Multi-Label Answer Aggregation based on Joint Matrix Factorization

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Abstract—Crowdsourcing is a useful and economic approach to data annotation. To obtain annotation of high quality, various aggregation approaches have been developed, which take into account different factors that impact the quality of aggregated answers. However, existing methods generally focus on single-label (multi-class and binary) tasks, and they ignore the intercorrelation between labels, and thus may have compromised quality. In this paper, we introduce a Multi-Label answer aggregation approach based on Joint Matrix Factorization (ML-JMF). ML-JMF selectively and jointly factorizes the sample-label association matrices collected from different annotators into products of individual and shared low-rank matrices. As such, it takes advantage of the robustness of low-rank matrix approximation to noise, and reduces the impact of unreliable annotators by assigning small (zero) weights to their annotation matrices. In addition, it takes advantage of the correlation among labels by leveraging the shared low-rank matrix, and of the similarity between annotators using the individual low-rank matrices to guide the factorization. ML-JMF pursues the low-rank matrices via a unified objective function, and introduces an iterative technique to optimize it. ML-JMF finally uses the optimized low-rank matrices and weights to infer the ground-truth labels. Our experimental results on multi-label datasets show that ML-JMF outperforms competitive methods in inferring ground truth labels. Our approach can identify unreliable annotators, and is robust against their misleading answers through the assignment of small (zero) weights to their annotation.

Index Terms—Crowdsourcing, Multi-Label Learning, Joint Matrix Factorization, Spammers

I. INTRODUCTION

With the emergence of the Internet of Things, a large amount of unlabeled data can be easily and cheaply collected. However, annotating such a vast amount of unlabeled data is a difficult challenge, because annotating data with correct and complete labels is time-consuming and often impractical, generally requiring expert knowledge. Crowdsourcing [1] provides an effective and economic solution to collect labels for data from non-expert workers in the open Internet. Label quality of training data plays a crucial role for the performance of machine learning algorithms, and high-quality labels contribute to reliable performance. Due to significant differences among the crowders (or workers) in their knowledge levels, dedications, and evaluation criteria when crowdsourcing, the quality of crowdsourced labels (answers) may be quite different [2], [3]. Furthermore, some workers may simply submit random answers as a mean to earn easy money. Therefore, how to aggregate high-quality answers is a key pursuit in crowdsourcing [4].

To aggregate high-quality answers, one typical solution is repeated labeling, which involves the annotation of the same samples by different workers. Preliminary studies [5] show that the quality of answers can be improved to some degree by integrating repeated labels. Label integration is accomplished by a ground-truth answer inference algorithm on crowdsourced labels, without knowing the features of samples. Many researchers worked on the development of methods to derive high-quality answers from different perspectives, such as the reliability [6], intention [7], difficulty of samples [8], bias of workers [9], and so on.

The aforementioned answer aggregation methods focus on single-label tasks, in which a worker is expected to assign a single label to each sample. However, for many real-world crowdsourcing applications (i.e., image annotations and medical diagnosis [10], [11]), it’s common for a sample to be simultaneously associated with several labels. In other words, workers are expected to provide a set of relevant labels for each sample. Assigning several labels to each sample increases the level of noise and bias of crowdsourced labels. In addition, workers no longer either completely agree or disagree on the crowdsourced labels, and thus the consensus becomes partial. Consequently, it’s difficult to assess the reliability of workers, since they may provide partially correct and incorrect answers at the same time [12]. Furthermore, the number of possible label combinations is affected by a combinatorial explosion in the case of multi-label samples. For these reasons, multi-label answer aggregation is intrinsically more challenging than its single-label counterpart. One simple solution is to independently treat each label and transform the task into multiple binary tasks. Such binary solutions completely ignore the correlation between labels, whose appropriate usage can significantly improve the performance of multi-label learning [13], [14].

To the best of our knowledge, the problem of multi-label answer aggregation in crowdsourcing remains a largely
unexplored topic [12], [15]. In this paper, we propose a Multi-
Label answer aggregation approach based on Joint Matrix
Factorization (ML-JMF). To take advantage of the robustness
of low-rank matrix factorization to noise [16], [17], ML-
JMF jointly factorizes the sample-label association matrices
of respective workers into a set of low-rank matrices for indi-
vidual workers and a shared low-rank matrix for labels. ML-
JMF assigns different weights to these association matrices to
further reduce the impact of low quality workers. In addition,
ML-JMF defines a term on the shared matrix to employ the
correlation between labels and another term on the individual
matrices to employ the similarity between workers. These two
terms and the objective of matrix factorization are integrated
into a unified objective function to guide the factorization.
We introduce an iterative solution to optimize the weights,
individual low-rank matrices of workers, and the shared low-
rank matrix. In the end, ML-JMF uses these weights and
optimized low-rank matrices to infer the ground-truth labels.

The main contributions of this paper are summarized as
follows:
(1) Our proposed ML-JMF can simultaneously take into ac-
count the quality of workers, the noise of crowdsourced labels,
correlations between labels, and connections between workers
for multi-label answer aggregation.
(2) We introduce an iterative technique to optimize the weights
assigned to workers and to pursue the joint matrix factoriza-
tion.
(3) Our empirical study on benchmark datasets shows that ML-
JMF outperforms state-of-the-art competitive methods [18]–
[20] for answer aggregation by up to 95% in accuracy, while
being robust against spammers. In addition, it can automati-
cally identify low quality workers.

The remainder of this paper is organized as follows. We
briefly review related work in Section II and then elaborate
on the proposed algorithm and its optimization in Section III
Section IV provides the experimental results and analysis, and
Section VI gives the conclusions and future work.

II. RELATED WORK

The simplest and most efficient answer aggregation method
is majority voting (MV) [21]. MV works very well under
two prerequisites: 1) the overall accuracy of most workers
is larger than 50% in binary labeling tasks, and 2) the error
of each worker is uniformly distributed over all class labels.
However, these prerequisites do not hold in complicated real-
world applications. Due to the lack of expert knowledge, most
workers tend to make shallow answers using common sense
or simply repeat what others say.

Besides the straightforward MV, researchers are dedicated
to many other aggregation solutions from different perspec-
tives [22]. To name a few, Dawid and Skene [23] applied
expectation maximization (EM) to model the confusion matrix
of each worker and to conduct aggregation from a set of
noisy labels. This EM based aggregation algorithm iteratively
estimates the labels that are most likely true classes, and then
uses these labels to estimate the error rate of each worker
and the label distribution. Raykar et al. [24] assumed that
annotators have biases toward the positive class and negative
class, and introduced a Bayesian approach by adding a specific
prior for each class. Whitehill et al. [25] proposed GLAD
(Generative model of Labels, Abilities, and Difficulties) to
model both the expertise levels of workers and the difficulties
of samples using EM. GLAD treats the probability of a sample
being positive as a latent variable, and it can produce high
quality results even with many adversarial labelers. Zhang
et al. [26] proposed a Positive Label frequency Threshold
(PLAT) algorithm to solve the imbalanced labeling problem
caused by the bias of workers via dynamically adjusting the
threshold to determine the class membership of an example
[26]. Zhang et al. [20] introduced adaptive weighted majority
voting (AWMV) to utilize the frequency of positive labels in
the multiple noisy label sets of each example to estimate a
bias rate, and then to assign weights derived from the bias
rate to negative and positive labels.

Some researchers have focused on worker behavior or task
assignment to improve label quality. Demartini et al. [6] as-
sumed that workers act independently and aggregated labels by
solving a maximum likelihood estimation problem. Raykar and
Yu [27] developed an empirical Bayesian algorithm based on
EM to iteratively estimate the ground-truth label and eliminate
spammers. Karger et al. [28] proposed a belief propagation
model to decide which tasks to assign to which workers. This
belief model uses task messages to iteratively update worker
messages, and vice versa. Next, the true classes are estimated
from the information contained in the task messages. Ho and
Vaughan [29] developed a two-phase exploration-exploitation
algorithm for assigning heterogeneous items to workers with
different qualities. Wang et al. [30] proposed an approach to
obtain high-quality labels from the crowds by distinguishing
easy and hard items prior to assigning them to workers.

The multi-label answer aggregation problem has been much
less explored [12], [31] than single-label aggregation solutions.
Nowak et al. [31] studied inter-annotator agreement for multi-
label image annotation and found that using the majority vote
strategy to generate one annotation set from several annotation
sets can filter out noisy judgments of non-experts to some
extent. To address the problem of different taxonomies being
used in a multi-label domain, Duan et al. [15] proposed
a probabilistic cascaded method called cascaded estimation
with Dawid-Skene (C-DS). C-DS maps label sets in a source
taxonomy to label sets in a target taxonomy in terms of
the semantic distance between them. Yoshimura et al. [19]
incorporated GLAD [25] into RAiEL (RAandom k-labELsets)
[32] and proposed RAiEL-GLAD to balance the estimation
accuracy and computational complexity in multi-label answer
aggregation. Hung et al. [12] extended the clustering based
Bayesian combination of classifiers method [33] for multi-
label answer aggregation. This extended solution additionally
models the co-occurrence dependency between labels by latent
label clusters and the partial consensus between workers by
grouping workers with similar answers.

The aforementioned single-label aggregation approaches ig-
nore the interdependence between labels; some of them cannot perform as well as on binary setting, while some other may fail to adapt to multi-label scenarios [34]. On the other hand, multi-label aggregation methods do not differentiate among different types of workers, they do not account for potential noisy annotations and the different biases of individual workers. It’s recognized that both label correlation and the types of workers contribute to answer aggregation [19], [35]. Given these observations, we propose an approach called ML-JMF to simultaneously account for label correlations, noisy labels, and quality of individual workers. ML-JMF can differentiate the quality of workers by assigning different weights to their annotations matrices, and reduce noise through low-rank matrix factorization. It further exploits correlation between labels and the similarity between workers to guide the low-rank matrix and weight optimization. Our empirical study shows that ML-JMF achieves superior aggregated labels than other inference algorithms [18]–[20]. ML-JMF can also identify spammers and can selectively aggregate annotations of workers.

### III. PROBLEM FORMULATION

In this section, we first discuss an image annotation task to illustrate the intrinsic challenges of multi-label answer aggregation. Then, we elaborate on ML-JMF and its optimization.

#### A. Motivation

Table I lists the crowdsourced labels of four images (i1 - i4) provided by five workers (w1 - w5). For simplicity, these labels are denoted with numbers from 1 to 5. In particular, ‘-’ denotes the fact that the worker thinks the image should not be annotated with the corresponding label.

<table>
<thead>
<tr>
<th>w1</th>
<th>w2</th>
<th>w3</th>
<th>w4</th>
<th>w5</th>
<th>ground truth</th>
<th>Majority Voting</th>
</tr>
</thead>
<tbody>
<tr>
<td>i1</td>
<td>2,3,4</td>
<td>2,3</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>(2)</td>
</tr>
<tr>
<td>i2</td>
<td>3,4</td>
<td>2,3,4</td>
<td>2</td>
<td>3</td>
<td>1,3,4</td>
<td>(3,4)</td>
</tr>
<tr>
<td>i3</td>
<td>3,5</td>
<td>-1,4</td>
<td>3</td>
<td>4,5</td>
<td>4,5</td>
<td>(4)</td>
</tr>
<tr>
<td>i4</td>
<td>(1,2,3)</td>
<td>3,4</td>
<td>5</td>
<td>3</td>
<td>(2,3,4)</td>
<td>(3)</td>
</tr>
</tbody>
</table>

1: grass; 2: lion; 3: sun; 4: tree; 5: river

A straightforward and widely adopted approach to derive aggregated labels is majority voting (MV) [18], [31], which separately considers the five labels. If the number of ‘votes’ for a particular label of a sample from all workers is the largest (or greater than half workers), this label is included in the aggregated label set. Considering ground truth labels (in practice, often unknown), we have two observations: the aggregated results obtained using MV are (i) partially incorrect (e.g., label 3 should not be assigned to i1); and (ii) partially incomplete (e.g., labels 1 and 4 should also be assigned to i4).

This is due to the fact that MV considers all answers as equally important and MV ignores the correlation between labels. In other words, MV assumes that all workers have similar biases and produce answers of equal quality. But in practice, they don’t. Kazai et al. [35] categorized workers into five groups: (i) Diligent workers (reliable workers) take care of their tasks and may be characterized by a high ratio of useful labels; (ii) Normal workers have general knowledge to give correct answers, but make mistakes occasionally; (iii) Sloppy workers care little about the quality of their work, they may still provide a high fraction of useful labels but with low accuracy; (iv) Incompetent workers lack professional skills or competence, resulting in low accuracy; (v) Spammers may come in different shapes and forms, e.g., they give the same answer to all questions or give random answers. Given the data in Table II, w5 might be a reliable worker who assigns correct labels; w1 and w2 may be normal workers who can give some correct answers; and w3 and w4 are spammers. Unlike single-label data, labels of multi-label samples are correlated. For example, we can see that labels 3 and 4 are often assigned to the same images.

From the illustrative example given in Table II we can conclude that: (i) if the spammers (w3 and w4) are removed, the aggregated results for i1 and i3 will be correct; (ii) label 4 can be assigned to the same image already tagged with label 3, thus image i4 can be annotated correctly with its ground truth labels. This illustrates that both quality of workers and correlation among labels should be considered in multi-label answer aggregation.

#### B. The Proposed Algorithm

Suppose there are m workers providing labels for n samples, each of which can be annotated with one or more labels, and \( \mathcal{L} = \{1, \ldots, c\} \) is the label set. Each collection of labels provided by a worker as annotation for a sample is a subset of \( \mathcal{L} \). Thus, each worker provides a sample-label association matrix for n samples and c distinct labels as follows:

\[
A_{w} \triangleq \begin{pmatrix}
    a_{11}^{w} & \cdots & a_{1c}^{w} \\
    \vdots & \ddots & \vdots \\
    a_{nc}^{w} & \cdots & a_{nc}^{w}
\end{pmatrix}
\]

where \( a_{il}^{w} \in \{-1,0,1\} \), \( a_{il}^{w} = 1(-1) \) states that the \( w \)-th worker annotated the \( i \)-th sample with (or without) the \( l \)-th (1 \( \leq l \leq c \)) label, and \( a_{il}^{w} = 0 \) means that the worker did not provide an answer for the corresponding label and sample.

The feature information of the \( n \) samples may often be shielded from the inference algorithm due to privacy issues [6]. To obtain high-quality aggregated labels from \( \{A_{w}\}_{w=1}^{m} \), we advocate the use of the shared information of workers, and also the intrinsic characteristics of individual workers. To this end, and motivated by the robustness to noise of a low-rank approximation of a matrix [17], [36], we jointly factorize \( \{A_{w}\}_{w=1}^{m} \) as follows:

\[
\min_{U,V>0} \sum_{w=1}^{m} \mu_{w} \left\| A_{w} - U_{w}SV^{T} \right\|_{F}^{2}
\]

s.t. \( \sum_{w=1}^{m} \mu_{w} = 1, \mu_{w} \geq 0 \)

where \( \left\| \cdot \right\|_{F}^{2} \) is the Frobenius norm, \( U_{w} \in \mathbb{R}^{n \times k} \) and \( V \in \mathbb{R}^{c \times k} \) are the individual matrix for the \( w \)-th worker and the shared low-rank matrix for \( c \) labels across \( m \) workers;
Thus, we define a regularization on individual matrices $U_w$ to take advantage of worker profiles as follows:

$$\min_{U_w \geq 0} \frac{1}{2} \sum_{w \neq p} R_{wp} \| U_w - U_p \|_F^2$$

$$= \sum_{w \neq p} R_{wp} \text{tr}((U_w - U_p)^T (U_w - U_p))$$

$$R_{wp} = \frac{\text{tr}(\tilde{A}_w \tilde{A}_p)}{\sqrt{\text{tr}(A_w A_w^T) \text{tr}(A_p A_p^T)}}$$

(5)

where $R_{wp}$ is the similarity between worker $w$ and worker $p$; it’s measured using the modified RV-coefficient [38]. The modified RV-coefficient ($R_{wp}$) is suggested to measure the common information of high-dimensional data matrices; it can probe the similarity between pairs of datasets (or data matrices) in a simple and comprehensive way [39]. The value of $R_{wp}$ is between 0 and 1: the larger the value, the larger the similarity between the two workers is. $R_{wp}$ can also be computed via Pearson correlation or cosine similarity. Our investigation shows that ML-JMF combined with the RV-coefficient to estimate worker similarity can achieve a better accuracy than ML-JMF combined with cosine similarity or Pearson correlation.

We combine the constraints on $V$ and $U_w$ with the joint matrix factorization in Eq. (2), and form the objective function of ML-JMF as follows:

$$\Phi(U_w, S, V, \mu) = \arg \min_{U,V>0} \sum_{w=1}^{m} \mu_w \left\| A_w - U_w S V^T \right\|_F^2$$

$$+ \lambda \| \mu \|_F^2 + \alpha \text{tr}(V^T L V)$$

$$+ \beta \sum_{w \neq p} R_{wp} \text{tr}((U_w - U_p)(U_w - U_p)^T)$$

(6)

where the parameters $\alpha$ and $\beta$ weight the constraints in Eq. (4) and Eq. (5), respectively. Besides the joint matrix factorization, we employ two constraints to guide the pursue of $U_w$, $V$, and $\mu_w$, and thus to improve the accuracy and reliability of multi-label answer aggregation. The experiments confirm the advantage of using these two constraints.

After optimizing $\mu_w$, $U_w$, $S$, and $V$, ML-JMF selectively aggregates the annotation matrices of $m$ workers as follows:

$$A^* = \sum_{w=1}^{m} \mu_w U_w S V$$

(7)

The inferred sample-label association matrix $A^*$ not only can reduce the impact of too noisy annotation matrices by assigning $\mu_w = 0$ to them, but also removes partially noisy annotations in $A_w$ of selected workers by low-rank matrix approximation.

C. Optimization

The proposed objective function of ML-JMF in Eq. (6) is not convex for all variables $V$, $\mu_w$, $S$, and $U_w$
1, 2, \cdots, m) at the same time. Therefore, it is unrealistic to expect to find the global optimum simultaneously. Here, we introduce an alternative update strategy to optimize \( V, \mu, S, S \), and \( U_w \). Particularly, we will optimize one variable while fixing the other variables as constants.

1) **Optimizing \( V \)**: By fixing \( U_w, \mu, (\forall w) \), and \( S \), we can optimize \( V \) as follows:

\[
\min \Phi_1(V) = \sum_{w=1}^{m} \mu_w \| A_w - U_w SV^T \|_F^2 + \alpha \text{tr}(V^T LV) \tag{8}
\]

subject to \( V \geq 0 \)

The derivative of \( \Phi_1(V) \) with respect to \( V \) is

\[
\frac{\partial \Phi_1}{\partial V} = \sum_{w=1}^{m} \mu_w (2V^T U_w^T U_w S - 2A_w^T U_w SV^T) + 2\alpha LV \tag{9}
\]

Using the Karush-Kuhn-Tucker (KKT) complementary condition \([40]\) for the nonnegativity of \( V \), we can obtain:

\[
\left( \sum_{w=1}^{m} \mu_w (V S^T U_w^T U_w S - A_w^T U_w SV^T) + \alpha LV \right)_{ij} \geq 0 \tag{10}
\]

Considering \( V \geq 0, S \), and \( L \) may take a positive or negative sign, we decompose them as \( A_w^T U_w S = (A_w^T U_w S)^+ - (A_w^T U_w S)^- \) and \( S V^T U_w - S V^T U_w S = (S V^T U_w S)^+ - (S V^T U_w S)^- \), where the matrices with positive and negative symbols are defined as \( O^+ = \frac{|O| + O}{2} \) and \( O^- = \frac{|O| - O}{2} \). Then, we can obtain the following updating formula for \( V \):

\[
V \leftarrow V + \frac{\sum_{w=1}^{m} \mu_w (A_w^T U_w S)^+ + \sum_{w=1}^{m} \mu_w V (S V^T U_w S)^+ + \alpha L V}{\sum_{w=1}^{m} \mu_w (A_w^T U_w S)^- + \sum_{w=1}^{m} \mu_w V (S V^T U_w S)^- + \alpha L V} \tag{11}
\]

2) **Optimizing \( U_w \)**: Similarly, we can update the \( U_w \), one by one. For a \( U_w \), given \( V, S, \mu, W, \), and \( U_p, p \in \{1, 2, \cdots, m\}, w \neq p \), the objective function for optimizing \( U_w \) is:

\[
\min \Phi_2(U_w) = \mu_w \| A_w - U_w SV^T \|_F^2 + \beta \sum_{w \neq p} R_{w p} \text{tr}((U_w - U_p)(U_w - U_p)^T) \tag{12}
\]

subject to \( U_w \geq 0 \)

The derivative of \( \Phi_2 \) with respect to \( U_w \) is

\[
\frac{\partial \Phi_2}{\partial U_w} = \mu_w (2U_w SV^T VS^T - 2A_w SV^T) + 2\beta \sum_{w \neq p} R_{w p} (U_w - U_p) \tag{13}
\]

Using the KKT complementary condition \([40]\) for the nonnegativity of \( U_w \), we can obtain:

\[
(\mu_w (U_w SV^T VS^T - A_w SV^T) + \beta \sum_{w \neq p} R_{w p} (U_w - U_p))_{ij} (U_w)_{ij} = 0 \tag{14}
\]

Since \( S \) may take any sign, similarly to the computation of Eq. \([10]\), we let \( A_w SV^T = (A_w SV^T)^+ - (A_w SV^T)^- \) and \( SV^T VS^T = (SV^T VS^T)^+ - (SV^T VS^T)^- \). Thus, Eq. \([13]\) leads to the following update formula for \( U_w \):

\[
U_w \leftarrow U_w + \frac{\mu_w (A_w SV^T)^+ + \mu_w (V S V^T)^+ + 2\beta \sum_{w \neq p} R_{w p} U_p}{\mu_w (A_w SV^T)^- + \mu_w (V S V^T)^- + 2\beta \sum_{w \neq p} R_{w p} U_w} \tag{15}
\]

3) **Optimizing \( S \)**: With \( U_w, V, \) and \( \mu \) known, optimizing Eq. \([6]\) with respect to \( S \) is equivalent to optimize

\[
\min \Phi_3(S) = \sum_{w=1}^{m} \mu_w \| A_w - U_w SV^T \|_F^2 \tag{16}
\]

Letting \( \frac{\partial \Phi_3}{\partial S} = 0 \) leads to the following updating formula:

\[
S = (\sum_{w=1}^{m} \mu_w (U_w^T U_w))^{-1} (\sum_{w=1}^{m} \mu_w (U_w^T A_w V) (V^T V)^{-1}) \tag{17}
\]

4) **Optimizing \( \mu \)**: Next, we view \( V, U_w, S \) as known, and define the objective function with respect to \( \mu \) as follows:

\[
\min \Phi_4(\mu) = \sum_{w=1}^{m} \mu_w \| A_w - U_w SV^T \|_F^2 + \lambda \| \mu \|_F^2 \tag{18}
\]

where \( \lambda \geq 0 \) and \( \gamma \geq 0 \) are the introduced Lagrange multipliers for constraints \( \mu_w \geq 0 \) and \( \sum_{w=1}^{m} \mu_w = 1 \). Let \( h_w = \| A_w - U_w SV^T \|_F^2 \), be the approximation loss for \( A_w, \mu \) = \[h_1, h_2, \cdots, h_m\]. The partial derivative of \( \Phi_4(\mu) \) with respect to \( \mu \) is

\[
\frac{\partial \Phi_4}{\partial \mu} = h + 2\lambda \mu - \zeta - \gamma \tag{19}
\]

The optimal \( \mu \) should satisfy the following four conditions \([40]\):

1) Complementary slackness condition: \( \zeta \mu_w = 0 \);
2) Stationary condition: \( h_w + 2\lambda \mu_w - \zeta - \gamma = 0 \);
3) Feasible condition: \( \sum_{w=1}^{m} \mu_w = 1, \mu_w \geq 0 \);
4) Dual feasibility condition: \( \forall \zeta \geq 0 \).

From the stationary condition, \( \mu_w \) can be computed as:

\[
\mu_w = \frac{\zeta_w + \gamma - h_w}{2\lambda} \tag{20}
\]

From Eq. \([18]\), we can see that \( \mu_w \) depends on \( \zeta_w \) and \( \gamma \), both of which can be analyzed via the following cases:

1) if \( \gamma > h_w \), then \( \mu_w > 0 \), because of the complementary slackness \( \zeta_w \mu_w = 0 \) and the dual feasibility \( \forall \zeta \geq 0 \), \( \zeta_w = 0 \) and \( \mu_w = \frac{\gamma - h_w}{2\lambda} \).
2) if \( \gamma = h_w \), because of \( \zeta_w \mu_w = 0 \) and \( \mu_w = \frac{\zeta}{2\lambda}, \zeta_w = 0 \) and \( \mu_w = 0 \).
3) if \( \gamma < h_w \), since \( \mu_w \geq 0 \), it requires \( \zeta_w > 0 \); because \( \zeta_w \mu_w = 0 \), then \( \mu_w = 0 \).

From the above analysis, we can set \( \mu_w \) as:

\[
\mu_w = \frac{\gamma - h_w}{2\lambda}, \quad \gamma > h_w
\]

\[
\mu_w = 0, \quad \gamma \leq h_w \tag{21}
\]

Suppose \( \overrightarrow{h} \) stores the entries of \( h \) in ascending order. For a predefined \( \lambda \) not too large, there exists \( q \in \{1, 2, \cdots, m\} \) with \( \overrightarrow{h}_q \leq \gamma \) and \( \overrightarrow{h}_{q+1} \geq \gamma \), satisfying \( \sum_{w=1}^{q} \frac{\gamma - \overrightarrow{h}_w}{2\lambda} = 1 \). Then \( \mu_w \) has the following explicit solution:

\[
\mu_w = \frac{\gamma - \overrightarrow{h}_w}{2\lambda}, \quad w < q
\]

\[
\mu_w = 0, \quad w > q \tag{22}
\]

From \( \sum_{w=1}^{m} \mu_w = \sum_{w=1}^{q} \frac{\gamma - \overrightarrow{h}_w}{2\lambda} = 1 \), we can get the value for \( \gamma \) as:
From the solution of $\mu$, we find that, if $\overrightarrow{H}_{r} > \overrightarrow{H}_{p}$ and $\gamma \geq \overrightarrow{H}_{r}$, the $p$-th worker will get a larger weight than the $r$-th worker. This is the case because the $r$-th worker may provide noisy annotations, which are inconsistent with other workers, and therefore resulting in a large approximation loss. Therefore, adding an $l_2$ norm to $\mu$ in Eq. (2) can not only remove noisy (irrelevant) answer matrices by assigning zero weights to them, but also can reduce the impact of partially noisy annotation matrices by crediting reduced weights to them. In addition, because of the robustness of low-rank matrix approximation to noises [17], [36], [41], the joint matrix factorization can also remove noisy annotations, and thus further improve the quality of aggregated labels.

From Eq. (22) and Eq. (23), we see that if $\lambda$ is set to a very small positive value, $\gamma \approx \sum_{q=1}^{\lambda} \overrightarrow{H}_{q}/q$, and then ML-JMF will select at least one annotation matrix. On the other hand, if $\lambda$ is fixed to a very large value, then all the annotation matrices will be used and credited nearly equal weights. To find a value of $q$ that satisfies Eq. (22), we decrease $q$ from $m$ to 1, step by step, and specify the search procedure in Algorithm 1. The whole ML-JMF approach is summarized in Algorithm 2.

**Algorithm 1** A method to seek $q$ and compute $\mu_{w}$

**Input:** Sorted $\overrightarrow{H}_{w}$, $w \in \{1, 2, \ldots, m\}$ in ascending order, $\lambda$

**Output:** $q$, $\mu_{w}$

1. Initialize $q = m$, $\gamma = 0$.
2. While $q > 0$ do
3. $\gamma = \frac{\lambda + \sum_{q=1}^{q} \overrightarrow{H}_{w}}{q}$
4. If $\gamma - \overrightarrow{H}_{q} > 0$ then
5. $q \leftarrow q - 1$.
6. Else
7. $q \leftarrow q - 1$.
8. End If
9. $\mu_{w} \leftarrow \frac{\overrightarrow{H}_{w}}{q}$, for $w = 1, \ldots, q$.
10. $\mu_{w} \leftarrow 0$, for $w = q + 1, \ldots, m$.
11. \textbf{end while}

**Algorithm 2** ML-JMF: Multi-label Answer Aggregation based on Joint Matrix Factorization

**Input:**

$\{A_{w}\}_{w=1}^{m}$: Annotation matrices of $m$ workers;
$\alpha$, $\beta$, $\gamma$: input parameters of ML-JMF;
$tol$: tolerance threshold for iterative optimization;
$maxIter$: maximum number of iterations.

**Output:**

$\mu$: weights assigned to $m$ workers;
$A^*$: the aggregated sample-label association matrix;
1. Initialize $U_{w}$, $V$, and $\mu$ in random;
2. Compute $R$ via Eq. (3) and $C$ via cosine similarity;
3. Compute the initial value of $\Phi^t(U_{w}, V, \mu)$ via Eq. (6);
4. $t = 0$, $\Phi^{0} = 0$;
5. While $|\Phi^{t} - \Phi^{t+1}| > tol$ and $t < maxIter$
6. $t \leftarrow t + 1$;
7. Update the matrix $U_{w}, V$ via Eq. (15), Eq. (11), Eq. (17), respectively;
8. Update $\mu$ using Algorithm 1;
9. Compute the value of $\Phi^{t} = (\Phi_{w} U_{w}, V, \mu)$;
10. \textbf{end while}
11. Return the aggregated label matrix $A^*$ via Eq. (7)

**Datasets:** To study the performance of ML-JMF in aggregating crowdsourced labels of multi-label samples, we carry out experiments on five real-world datasets. The statistics of these datasets are listed in Table IV. Movie is a movie category classification dataset used in [19]. The other four real-world datasets were used by Duan et al. [42] in emotion classification. The candidate label sets are taken from the Ekman’s taxonomy [43] and the Nakamura’s taxonomy [44].

**Comparing Algorithms:** We compare ML-JMF against two state-of-the-art multi-label methods RAEL-GLAD [19] and C-DS [15], the classical MV [18], and two representative single-label methods AWMV [20] and PLAT [26] (all discussed in the related work Section). RAEL-GLAD has two parameters: $k$ (number of labels in a label subset) and $M$ (number of random label subsets); we set $k = 2$ and $M = c \times (c-1)/2$ for the experiments. To facilitate the comparison with AWMV and PLAT, we decompose the multi-label answer aggregation problem into multiple binary-label aggregation problems. For example, the “Apple” Ekman dataset has six labels, AWMV is separately applied on each label and each label has 2340 tasks (30 workers/Instance $\times 78$ Instances). In addition, we introduce ML-JMF(A), a variant of ML-JMF, which uses the optimized weights, but also the original annotation matrices to infer labels; namely, $A^* = \sum_{w=1}^{m} \mu_{w} A_{w}$. The input parameters of MV, AWMV and PLAT, and C-DS are specified or optimized as the authors suggested in their code or papers. ML-JMF and its variant set $\alpha = 10^{3}$, $\beta = 0.01$, $\gamma = 10^{4}$, and $k = \lfloor c/2 \rfloor + 1$. The parameter sensitivity analysis for ML-JMF is also presented.

**Evaluation Metrics:** In multi-label answer aggregation, results can be partially correct. We therefore rely on the set-based definition of Accuracy to evaluate the individual correctness on $n$ samples. The accuracy is defined as follows [12].

$$\text{Accuracy} = \frac{1}{n} \sum_{i=1}^{n} \frac{|T_{i} \cap T_{i}^*|}{|T_{i}|}$$

where $T_{i}$ and $T_{i}^*$ are the set of true labels and the set of aggregated labels of the $i$-th sample, respectively.

We also use the RankingLoss, a representative evaluation metric in multi-label learning, to evaluate the average fraction of label pairs that are not correctly ranked for the sample. The formal definition of RankingLoss is:

$$\text{RankingLoss} = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{c} \sum_{c_{1}=1}^{c} \frac{|R_{i}|}{|T_{i}||T_{i}^*|}$$

where $R_{i} = \{(c_{1}, c_{2}) \in T_{i} \times T_{i}^*: A^*(i, c_{1}) \leq A^*(i, c_{2})\}$, $T_{i} \in L$ is the set of labels associated with the $i$-th sample, $T_{i}$ is the complementary set of $T_{i}$ in $L$, $c_{1} \in L$, $c_{2} \in T_{i}$ are the relevant labels and irrelevant labels, respectively. The smaller the RankingLoss is, the better the performance is. The performance is perfect when the RankingLoss is zero [13].

To be consistent with Accuracy, we report the 1-RankingLoss in the following experiments. The initially obtained aggregated labels are expressed as real-numbers and need to be converted into binary labels for computing the Accuracy. In the experiments, we choose the labels with the highest probabilities as the aggregated labels of the sample.
according to the number of ground truth labels per sample for all methods. For example, if the $i$-th sample has two true labels, all the methods consider the two labels with the highest values of their respective label likelihood vectors as the aggregated labels. The RankingLoss directly uses the initially aggregated labels without such conversion.

### A. Results of Multi-Label Answer Aggregation

Table III shows the results of different answer aggregation methods on five real-world datasets. Since ML-JMF initializes the matrices $U_w$, $S$, and $V$ randomly, we independently run ML-JMF ten times and report the average results and variance. The five comparing methods are deterministic.

From Table III, we can clearly see that ML-JMF generally outperforms the comparing methods on different datasets. Both RA$k$EL-GLAD and ML-JMF consider label correlation of multi-label samples while C-DS not. RA$k$EL-GLAD generally obtains a better Accuracy than C-DS, but they all lose to ML-JMF. This is because ML-JMF takes into account the quality variance of workers and reduces the impact of noisy annotations via matrix factorization. This result corroborates the fact that the quality of workers should be considered when aggregating crowdsourcing labels. ML-JMF(A) assigns different weights to $A_w$; it outperforms MV, but always loses to ML-JMF. The achieved performance margin between ML-JMF and ML-JMF(A) provides support to the robustness of a low-rank matrix approximation, corroborating its use.

MV, PLAT, and AWMV convert the multi-label answer aggregation problem into multiple single-label problems. They ignore the correlation between labels; as such they are outperformed by ML-JMF and RA$k$EL-GLAD, which take advantage of label correlations. This comparison suggests that label correlations should be considered in multi-label answer aggregation. Although AWMV and PLAT are signal-label methods, AWMV achieves a better performance than PLAT. A possible reason is that AWMV assigns different weights to different types of labels. We also report the 1-RankingLoss of ML-JMF, RA$k$EL-GLAD, and C-DS in Table IV. We can see that ML-JMF generally has a larger 1-RankingLoss than these two comparing methods and ML-JMF(A).

In summary, these experimental results not only prove the effectiveness of ML-JMF in aggregating labels of multi-label samples in crowdsourcing, but also confirms that both label correlation and quality of workers should be considered in fusing crowdsourcing labels.

### B. Component Analysis of $\mu$

To account for the quality variance of different workers, ML-JMF attaches a weight $\mu_w$ to the $w$-th worker, which is expected to be small for a low-quality worker and large for a high-quality one. From the explicit solution of $\mu$ in Eq. (22), we can see that once $\lambda$ is specified, the weight assigned to $\mu_w$ can be derived from the approximation loss of $A_w$. To find a feasible value of $\lambda$, we vary $\lambda$ in $\{10^{-5}, 10^{-4}, \ldots, 10^0, 10^7\}$. Furthermore, to investigate the capability of ML-JMF of identifying spammers, we additionally append 20 spam workers, who randomly select a label for all the samples of AppleEkman and AppleNakamura datasets. The Accuracy of ML-JMF under each value of $\lambda$ is revealed in Figure 1. In practice, we also separately investigated the 20 spammers who assign a random label to each sample, and the 20 spammers who randomly assign the average number of annotations of all workers to samples of the dataset. These two investigations give the similar results as revealed in Figure 1.

ML-JMF has the highest Accuracy when $\lambda \approx 10^4$, the lowest Accuracy when $\lambda < 10$, and gradually reduced Accuracy when $\lambda \geq 10^5$. To further investigate these results, we take the AppleEkman dataset, and report the weights ($\mu_w$) assigned to all the annotation matrices when $\lambda = 10$, $\lambda = 10^4$, and $\lambda = 10^5$ in Figure 2. We have several interesting observations. (i) When $\lambda = 10$, only a very small portion of annotation matrices are selected; when $\lambda = 10^5$, all annotation matrices are selected and assigned nearly equal weights. This is expected from Eq. (6): a (too) small $\lambda$ value does not have a sufficient regularization effect on the weights assigned to individual matrices, and thus only few data matrices are selected. On the other hand, a (too) large $\lambda$ value inflicts a strong regularization effect and forces similar weight assignments to all matrices. (ii) Since complementary information is spread across the annotation matrices of different workers, ML-JMF with $\lambda = 10^5$ and with $\lambda = 10^4$ obtains a significantly better
performance than with $\lambda = 10$. (iii) Even if ML-JMF with $\lambda = 10^3$ combines the annotations of 20 spammers, it still obtains a better performance than ML-JMF with $\lambda = 10$; this is because the low-rank matrix approximation can eliminate the noise of annotation matrices. For a similar reason, ML-JMF with $\lambda = 10^4$ occasionally does not assign zero weights to several spammers.

In summary, these experimental results corroborate the fact that ML-JMF can identify spammers and can selectively integrate different annotation matrices via joint matrix factorization. Based on these experimental results, we adopt $\lambda = 10^4$ for the experiments.

C. Robustness to Spammers

Spammers always exist in crowdsourcing platforms. Previous studies show that the proportion of spammers could be up to 40% [3, 40]. As a result, it is important to investigate how each aggregation technique performs when the workers are not trustworthy. For this investigation, we artificially injected 10\%, 20\%, 30\%, 40\% spammers into the worker population and report the performance of the comparing methods under different ratios of spammers in Figure 5. Here, each spammer randomly selects a label from the label space, and then assigns the chosen label to all the samples of the dataset.

As the ratio of spammers increases, all the aggregation methods have reduced Accuracy. This pattern is expected, since more spammers bring in more noisy annotations, which may even surpass the correct ones and make the aggrega-

Table III: Accuracy of ML-JMF and comparing methods

<table>
<thead>
<tr>
<th></th>
<th>Movie</th>
<th>LoveNakamura</th>
<th>LoveEkman</th>
<th>AppleNakamura</th>
<th>AppleEkman</th>
</tr>
</thead>
<tbody>
<tr>
<td>MV</td>
<td>0.9275</td>
<td>0.8726</td>
<td>0.8697</td>
<td>0.8510</td>
<td>0.8622</td>
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<tr>
<td>PLAT</td>
<td>0.8968</td>
<td>0.8839</td>
<td>0.8818</td>
<td>0.8662</td>
<td>0.8868</td>
</tr>
<tr>
<td>AWMV</td>
<td>0.9326</td>
<td>0.9126</td>
<td>0.8857</td>
<td>0.8703</td>
<td>0.8953</td>
</tr>
<tr>
<td>RAKEL-GLAD</td>
<td>0.9430</td>
<td>0.9363</td>
<td>0.9202</td>
<td>0.9317</td>
<td>0.9295</td>
</tr>
<tr>
<td>C-DS</td>
<td>0.9423</td>
<td>0.9267</td>
<td>0.9023</td>
<td>0.9276</td>
<td>0.9363</td>
</tr>
<tr>
<td>ML-JMF(A)</td>
<td>0.9317±0.0017</td>
<td>0.8396±0.0021</td>
<td>0.8427±0.0000</td>
<td>0.8313±0.0124</td>
<td>0.8617±0.0031</td>
</tr>
<tr>
<td>ML-JMF</td>
<td>0.9458±0.0028</td>
<td>0.9505±0.0127</td>
<td>0.9235±0.0121</td>
<td>0.9550±0.0208</td>
<td>0.9513±0.0228</td>
</tr>
</tbody>
</table>

Table IV: 1-RankingLoss of ML-JMF and comparing methods

<table>
<thead>
<tr>
<th></th>
<th>Movie</th>
<th>LoveNakamura</th>
<th>LoveEkman</th>
<th>AppleNakamura</th>
<th>AppleEkman</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAKEL-GLAD</td>
<td>0.9978</td>
<td>0.9911</td>
<td>0.7249</td>
<td>0.9681</td>
<td>0.9701</td>
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<tr>
<td>C-DS</td>
<td>0.9932</td>
<td>0.9729</td>
<td>0.8174</td>
<td>0.9694</td>
<td>0.9687</td>
</tr>
<tr>
<td>ML-JMF(A)</td>
<td>0.9879±0.0000</td>
<td>0.9519±0.0000</td>
<td>0.9127±0.0017</td>
<td>0.9656±0.0001</td>
<td>0.9577±0.0000</td>
</tr>
<tr>
<td>ML-JMF</td>
<td>0.9979±0.0000</td>
<td>0.9791±0.0017</td>
<td>0.9358±0.0000</td>
<td>0.9703±0.0013</td>
<td>0.9716±0.0000</td>
</tr>
</tbody>
</table>

Figure 2: Weights assigned to 77 (57 workers + 20 spammers) annotation matrices of AppleNakamura dataset.

Figure 3: Accuracy under different ratios of spammers on AppleEkman, AppleNakamura and LoveEkman.

D. Parameter sensitivity analysis

The four parameters $\alpha$, $\beta$, $\lambda$, and the rank $k$ of $S$ may affect the performance of ML-JMF. We conduct additional experiments to study the sensitivity of ML-JMF with respect...
to $\alpha$, $\beta$, and $k$. The sensitivity of $\lambda$ was studied in Subsection V-B (see Figure 1 and Figure 2).

Figure 4: Accuracy of ML-JMF under different combinations of $\alpha$ and $\beta$.

Figure 5: Accuracy of ML-JMF under different low-rank sizes ($k$).

Figure 6: Convergence curve of ML-JMF on the AppleEkman dataset.

languages; as such, it’s not meaningful to compare their empirical runtime costs. Therefore, we give the theoretical computational complexity of three multi-label answer aggregation approaches (ML-JMF, RA$\&$EL-GLAD, and C-DS). RA$\&$EL-GLAD takes $O(mnc)$ to create a power set of each label and $O(mnM2^k)$ (where $k$ is the number of labels in a label subset, and $M$ is the number of random label subsets) to calculate the average likelihood of each label, so its complexity is $O(mnc+mnM2^k)$. The computational complexity of C-DS is $O(mnc^2+mc^3+c^3)$, C-DS takes $O(mnc^2+mc^3)$ to compute the joint distribution over the source label vectors and target labels, and $O(c^3)$ to compute the probability of each label for each sample. ML-JMF takes $O(mnk^2)$, $O(nck)$, $O(mnc)$ to iteratively update the low-rank matrices $V, U_w, S$, respectively, and $O(tm)$ to update $\mu$. Thus, the computational complexity of ML-JMF is $O(mnk^2+tmnck+tm)$, where $t$ is the number of iterations. The three single-label answer aggregation methods (MV, PLAT, and AWMV) have a lower complexity than multi-label methods, since they separately aggregate answers for each label. Since $k < \{c, n\}$, ML-JMF has a lower complexity than RA$\&$EL-GLAD and C-DS. The code of ML-JMF will be made publicly available.

V. CONCLUSION

This paper studies how to aggregate the labels of multi-label samples collected via crowdsourcing, and introduces a Multi-Label answer aggregation approach based on Joint Matrix Factorization (ML-JMF). ML-JMF jointly factorizes the sample-label association matrices obtained from different workers into the product of individual low-rank matrices and a shared low-rank matrix, and selectively integrates them by assigning different weights to their answer data matrices. It further integrates the correlation between labels based on the shared matrix and connections between workers by individual matrices to guide the matrix factorization and weights. Experimental results on five real-world datasets show that ML-JMF can identify spammers and achieve higher accuracy than related methods. Our study suggests that both label correlation and quality of workers should be considered in aggregating the labels of multi-label samples. The code of ML-JMF is available at [http://mlda.swu.edu.cn/codes.php?name=MLJMF](http://mlda.swu.edu.cn/codes.php?name=MLJMF)

Like existing solutions, ML-JMF currently depends on the input parameters; how to reduce the number of input parameters, and how to automatically determine their optimal values are future issues to be pursued.
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