Incentive Mechanisms for Sharing Resources in Networks
— A Smart Grid Application

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Motivation

Increasing energy consumption is painful to our society.

1. Environmentally: burning coals, natural gas or petroleum

Figure : Credit: http://www.consumeraffairs.com

2. Economically: For a townhouse in Virginia US, it cost around $200 per month.

Green energy

1. **More homes** are equipped with solar panels to generate and consume renewable solar energy.
2. **Power grid utility companies** are enforced/encouraged to generate and consume larger portion of renewable energy[1].

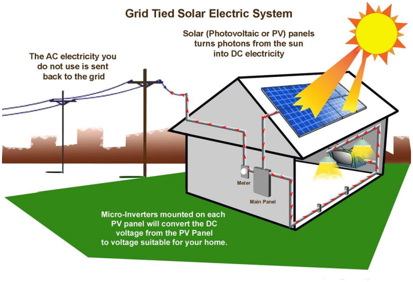
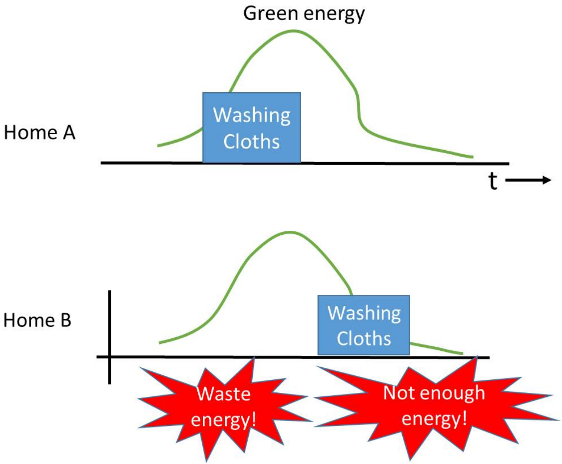


Figure : Credit: http://conservio.net

[1] "State of California executive order S-21-09", 2009, http://gov38.ca.gov/index.php?/executive-order/13269

Challenges

1. **Energy mismatch** between the supply and demand for individual users



2. **The more consumers, the better (for consumers and power grids)**?

Our objective

Intuition

Build up a system that allows a user to 'bank' his surplus if the local energy supply exceeds the local energy demand and to 'borrow' some energy.

Fair renewable energy trade

Design an **incentive** and **fair energy trading mechanism** that benefits all the users as well as the grid. A grid providing fair energy trading mechanism attracts users and improves surplus energy throughput.

Related work

1. Solve the **mismatch problem**
   1.1 Store surplus energy using local batteries.
      - Require large batteries.

   1.2 Trade surplus energy with smart grid (used by power companies, usually at the price of spot market).
      - Destabilize the grid in energy storage.

   1.3 Shift workload to be executed at later time.
      - Affect people's daily life quality.

2. **Share renewable energy**
   2.1 References[2],[3] propose to share surplus renewable energy among homes

[2] Zhu, et. al, "Sharing renewable energy in smart microgrids", ACM/IEEE ICCPS, 2013

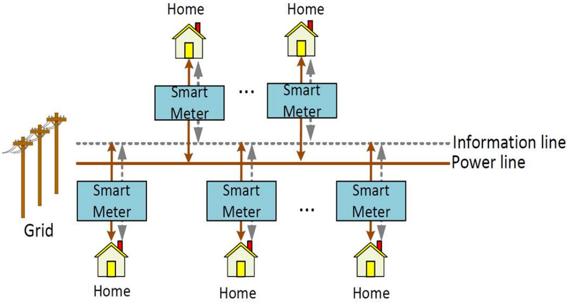
[3] Zhong et. al, "iDES: Incentivedriven distributed energy sharing in sustainable microgrids", IEEE IGCC, 2014

Model

1. **Powerline**: A home can push surplus energy to or pull energy from the smart grid.
2. **Information line**: A home can communicate with smart grid.
3. **Smart meter**: Monitor the amount of energy push to or pull from smart grid.

Assumption

A home is **not** equipped with a battery to store energy.



Questions to answer

Energy sharing are conducted at a window-based online manner.

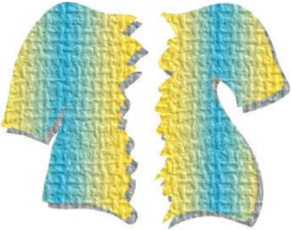
1. How do we define 'fairness'?

Problem

At a time, we have k users with their energy request demands $d_1, d_2, \ldots, d_k$. and the total surplus energy supply is H_s. How shall we partition H_s to the k requests in a fair manner?

2. Can we have exact energy requests and energy supplies known at a time?
   (Techniques are not related to game theory. Skipped in this talk.)
3. If we are 'fair' at a time for all users, how do we guarantee that we are still fair at a later time if the sets of users are different?
   (Techniques are not related to game theory. Skipped in this talk.)

THE DISPUTED GARMENT PROBLEM: THE MATHEMATICS OF BARGAINING & ARBITRATION



Richard Weber

Nicky Shaw Public Understanding of Mathematics Lecture

7 February, 2008

The Disputed Garment Problem

The **Babylonian Talmud** is the compilation of ancient law and tradition set down during the first five centuries A.D. which serves as the basis of Jewish religious, criminal and civil law. One problem discussed in the Talmud is the disputed garment problem.

> **"Two hold a garment; one claims it all, the other claims half. Then one is awarded $\frac{3}{4}$ and the other $\frac{1}{4}$."**

The idea is that half of the garment is not in dispute and can be awarded to the one who claims the whole garment. The other half of the garment is in dispute and should be split equally.
Thus one gets $\frac{1}{2} + \frac{1}{4}$ and the other gets $\frac{1}{4}$.

Sharing the Cost of a Runway

Suppose three airplanes share a runway.

- Plane 1 requires 1 km to land.
- Plane 2 requires 2 km to land.
- Plane 3 requires 3 km to land.

So a runway of 3 km must be built.

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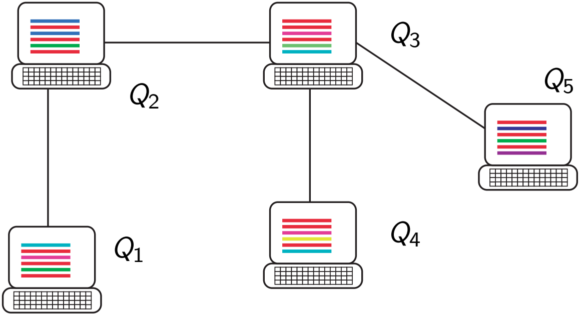
What proportion of the building cost should each plane pay?

Sharing Files

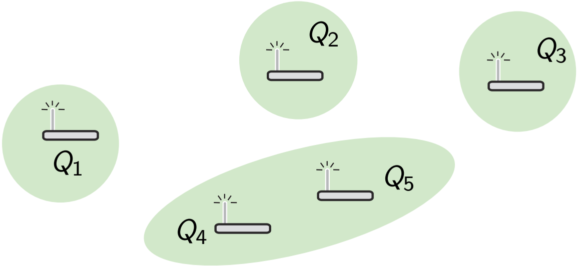
A file sharing system Peers contribute files to a shared library of files which they can access over the Internet.



Peer i shares Q_i files.
The benefit to peer j is $\theta_j u(Q_1 + \cdots + Q_5)$.

Sharing WLANS

A sharing of wireless LANS system Peers share their wireless Local Area Networks so that they can enjoy Internet access via one another's networks whenever they wander away from their home locations.



Peer i makes his WLAN available for a fraction Q_i of the time.
The benefit to peer j is $\theta_j u(Q_1 + \cdots + Q_5)$.

Example: A Bridge

A bridge may or may not be built. There are 2 potential users.

Mathematical Bridge, Queens' College, Cambridge

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What is the best fee mechanism, $p_1(\theta_1, \theta_2)$ and $p_2(\theta_1, \theta_2)$?

Tractate Kethuboth

The Baylonian Talmud also gives instructions about dividing estates.

MISHNAH 93. IF A MAN WHO WAS MARRIED TO THREE WIVES DIED, AND THE KETHUBAH OF ONE WAS A MANEH, OF THE OTHER TWO HUNDRED ZUZ, AND OF THE THIRD THREE HUNDRED ZUZ AND THE ESTATE [WAS WORTH] ONLY ONE MANEH [THE SUM] IS DIVIDED EQUALLY. IF THE ESTATE [WAS WORTH] TWO HUNDRED ZUZ [THE CLAIMANT] OF THE MANEH RECEIVES FIFTY ZUZ [AND THE CLAIMANTS RESPECTIVELY] OF THE TWO HUNDRED AND THE THREE HUNDRED ZUZ [RECEIVE EACH] THREE GOLD DENARII. IF THE ESTATE [WAS WORTH] THREE HUNDRED ZUZ, [THE CLAIMANT] OF THE MANEH RECEIVES FIFTY ZUZ AND [THE CLAIMANT] OF THE TWO HUNDRED ZUZ [RECEIVES] A MANEH WHILE [THE CLAIMANT] OF THE THREE HUNDRED ZUZ [RECEIVES] SIX GOLD DENARII.

The Marriage Contract Problem

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What happens when the man dies with less than 600 zuz?

The Marriage Contract Problem

The Talmud gives recommendations.

	Debt		
Estate	100	200	300
100	33.33	33.33	33.33

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200	50	75	75
300	50	100	200

100: equal division
300: proportional division
200: ?

The Bankruptcy Game

Two creditors have claims for \$30 million and \$70 million against a bankrupt company. The company only has \$60 million.

The players must reach an agreement about how to divide the money between them, i.e., to choose a_1, a_2, such that

Creditor 1 gets a_1;
Creditor 2 gets a_2;
with $a_1 + a_2 \leq 60$.

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What would be a ‘fair’ division of the money?

Possible Lawyer's Arguments

Creditors 1 and 2 have valid claims for 30 and 70. But there is only 60 to divide. Some possibilities:

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(c) A 30 : 60 split, proportional to the amounts they could get if the other was not a creditor: $\mathbf{(20, 40)}$.

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(d) The disputed garment principle. Creditor 2 should be awarded at least 30, since this is what would be left for him if he first paid Creditor 1's entire claim,

$$30 = 60 - 30 \text{ (Creditor 1's entire claim)}.$$

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$$30 = 60 - 30 \text{ (Creditor 1's entire claim).}$$

30 is in dispute, and is split equally, giving $\mathbf{(15, 45)}$.
Each suffers the same loss compared to what he would get if he were the only creditor, i.e., $\mathbf{(30, 60) - (15, 45) = (15, 15)}$.

John Nash, 1928–

Equilibrium Points in *N*-person Games, *Proceedings of the National Academy of Sciences* 36 (1950).

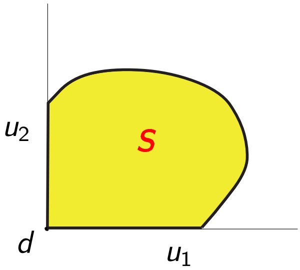
'The Bargaining Problem', *Econometrica* 18 (1950).

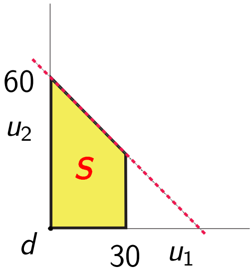
'Two-person Cooperative Games', *Econometrica* 21 (1953).

Nobel Prize in Economics (1994)

Nash's Bargaining Game

We can represent the bargaining game in the following picture.





Two players attempt to agree on a point $u = (u_1, u_2)$, in the set S.
If they agree on $u = (u_1, u_2)$ their 'happinesses' are u_1 and u_2 respectively.
If cannot agree they get $d = (d_1, d_2)$ (the 'disagreement point').

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3. **Independence of Irrelevant Alternatives**.
   You and I are deciding upon a pizza to order and share. We decide on a pepperoni pizza, with no anchovies. Just as we are about to order, the waiter tells us that the restaurant is out of anchovies. Knowing this, it would now be silly to decide to switch to having a mushroom pizza.
   The fact that anchovies are not available is irrelevant, since we did not want them anyway.

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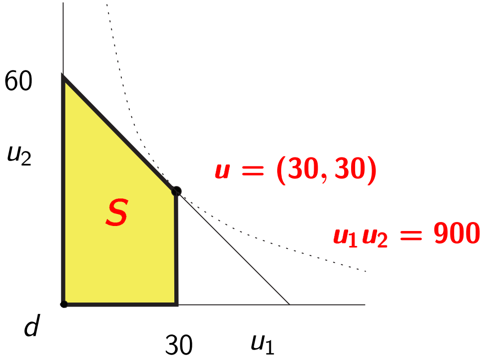
$$u_1' = a_1 + b_1 u_1$$
$$u_2' = a_2 + b_2 u_2$$

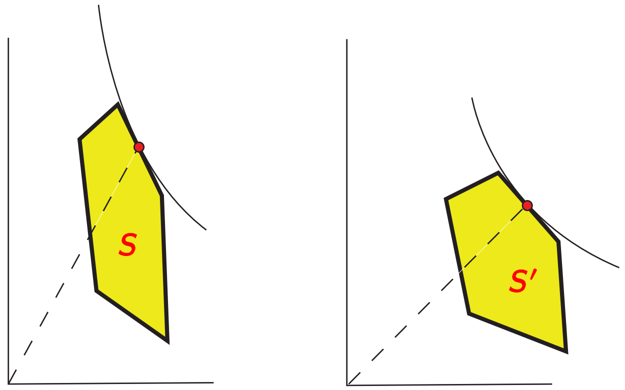
and $(\bar{u}_1, \bar{u}_2)$ is the solution to the game played in bargaining set S and with disagreement point $d = (d_1, d_2)$, then $(a_1 + b_1 \bar{u}_1, a_2 + b_2 \bar{u}_2)$ is the solution to that game played in bargaining set $S' = \{(v_1, v_2) \ : \ v_i = a_i + b_i u_i \, , (u_1, u_2) \in S\}$ and with disagreement point $d' = (a_1 + b_1 d_1, a_2 + b_2 d_2)$.

Nash Solution of the Bargaining Game

Theorem 1 *There is one and only one way to satisfy the Nash bargaining axioms. It is to choose the point in S which maximizes $(u_1 - d_1)(u_2 - d_2)$.*

Creditors 1 and 2 have valid claims for 30 and 70. But there is only 60 to share. The Nash bargaining solution is $u = (30, 30)$.

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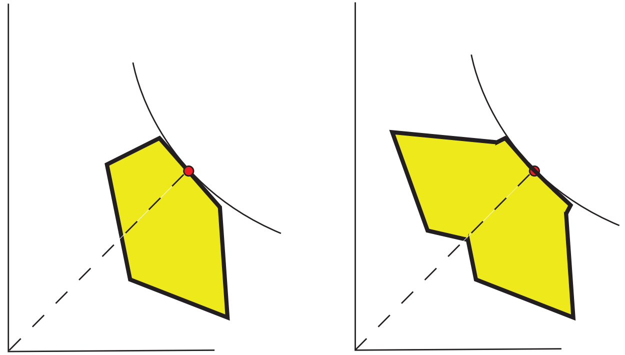

Objections and Counterobjections

Player 1 has an objection to (u_1, u_2) if there is probability p_1 that he can force Player 2 to accept some (v_1, v_2) (otherwise negotiations breakdown) and

$$p_1 v_1 + (1 - p_1) d_1 \geq u_1 \iff p_1 \geq \frac{u_1 - d_1}{v_1 - d_1}$$

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Suppose every objection to u has a valid counterobjection.
This requires that for all (v_1, v_2),

$$\frac{u_1 - d_1}{v_1 - d_1} \geq p_2 \geq \frac{v_2 - d_2}{u_2 - d_2}$$
$$\implies (u_1 - d_1)(u_2 - d_2) \geq (v_1 - d_1)(v_2 - d_2)$$

Back to the Marriage Contract Problem

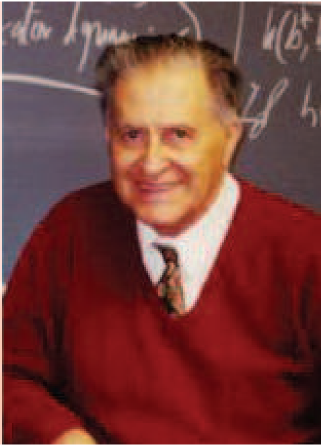
Recall that the Talmud recommends:

	Debt		
Estate	100	200	300
100	33.33	33.33	33.33
200	50	75	75
300	50	100	150

This baffled scholars for two millennia. In 1985, it was recognised that the Talmud anticipates the modern game theory.

The Talmud's solution is equivalent to the nucelolus of an appropriately defined cooperative game. The nucleolus is defined in terms of objections and counterobjections.

Robert Aumann and Michael Maschler



Game-Theoretic Analysis of a Bankruptcy Problem from the Talmud, *Journal of Economic Theory* (1985).