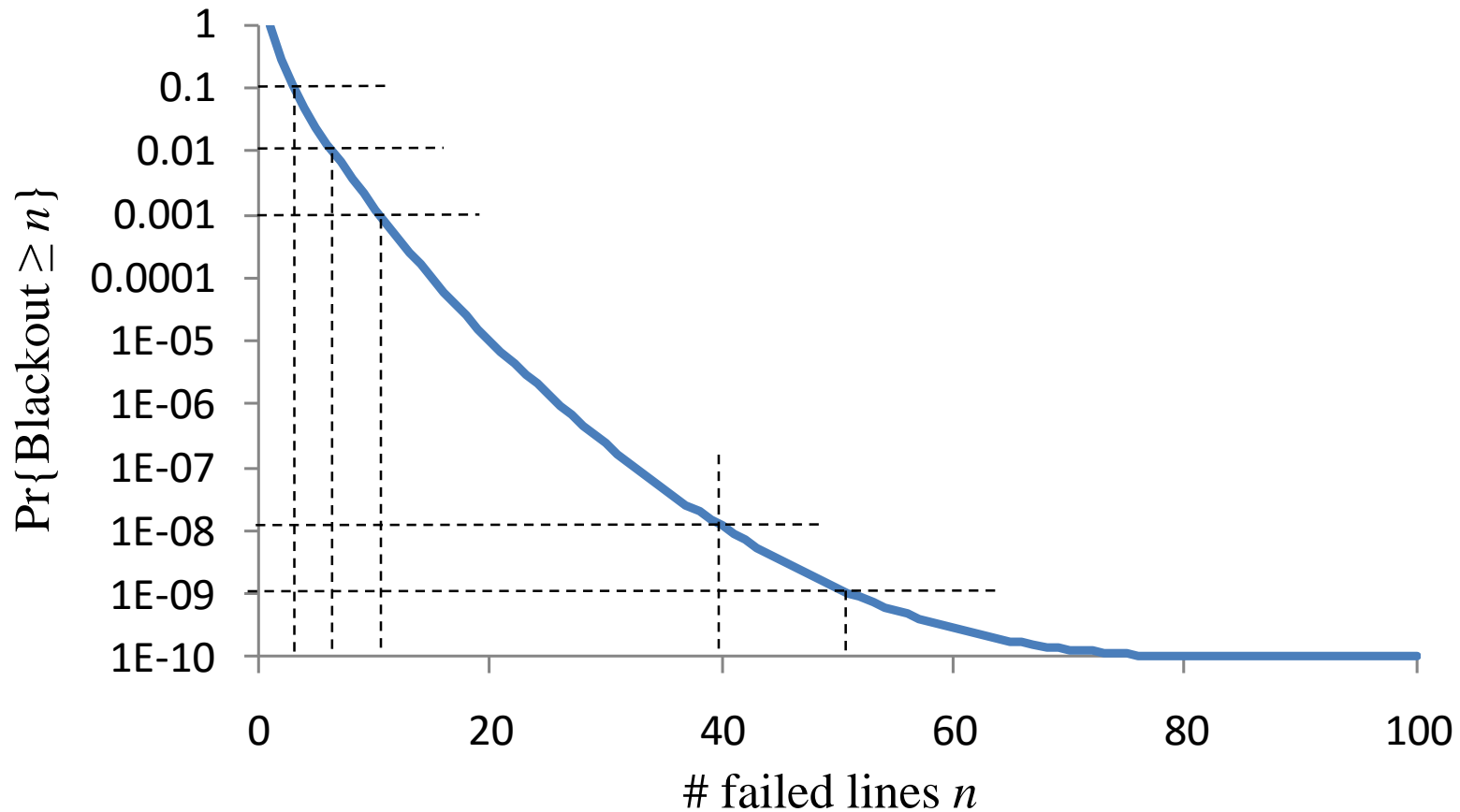
# Cascading Blackout Model



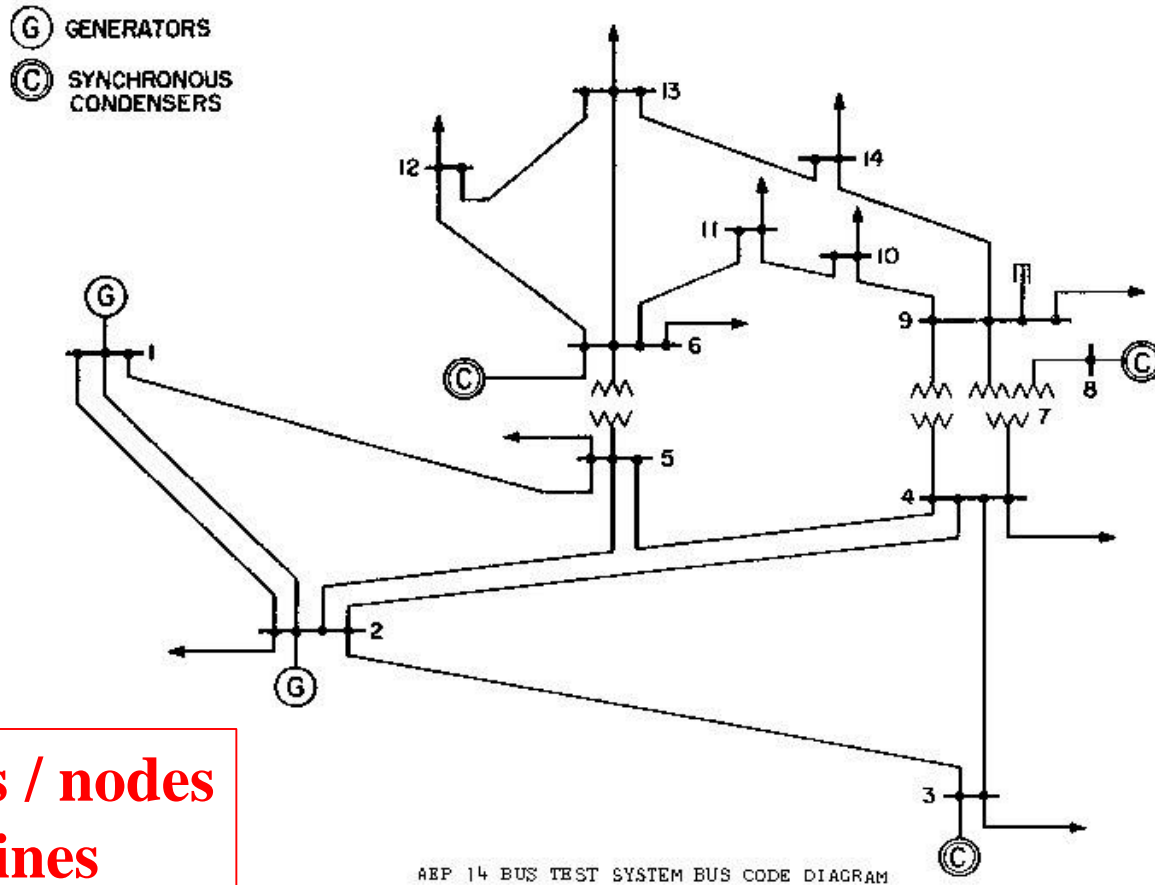
Similar to some models in literature, for example:

- Chen, J., J. Thorp, I. Dobson. 2005. Cascading dynamics and mitigation assessment in power system disturbances via a hidden failure model. *Electrical Power & Energy Systems*, 27, 318-326.
- Bae, K. J. Thorp. 1999. A stochastic study of hidden failures in power system protection. *Decision Support Systems*, 24, 259-268.

# Blackout-size Distribution

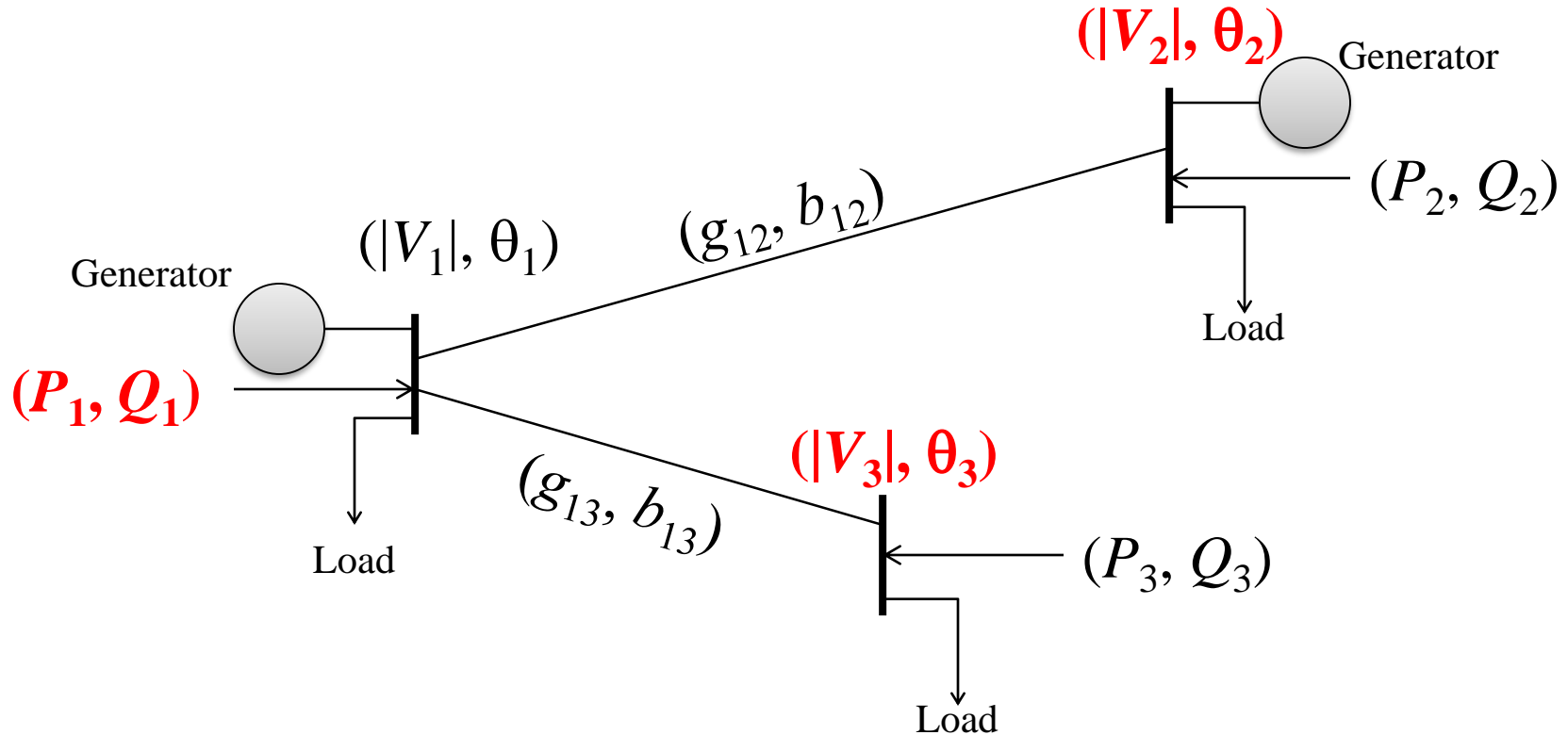


# IEEE 14-Bus Grid



**14 buses / nodes**  
**20 lines**

# Power-Flow Equations



## Notation

- $P_i, Q_i = net$  real, reactive power injected into bus  $i$  (negative for load bus)
- $|V_i| =$  root-mean-square voltage at bus  $i$
- $\theta_i =$  phase angle of voltage at bus  $i$
- $g_{ik}, b_{ik} =$  conductance, susceptance for link  $i-k$

**Black = given**

**Red = unknown variable** 8



# Power Flow Equations

- Power-flow equations

$$P_i = \sum_{k=1}^n |V_i| |V_k| [g_{ik} \cos(\theta_i - \theta_k) + b_{ik} \sin(\theta_i - \theta_k)] \quad i = 1, \dots, n$$

$$Q_i = \sum_{k=1}^n |V_i| |V_k| [g_{ik} \sin(\theta_i - \theta_k) - b_{ik} \cos(\theta_i - \theta_k)] \quad i = 1, \dots, n$$

- Linear equations

- Assume  $\theta_i$  small,  $|V_i| \approx 1$ ,  $g_{ik} \approx 0$

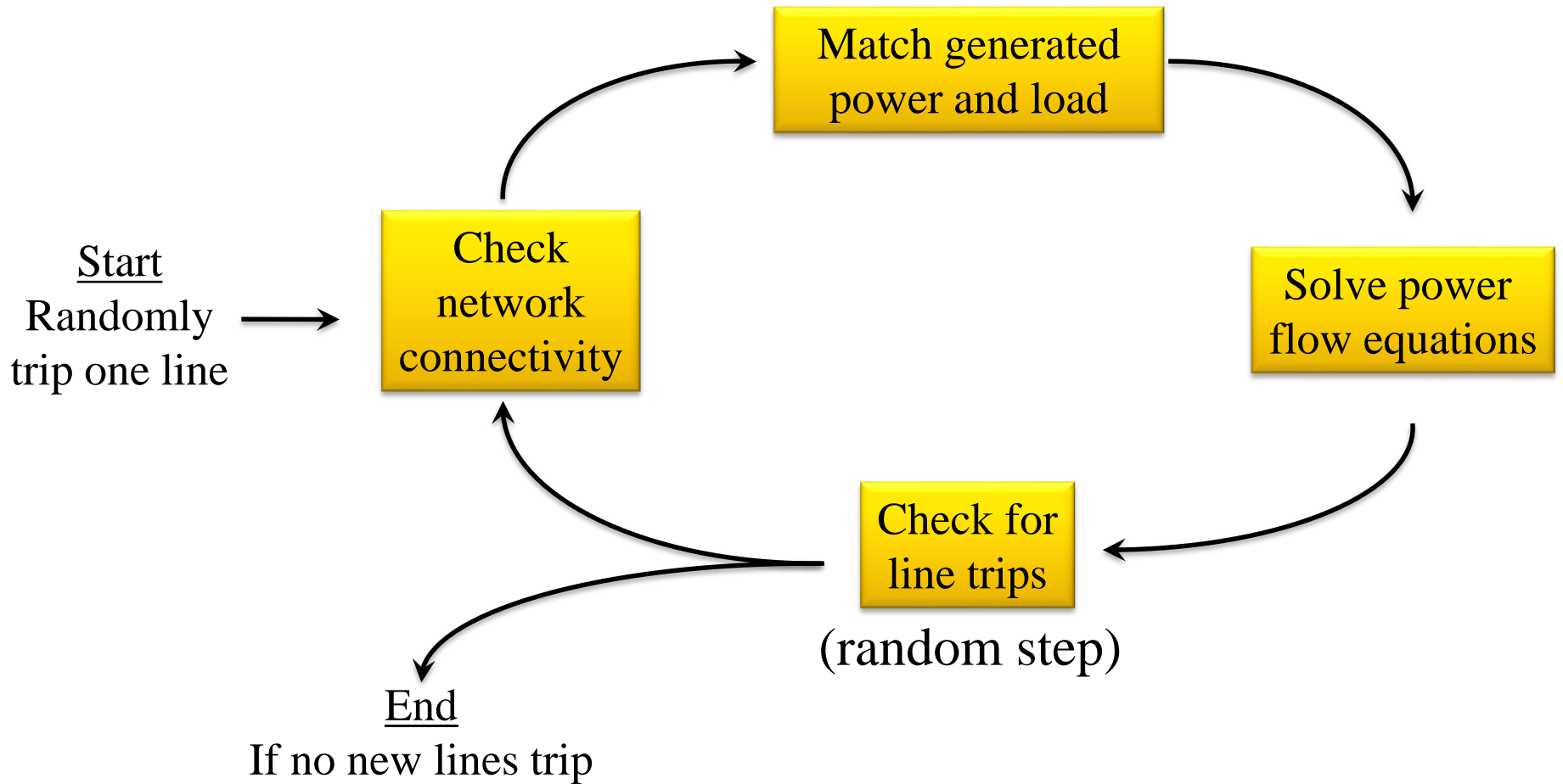
$$P_i = \sum_{k=1}^n b_{ik} (\theta_i - \theta_k)$$

- Return:  $P_1, \theta_2, \dots, \theta_n$

## Notation

- $P_i, Q_i$  = net real, reactive power injected into bus  $i$  (negative for load bus)
- $|V_i|$  = RMS of voltage at bus  $i$
- $\theta_i$  = phase angle of voltage at bus  $i$
- $g_{ik}, b_{ik}$  = conductance, susceptance for link  $ik$

# Cascading Blackout Model



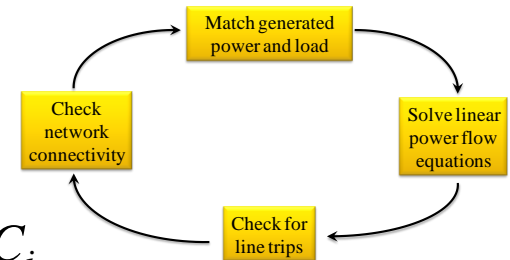
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# Stochastic Line Trip

- Definitions

- $C_j$  = capacity of line  $j$  (random)
- $F_j(x)$  = cumulative distribution function of  $C_j$
- $x_j$  = power flow on line  $j$
- $x_j'$  = largest previously observed power flow on line  $j$
- $[x]^+$  =  $\max(x, 0)$



- Probability of line trip

$$\Pr\{C_j \leq x_j \mid C_j > x_j'\} = \frac{[F_j(x_j) - F_j(x_j')]^+}{1 - F_j(x_j')}$$

- Notes

- Possible to have more than one line trip in each step
- Distinctive feature: Model accounts for additional load above largest previously observed load

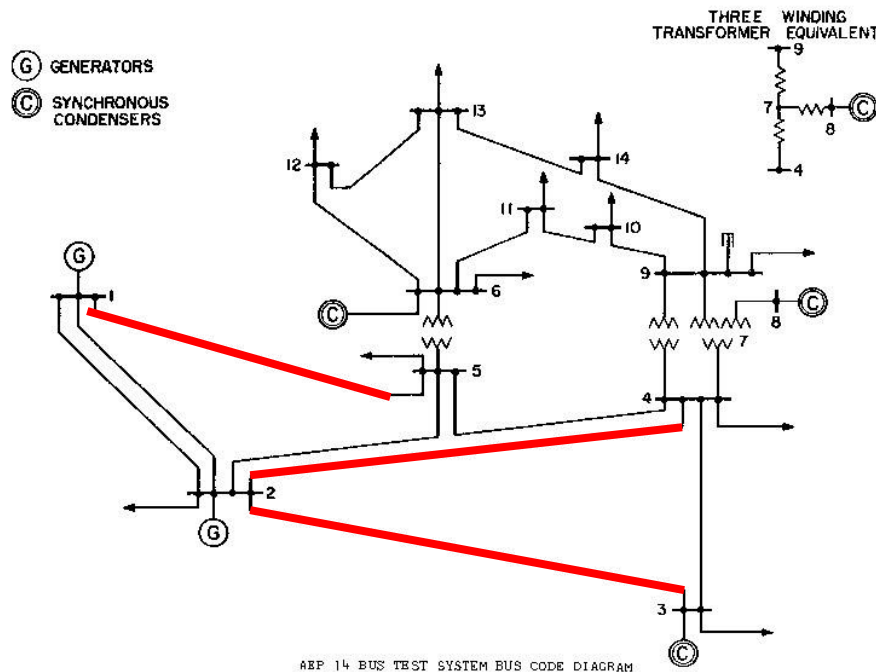
# Cascading Blackout Model

## Factors modeled

- Network structure
- Physics of power flows
- Dependent nature of cascade
- Stochastic failures, demand

## Factors **not** modeled

- Timing of events
- Transient effects
- Generator failures
- Human factors
- ...



# Problems Simulating Rare Events

$\gamma \equiv$  Probability of rare event

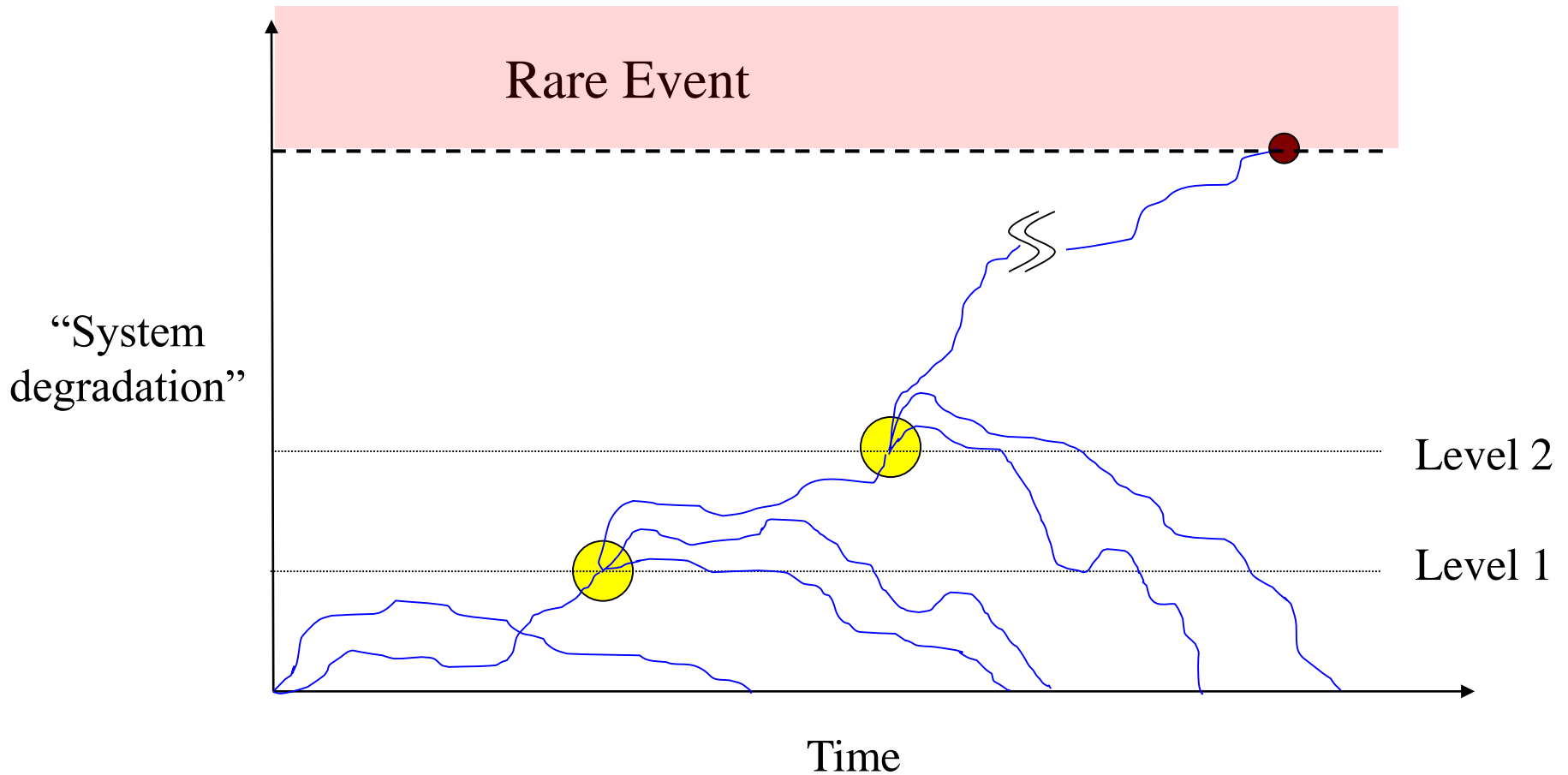
$\hat{\gamma}_n \equiv \frac{\text{\# replications in which rare event occurs}}{n}$

$$\text{Relative Error}[\hat{\gamma}_n] \equiv \frac{\sqrt{\text{var}[\hat{\gamma}_n]}}{E[\hat{\gamma}_n]} = \frac{\sqrt{\gamma(1-\gamma)/n}}{\gamma} \approx \frac{1}{\sqrt{n\gamma}}$$

Time Required to Achieve 1% Relative Error  
(Assumes 1,000 replications per second)

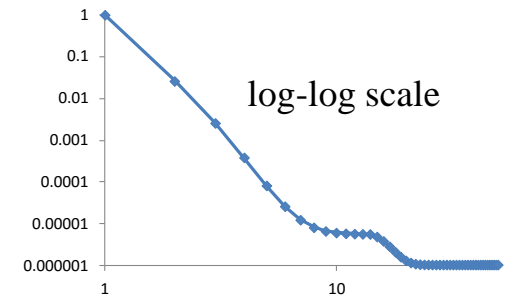
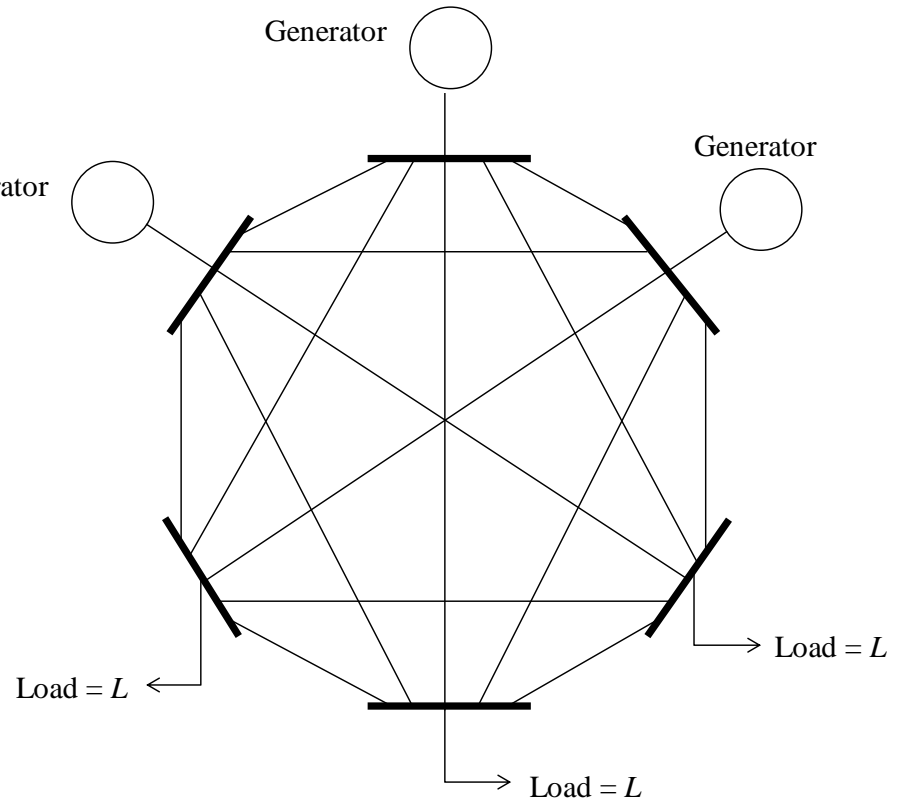
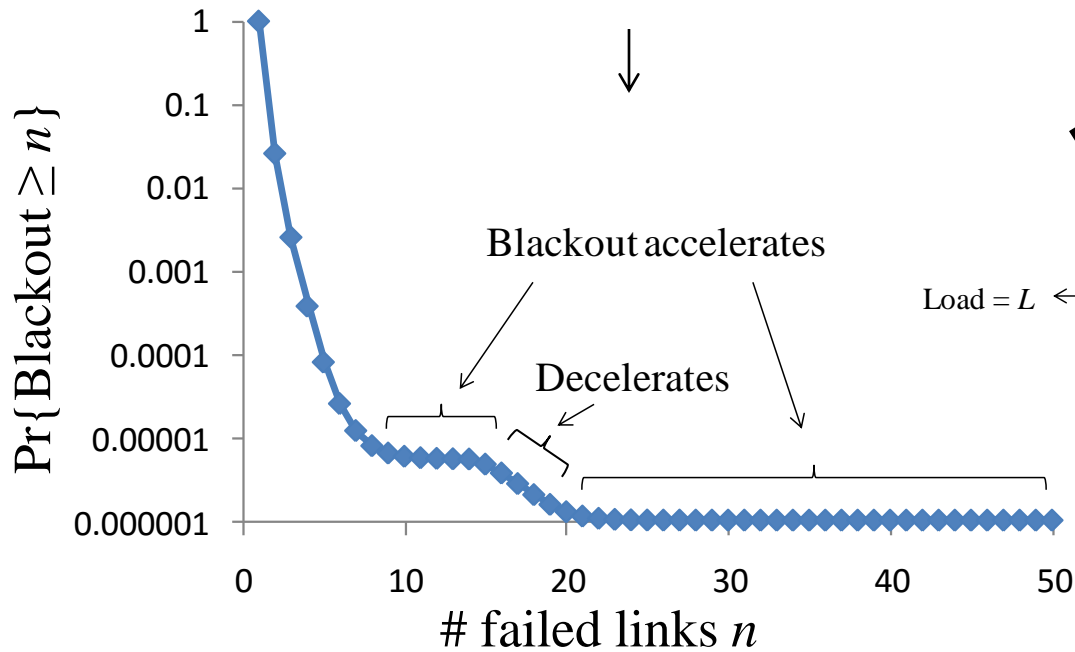
Rare Event Probability $\gamma$	Simulation Runs $n$	Required Time
$10^{-3}$	$10^7$	16.7 minutes
$10^{-5}$	$10^9$	1.2 days
$10^{-7}$	$10^{11}$	116 days
$10^{-9}$	$10^{13}$	31.7 years

# Splitting



# Example: Mesh Network

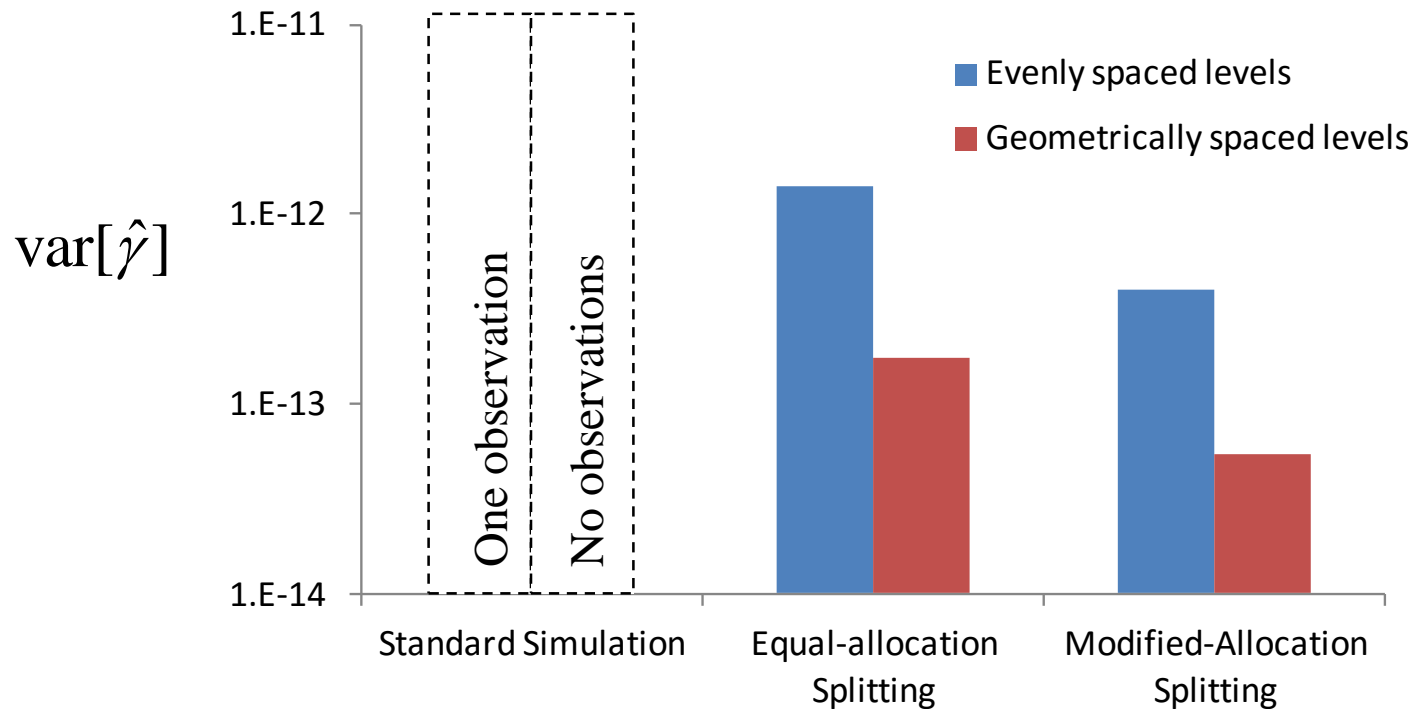
Example: 15-bus mesh network  
5 generators, 10 loads



Line capacities follow normal distribution with mean = twice flow of maximum flow on any line when all lines are working. Std. dev = 0.2 x mean.

# Simulation Efficiency

Objective: Estimate  $\gamma = \Pr\{50 \text{ lines fail}\}$



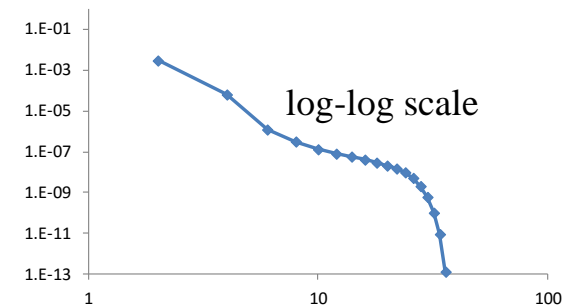
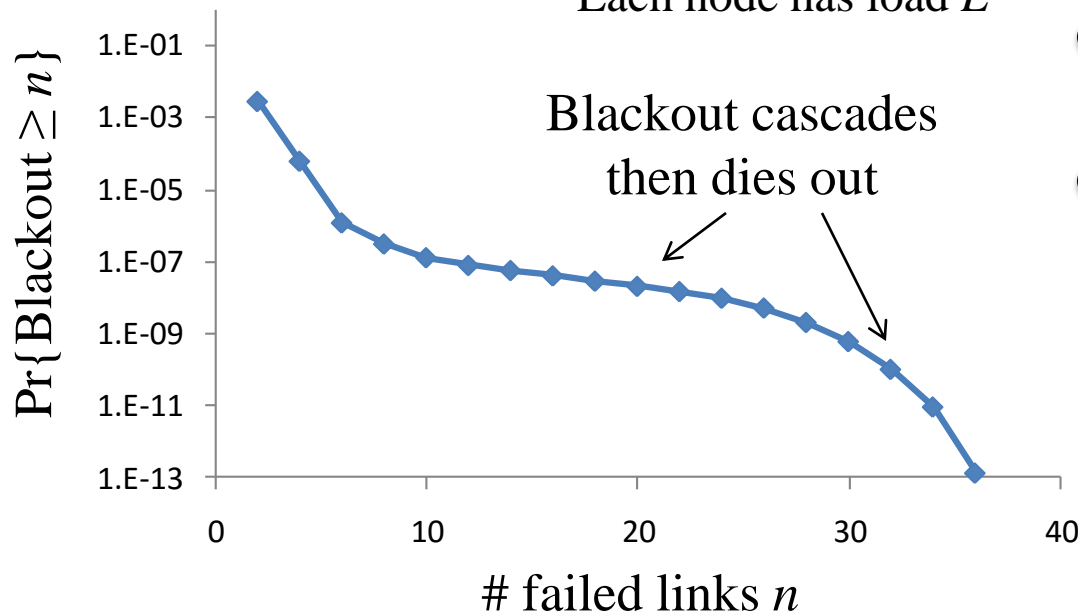
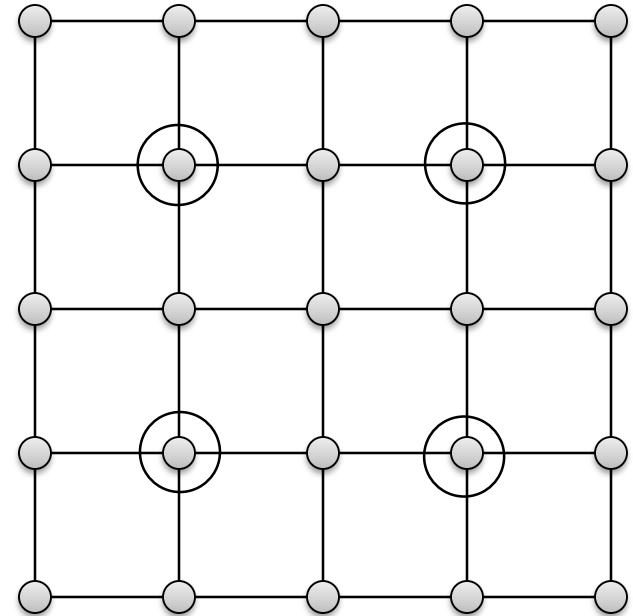


# Example: Grid Network

○ = Generator

Each node has load  $L$

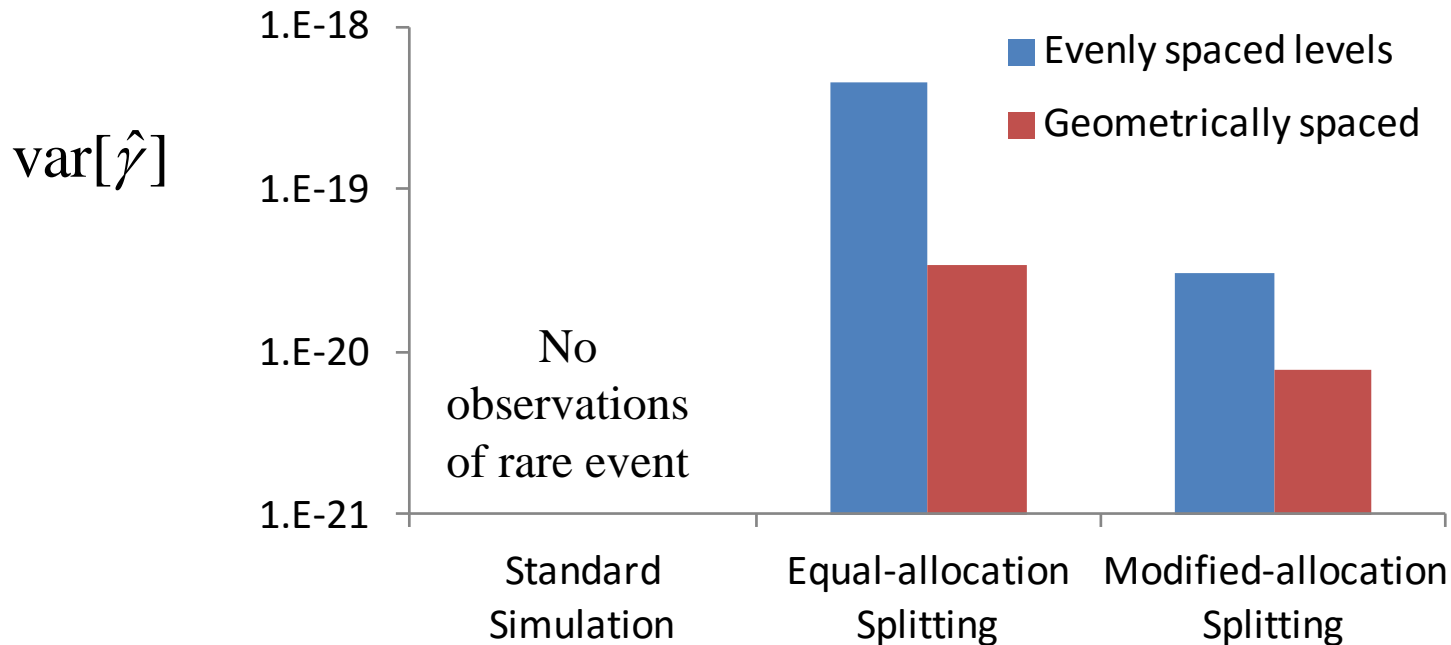
Blackout cascades  
then dies out



Line capacities follow normal distribution with mean = twice flow on any line after considering all single line failures.  
Std. dev = 0.2 x mean.

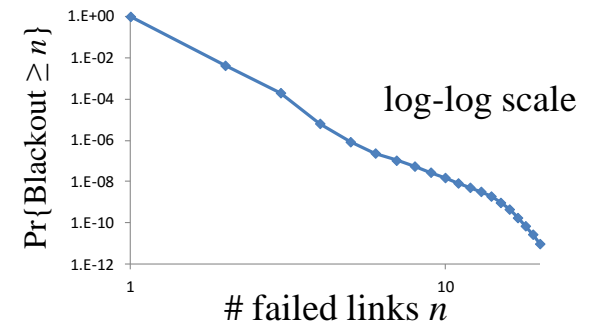
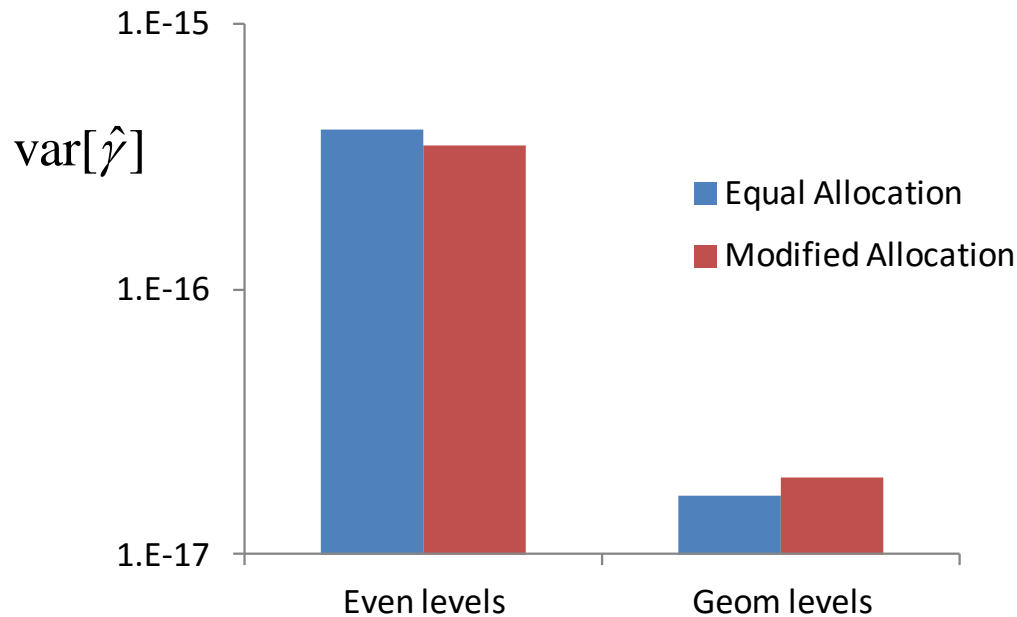
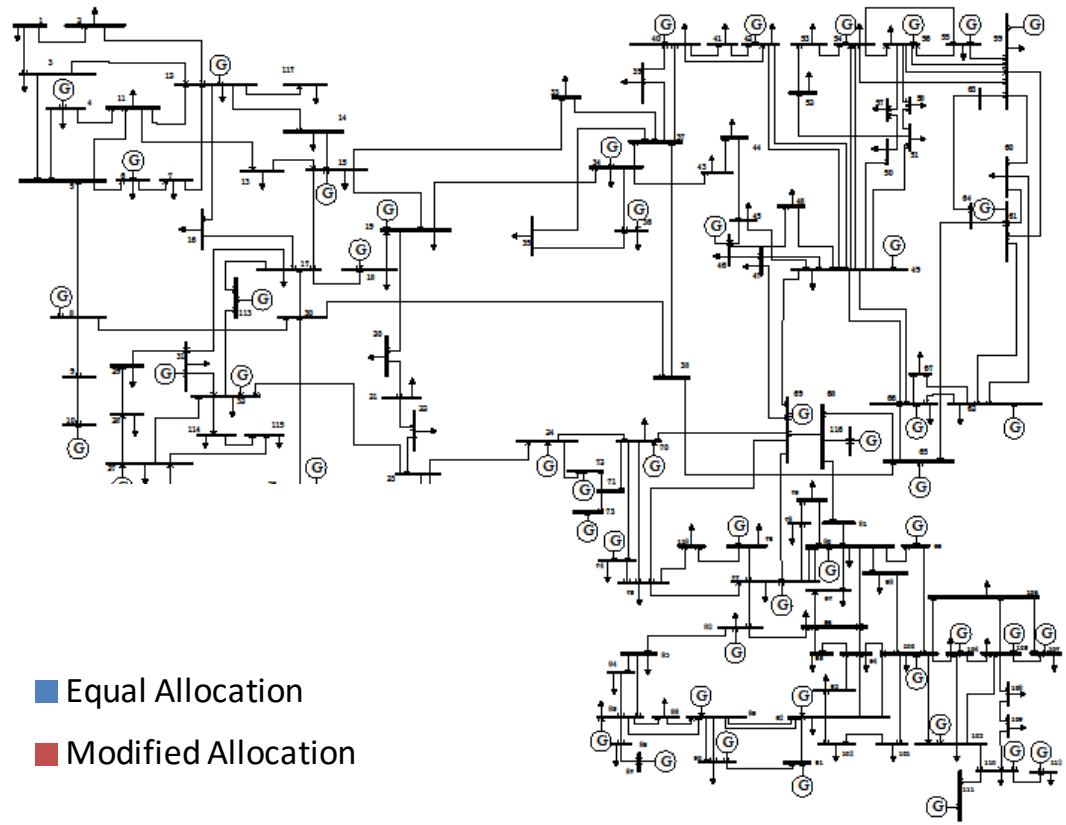
# Simulation Efficiency

Objective: Estimate  $\gamma = \Pr\{32 \text{ lines fail (out of 40)}\}$



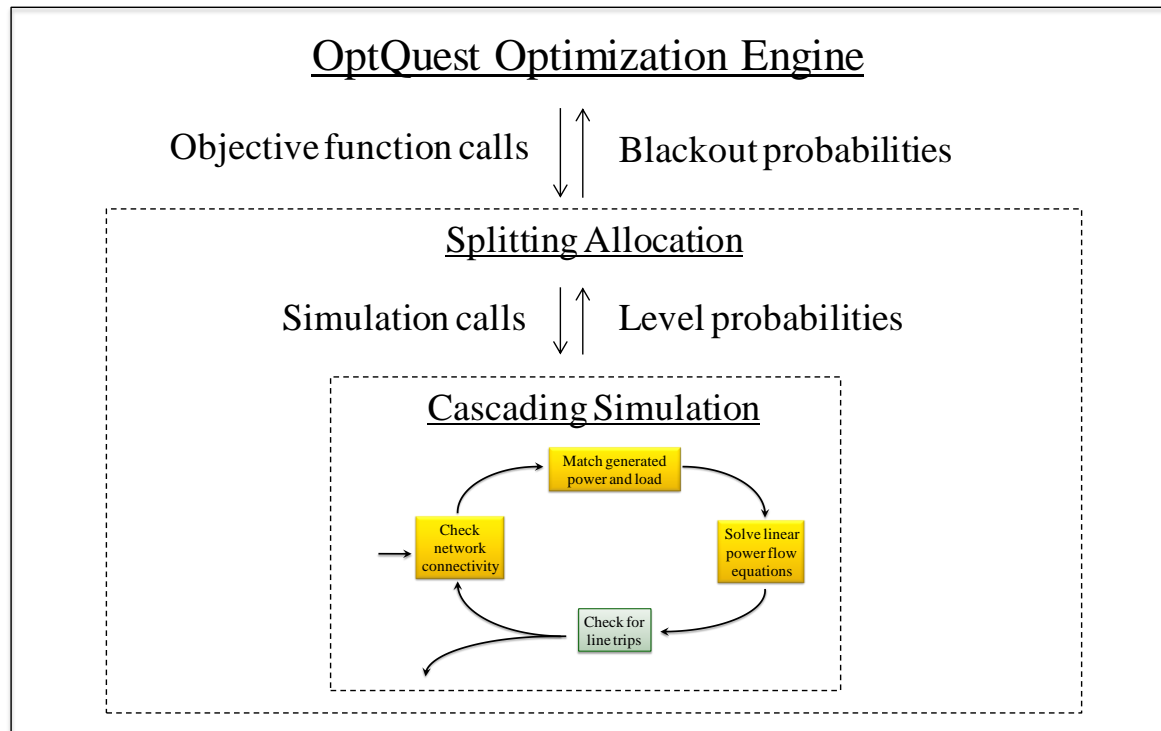
# Example: 118-bus System

Line capacities follow normal distribution with mean = 1.25 times flow on any line after considering all single line failures. Std. dev = 0.2 x mean. Computing budget = 8 minutes per replication, x35 replications



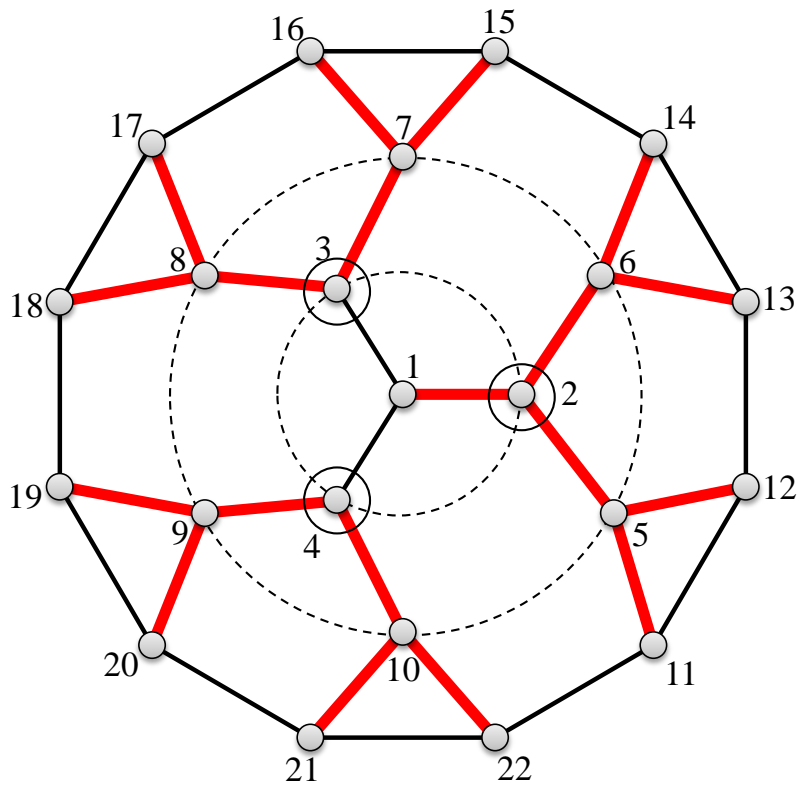
# Optimal Capacity Expansion

Minimize:  $\Pr\{\text{blackout size} \geq \text{threshold}\}$   
Such that:  $\text{expansion cost} \leq \text{budget}$

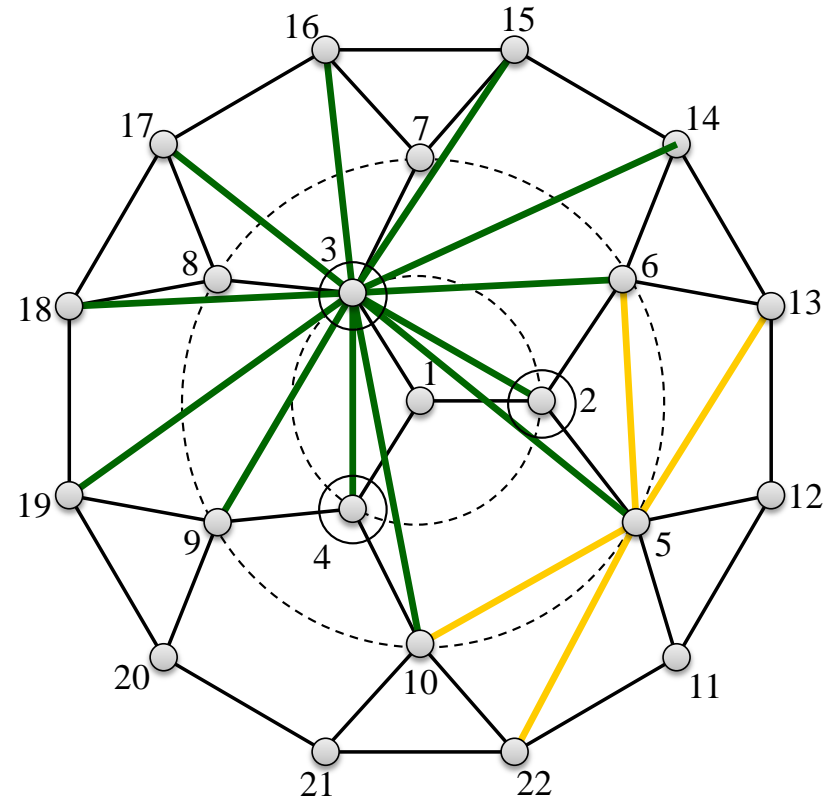


# Expansion: Tree Network

Large-scale Blackout



Medium-scale Blackout



# Summary

- Dependent nature of cascade. Line failures change failure probabilities of other lines
- Standard simulation may not be computationally tractable for estimating rare-event probabilities
  - Splitting is a technique that can make problem tractable
- Other work
  - Evaluation of splitting parameters
  - Capacity optimization favors expanding a single path to each node for high utilization
- Current work in stochastic “N-k” problem