Persistent Heat Signature for Pose-oblivious Matching of Incomplete Models

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Problem

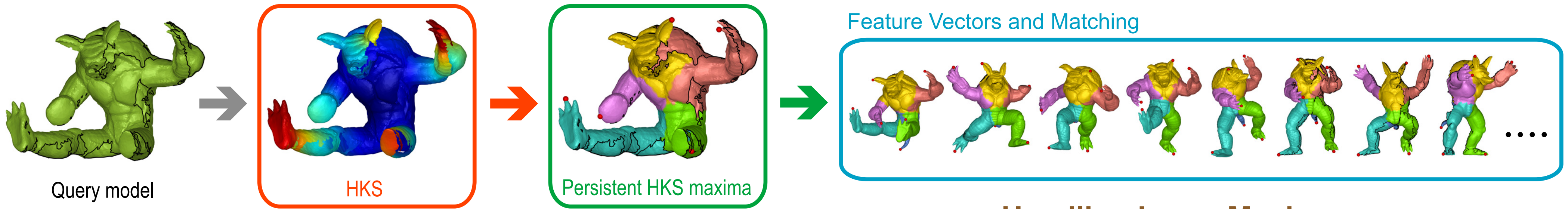
Shape matching is hard in a database that contains **partial** and **incomplete** models with **pose variations**.

Our Contribution

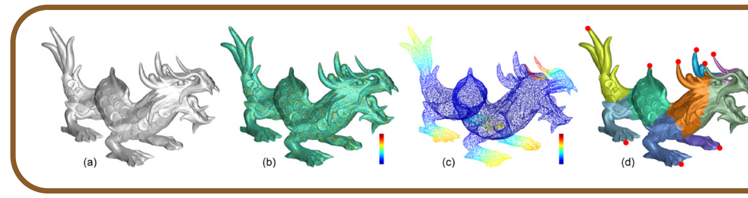
An efficient pose-oblivious shape matching algorithm for all models including partial, incomplete, and complete ones.

Algorithm

Heat Kernel Signature (HKS) and Persistent Homology



Handling Large Meshes

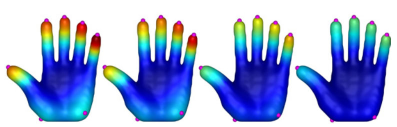


Heat Diffusion

Let M be a manifold. The heat diffusion is governed by

$$\Delta u(x,t) = \frac{-\partial u(x,t)}{\partial t}$$

where $u(x,t)$ denotes the heat at a point x in M at time t, and Δ is the **Laplace operator** of M.



Heat Operator

Let $f: M \to R$ be the initial heat on M, the heat operator $\mathbf{H}_t$ performed on f gives the heat distribution at time t. That is, $\mathbf{H}_t f = u(.,t)$, if $u(.,0) = f$. $\mathbf{H}_t f$ has the following form:

$$\mathbf{H}_t f(x) = \int_M \mathbf{h}_t(x,y) f(y)$$

where $\mathbf{h}_t : M \times M \to R$ is the **heat kernel** function.

Heat Kernel

◊ For two points x, y in M, heat kernel $\mathbf{h}_t(x,y)$ measures the heat that passes from y to x within time t.

$$\mathbf{h}_t(x,y) = \sum \rho_i \varphi_i(x) \varphi_i(y)$$

where ρ_i and φ_i are eigenvalues and eigenfunctions of the heat operator $\mathbf{H}_t$, i.e., $\mathbf{H}_t \varphi_i = \rho_i \varphi_i$

◊ Since $\mathbf{H}_t = e^{-t\Delta}$, thus $\rho_i = e^{-t\lambda_i}$, where $\Delta\varphi_i = \lambda_i \varphi_i$. So

$$\mathbf{h}_t(x,y) = \sum e^{-t\lambda_i} \varphi_i(x) \varphi_i(y)$$

Heat Kernel Signature (HKS)

$$HKS_t(x) = \mathbf{h}_t(x,x) = \sum e^{-t\lambda_i} \varphi_i(x)^2$$

$HKS_t(x)$ measures how much heat is left at time t at x if unit heat is placed at x at t=0. This is deterimied by the intrinsic geometry of M.

HKS as Shape Descriptor

◊ Invariant to isometric deformations, not sensitive to noise.

◊ **Multi-scale**: small t characterizes small local features of M, while large t describes global features of M.

◊ **Informative**: the family of HKS uniquely decides M up to isometry.

HKS Maxima

◊ The heat diffusion is governed by the intrinsic geometry of M: Heat tend to diffuse slower at points with positive curvature and faster with negative curvature.

◊ HKS **Maxima** correspond to protrusions of a shape, we thus use HKS maxima as candidates of feature points.

Feature Vectors and Matching

We use top 15 persistent HKS maxima as feature points of M. That is, the **region merging** stops when there are 15 regions left.

Feature Vector

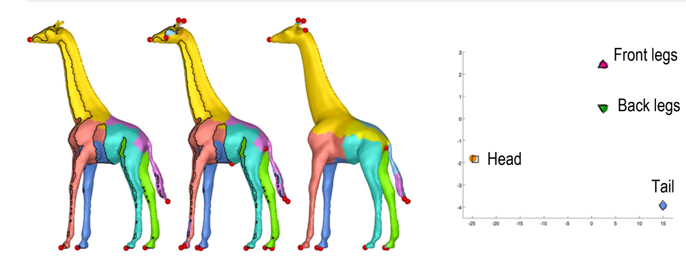
We use 15 time scales to compute a 15D feature vector at each feature point. The times are $t = \alpha\tau$, where τ is the time unit and α=5, 20, 40, 60, 100, 150, 200, 300, 400, 500, 600, 700, 800, 900, 1000.

Matching

◊ Let F_1 and F_2 be the set of feature vectors computed for two models M_1 and M_2. Then their **matching score** is

$$\sum_{f_1 \text{ in } F_1} \min_{f_2 \text{ in } F_2} \|f_1-f_2\| + \sum_{f_2 \text{ in } F_2} \min_{f_1 \text{ in } F_1} \|f_1-f_2\|$$

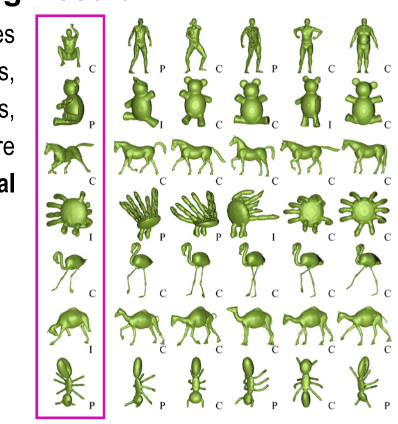
◊ M_1 is matched with M_2 in the database if their matching score is smaller than M_1 with other models in the database.



Experiments

Database and Matching Result

We build a database of shapes divided into different classes (dogs, horses, airplanes, chairs, humans, etc.), where each class has more than one shape including **partial** and **incomplete** versions.



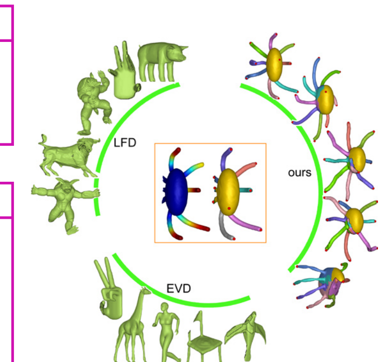
Comparison with EVD and LFD

Hit rate from a database of 300 models

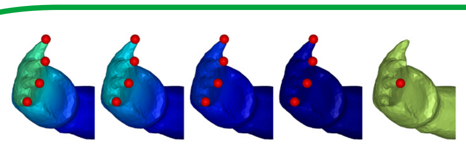
#query models	ours	EVD	LFD
32 incomplete	91%	62%	59%
18 complete	83%	100%	39%
50 total	88%	76%	52%

Timing on 2.4G CPU and 4G RAM

Model	#v	ours	EVD	LFD
Plier	4.8k	9.4	7.9	0.7
Hand	8.7k	19.8	8.1	1.0
Octopus	11.0k	25.2	8.5	1.3
Teddy	12.6k	30.7	8.6	1.4
Human	15.2k	42.8	8.9	1.5
Dragon	1000k	712.0	28.0	40.0
Retrieval	time	0.2	0.006	36.0



Persistent HKS Maxima



◊ A **region** is set of triangles of M forming a disk.

◊ Define the **persistence** of a region R as

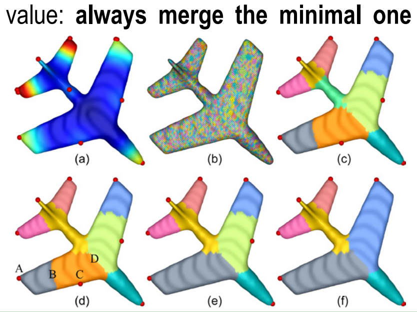
$$pr(R) = HKS(\sigma) - HKS(e)$$

where σ and e are R's central triangle and its paired edge.

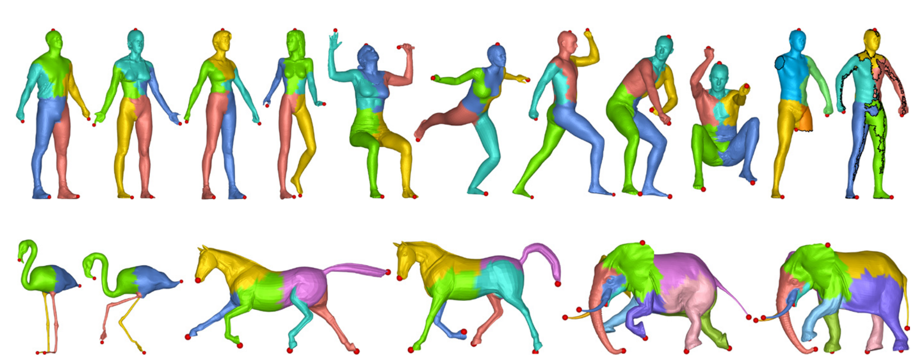
◊ For the matching purpose, we use **persistent homology** to select **feature points** as a subset of HKS maxima that have large persistence values.

Region Merging: $O(n \lg n)$

◊ Starting from initial mesh of M where each triangle is a region, we merge the regions iteratively based on their persistence value: **always merge the minimal one** at each step.



More Examples on Persistent HKS Maxima



References

1 Sun, J., Ovsjanikov, M., Guibas, L. J. 2009. A Concise and Provably Informative Multi-scale Signature based on Heat Diffusion. *Comput. Graphics Forum* 28, 5, 1383-1392 **(HKS)**

2 Edelsbrunner, H., Letscher, D., Zomorodian, A. 2002. Topological Persistence and Simplification. *Discrete Comput. Geom.* 28, 511-533 **(Persistence Homology)**

3 Jain, V., Zhang, H. 2007. A Spectral Approach to Shape-based Retrieval of Articulated 3D Models. *Comput. Aided Design* 39, 5, 398-407 **(EVD)**

4 Chen, D.-Y., Tian, X. P., Shen, Y.-T., Ouhyoung, M. 2003, On Visual Similarity based 3D Model Retrieval, *Comput. Graphics Forum* 22, 3, 223-232. **(LFD)**