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Honors Thesis
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Dimension in Graphs

My Honors Thesis Project will focus on structural problems in Graph Theory following on the latest results from the studies and problems on graph dimensions. Most of these problems have not found solution; in all cases the structural properties of graphs have been simplified in order to obtain relationships between graph dimensions. Finding new properties for-a general, not limited graph is posed for the future. There are three graph dimensions that relate to one-another in interesting and counter-intuitive ways:

- The Vapnik-Chernovenkis (VC) Dimension
- The Testing Dimension
- Maximum Average Degree (MAD) of Graphs

The main subtopic where Prof. Tamon and I started to work was the Testing dimension. This was related to the VC dimension and to the MAD of graphs. The VC dimension of graphs was investigated earlier by Anthony, Brightwell, Cooper, and Kranakis. There is more work related to the VC dimensions than to the testing dimension. This is related to the fact that there are still opened questions related to the testing dimension of graphs. As of now, there is no clear way to synthesize a minimal graph with a testing dimension of 3 or more. There is an abundance of conjectures offered for this problem and other ones related to it that offer substantial work for research.

Focusing on the VC dimension and its applications in the light of Graph Theory produces structural results mainly through characterization problems and computational questions as the speed of computing dimensions. Therefore, for my project I intend to understand fully the latest results about the VC, Testing dimensions and the MAD since the three are related to one another in many theorems. Parallel to obtaining knowledge on these three dimensions and the way they relate to one another, I intend to start experimenting with structural problems on graphs that should satisfy some restrictions. As an example, Prof. Tamon and I have started to look at how the conditions of connectivity and minimum number of vertices and edges affect and limit the structure of a graph.

The reason I am interested in these problems on VC, testing dimensions and MAD is that although having an appearance of purely theoretical, there are interesting applications of structural results and relations between the dimensions for random graphs in statistical learning theory. The VC dimension, in particular, can be used to provide bounds on the complexity of a learning problem on graphs. The applications of the VC dimension can also be found in Machine learning (Pattern Recognition is a very interesting sub field) and Computational Geometry typically in infinite/uncountable domains. It is worthy to note that the origin al definitions and applications of the VC dimension have been in finite/countable domains, a natural setting for graphs. Although there is no way to predict the course of the research, I intend to parallel the reading with concrete experimentation with the concepts mastered in order to learn on a need-to-know basis. I believe this is the best approach for me in order to pursue results in a limited time frame.