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Summer Lecture Series 2002

Dynamical Systems Model of the Simple Genetic Algorithm

Introduction to Michael Vose's Theory

Overview and References

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The last lecture of the summer series on evolutionary computation theory describes the basic theory of the Simple Genetic Algorithm (SGA) as developed by Michael Vose in the 1990's. It introduces the mathematical formalization in which the action of proportional selection, mutation, and recombination operators can be described and analyzed.

Overview

Part 1. Introduction to Vose's Dynamical Systems Model:

We start with a definition of a dynamical system and then describe the SGA as a dynamical system. Next, we represent the SGA populations as population vectors and describe their most important properties. Then, we define a broad class of algorithms called Random Heuristic Search from which Simple Genetic Algorithm emerges as a special case. Finally, we define mathematical framework for the analysis of the actions of proportional selection.

Part 2. Defining Mixing Matrices:

In this part we define the process of mixing and its two components: mutation and recombination. Next, we build a model of mutation operator and analyze mathematically convergence properties of the SGA using only proportional selection and mutation operators. Then, we define recombination operator for the simple case of one-point crossover and give an example of the actions of all three operators on a population vector. Finally, we discuss the interesting properties of mixing scheme and define the mixing matrix.

Part 3. Finite populations:

So far we were concerned with the infinite population model of the SGA. In this part we discuss the properties of the finite population the SGA and describe the algorithm as a Markov chain with its transition matrix. Then we describe the properties of the Markov chain model and present a way of visualizing the actions of heuristic function G. Finally, we define and discuss the importance of metastable states for the actions of the SGA.

Part 4. Conclusions:

In this part we summarize our findings and present some current outstanding conjectures.

The following list of references includes some good starting point papers and books for studies of the dynamical system model of SGA and also contains references to other relevant papers.

References

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