Nonhomogeneous Kleinberg's Small World Model: Cascades and Myopic Routing

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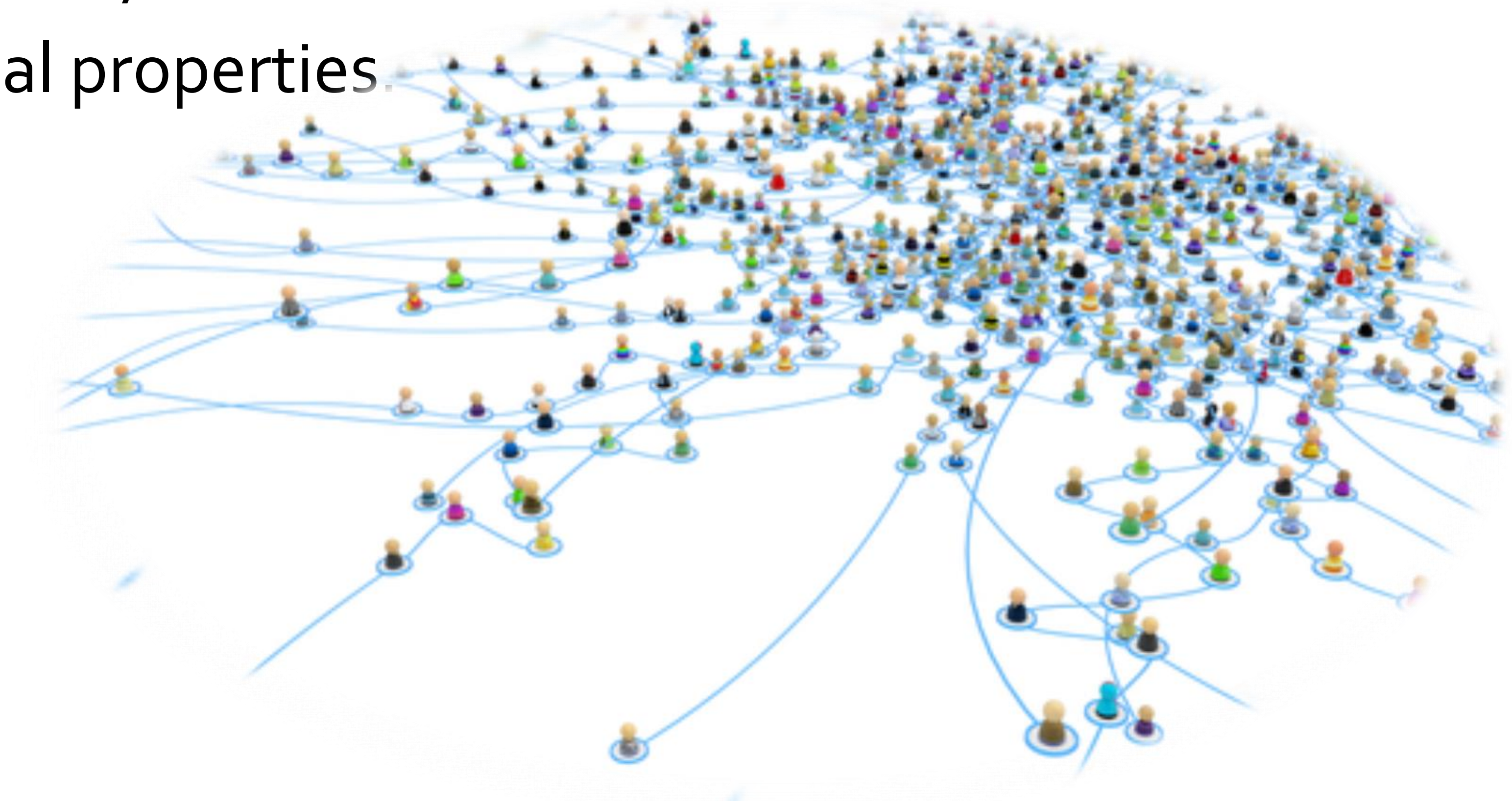
What is a social network?

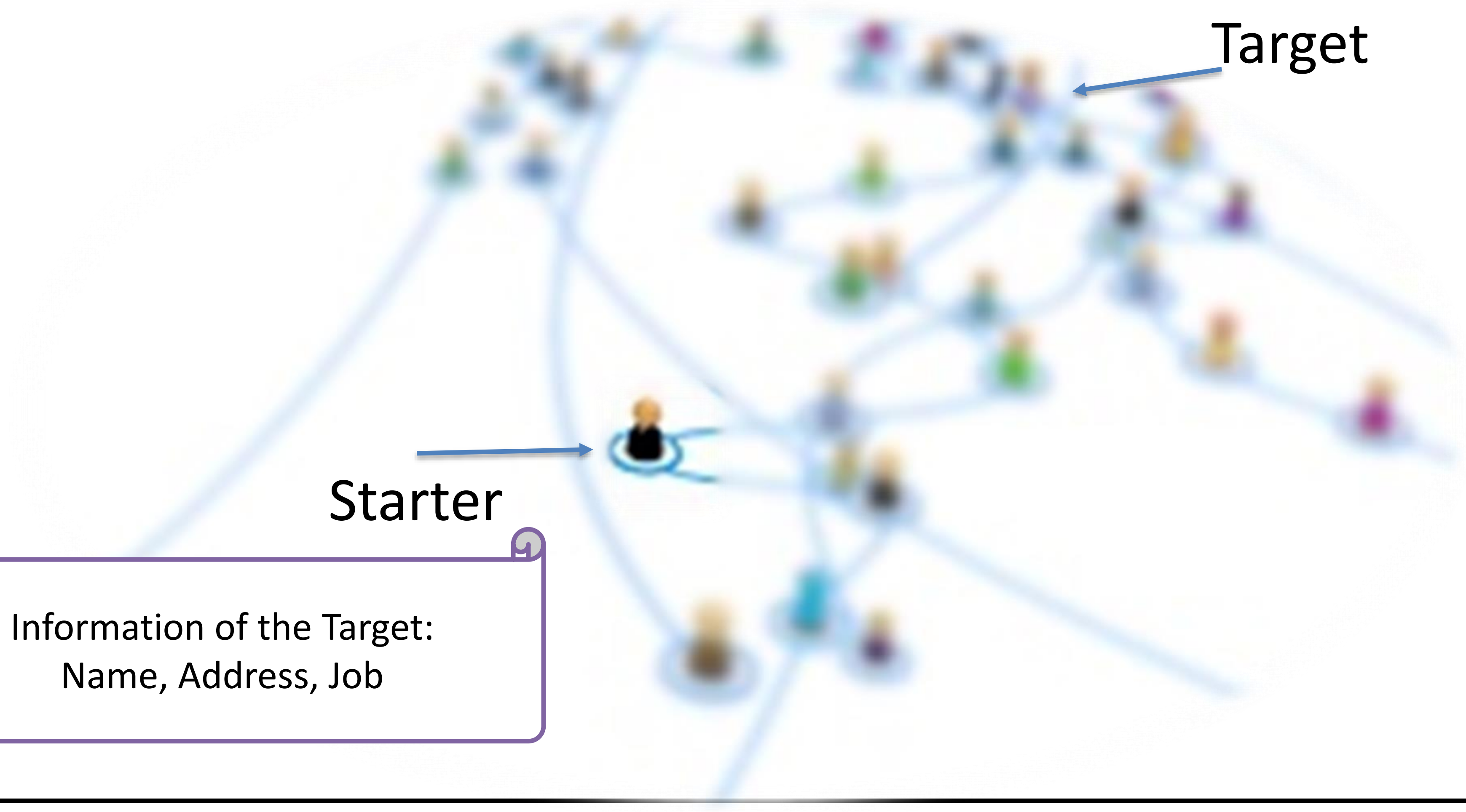
- Social network models interactions between individuals
  - Individuals behave freely.
  - Society shows special properties.



Outline

- Background
  - Milgram’s Experiment
  - Kleinberg's Small World Model
- Nonhomogeneous Kleinberg’s Small World Model
- Myopic Routing
  - Theorem
  - Proof Outline
- k-Complex Contagions Model

An Experiment by Milgram[1967]



Information of the Target:
Name, Address, Job

An Experiment by Milgram[1967]

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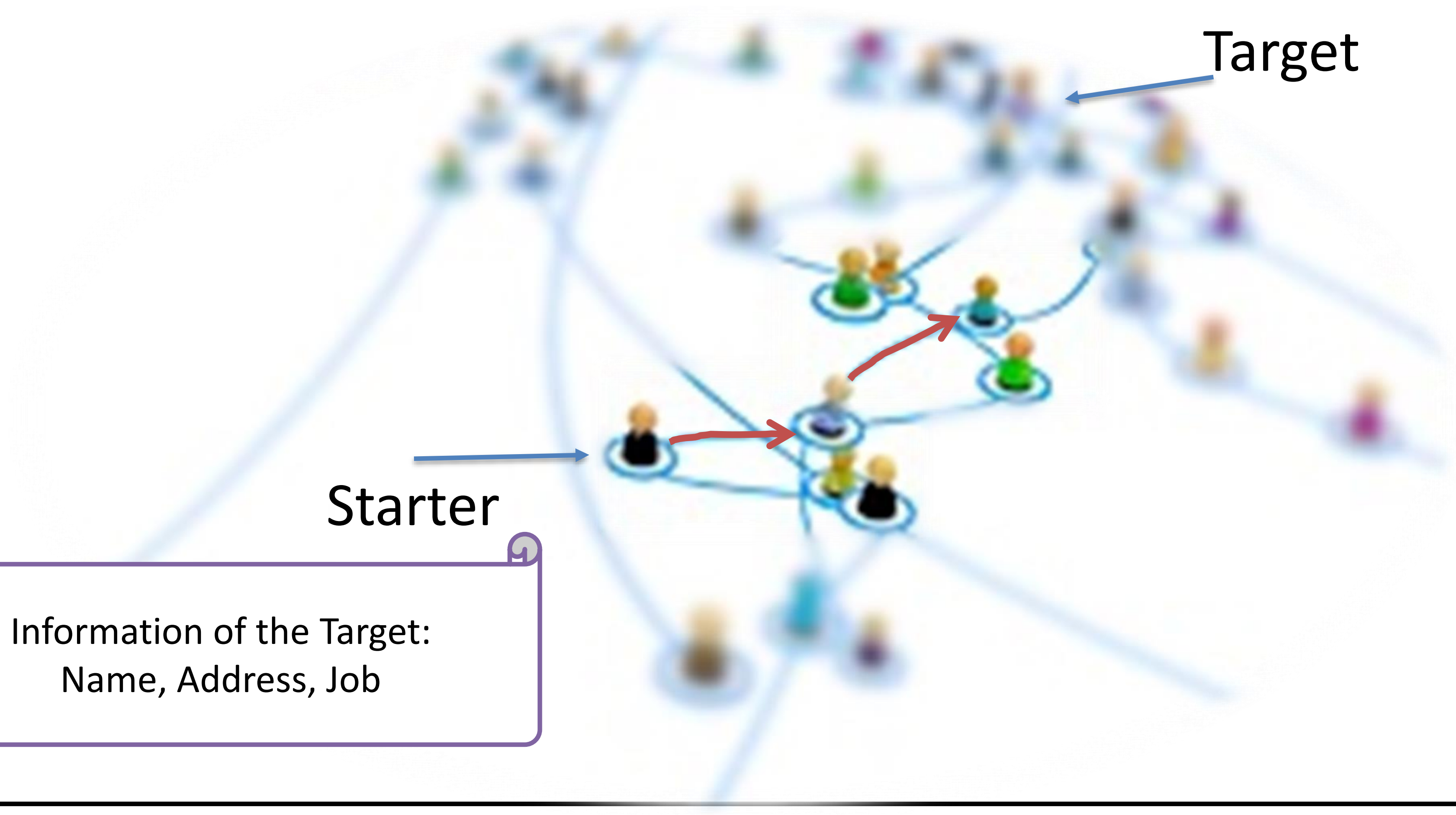

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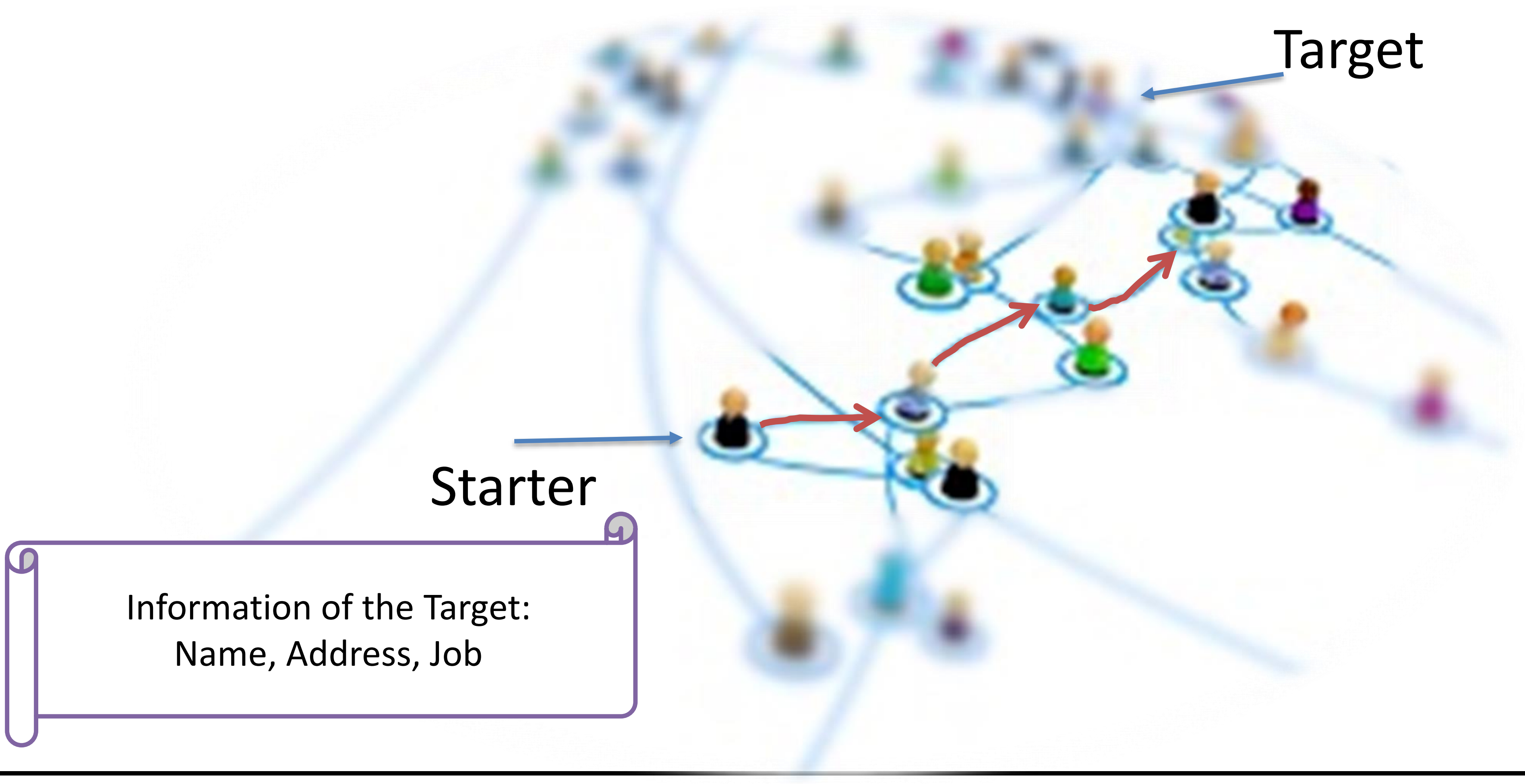
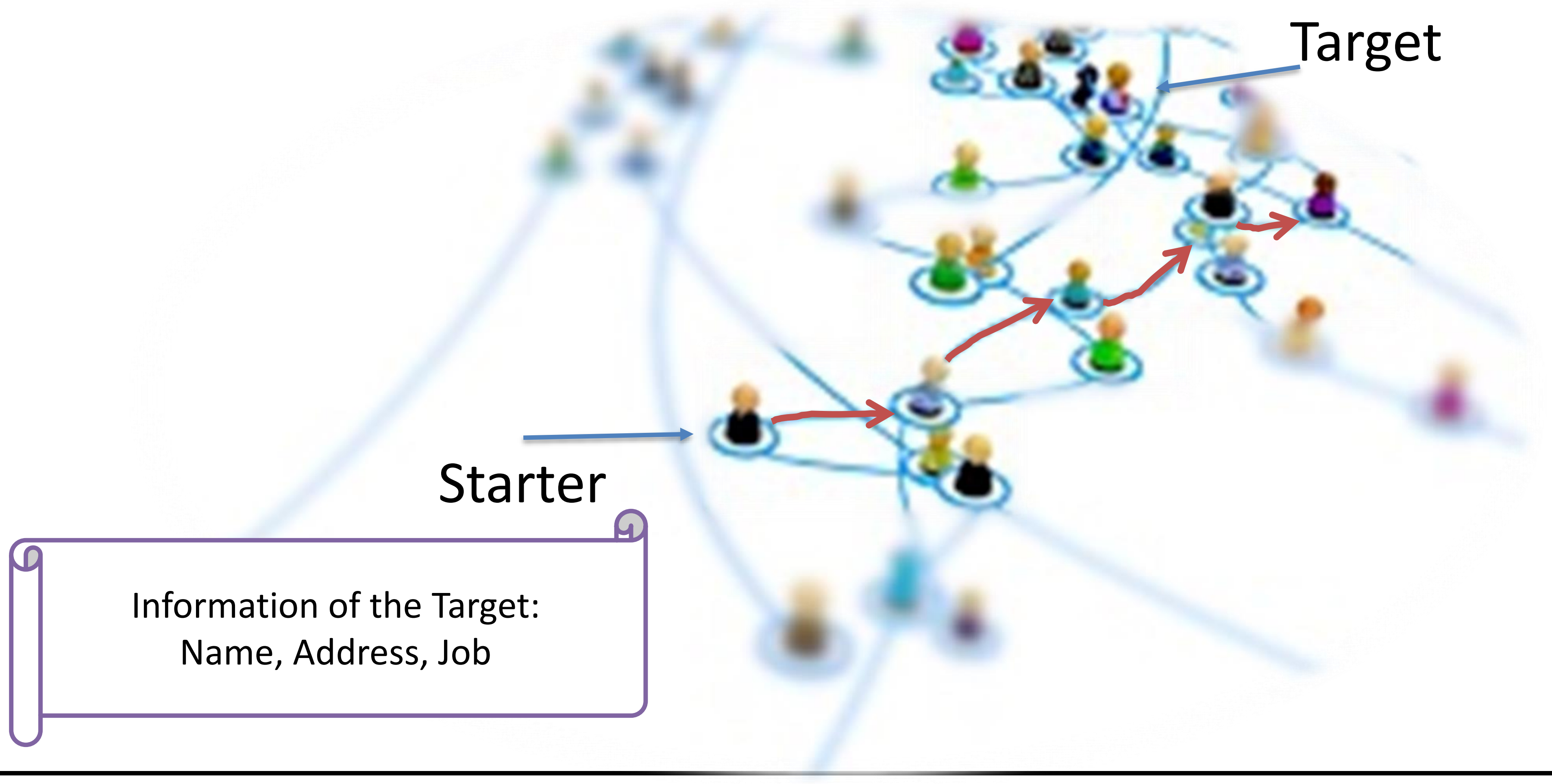
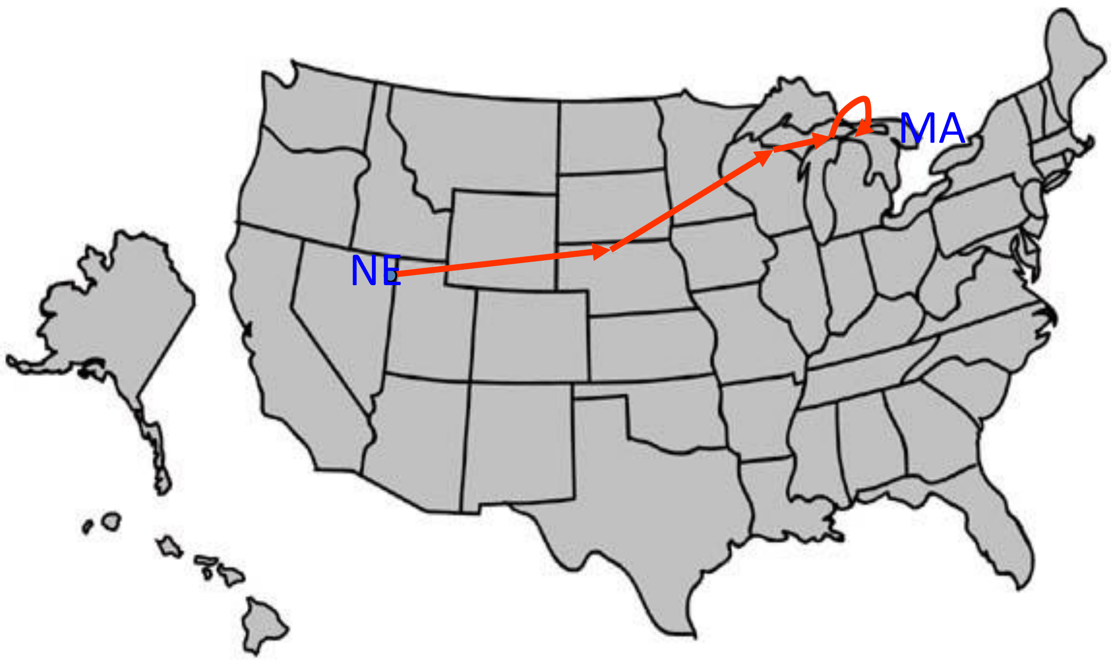


An Experiment by Milgram[1967]

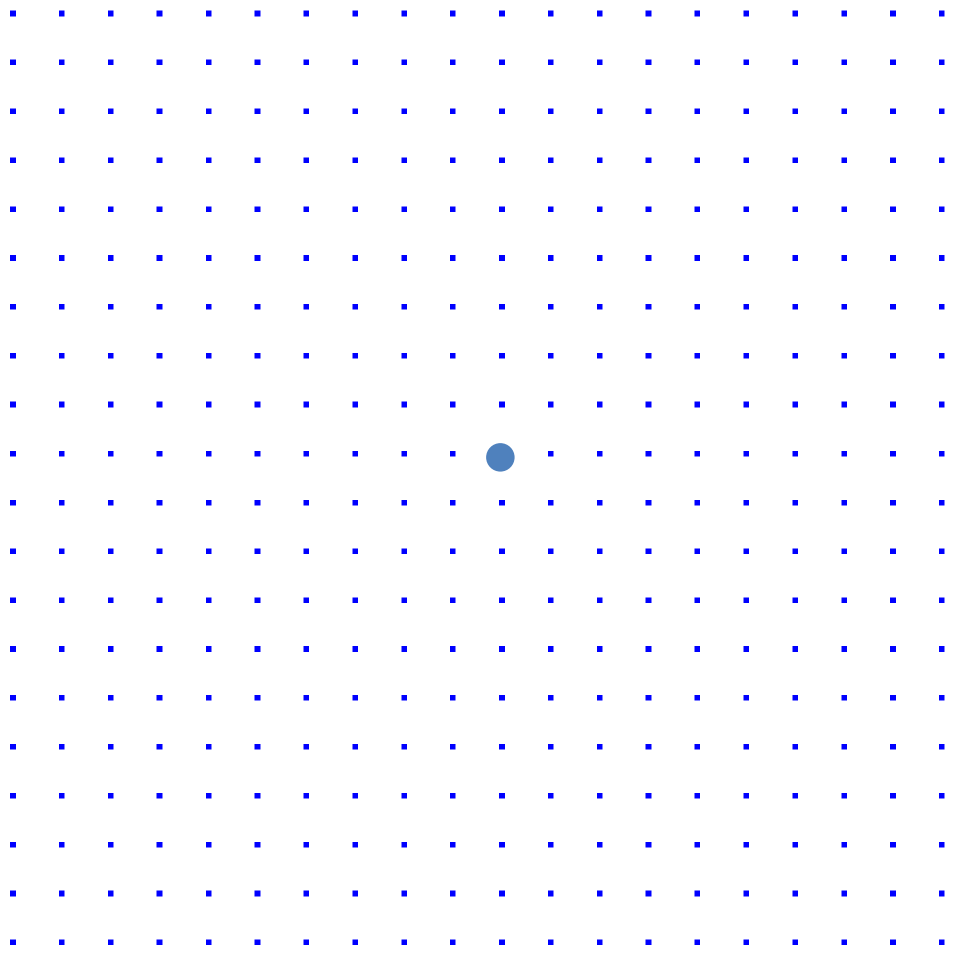


Small World Model

- Six degrees of separation--- very short paths between arbitrary pairs of nodes

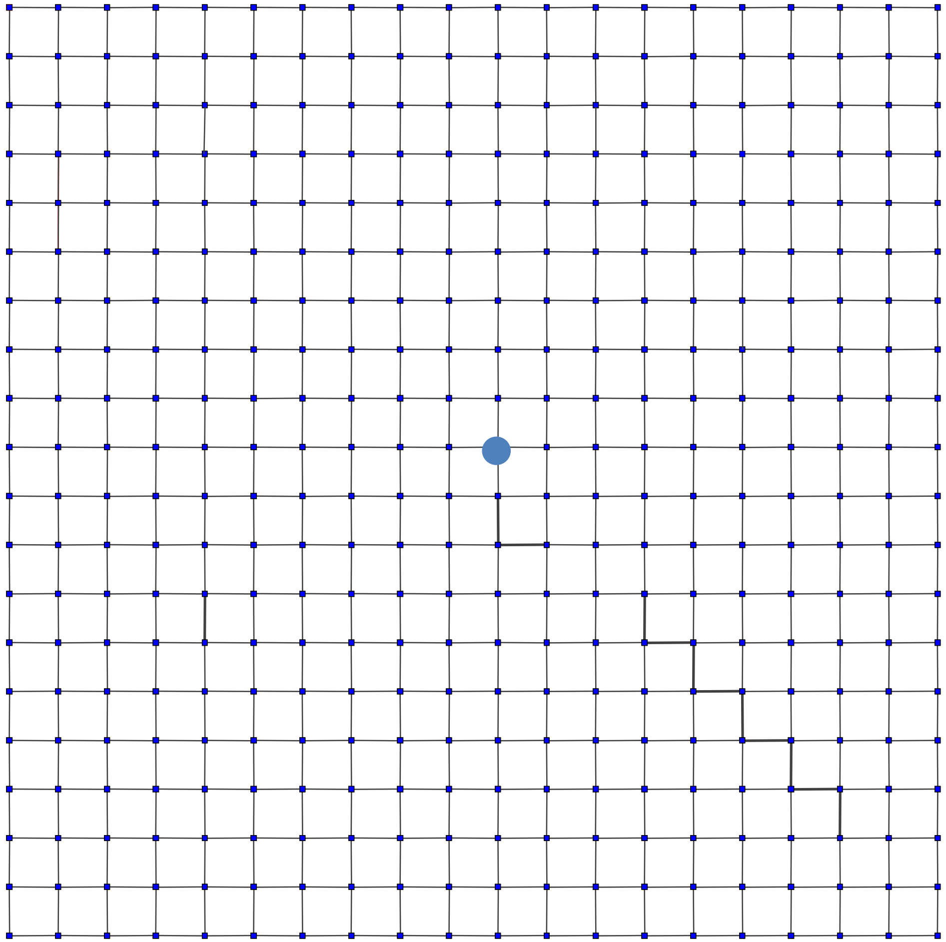


Watts/Strogatz model, Newman–Watts model



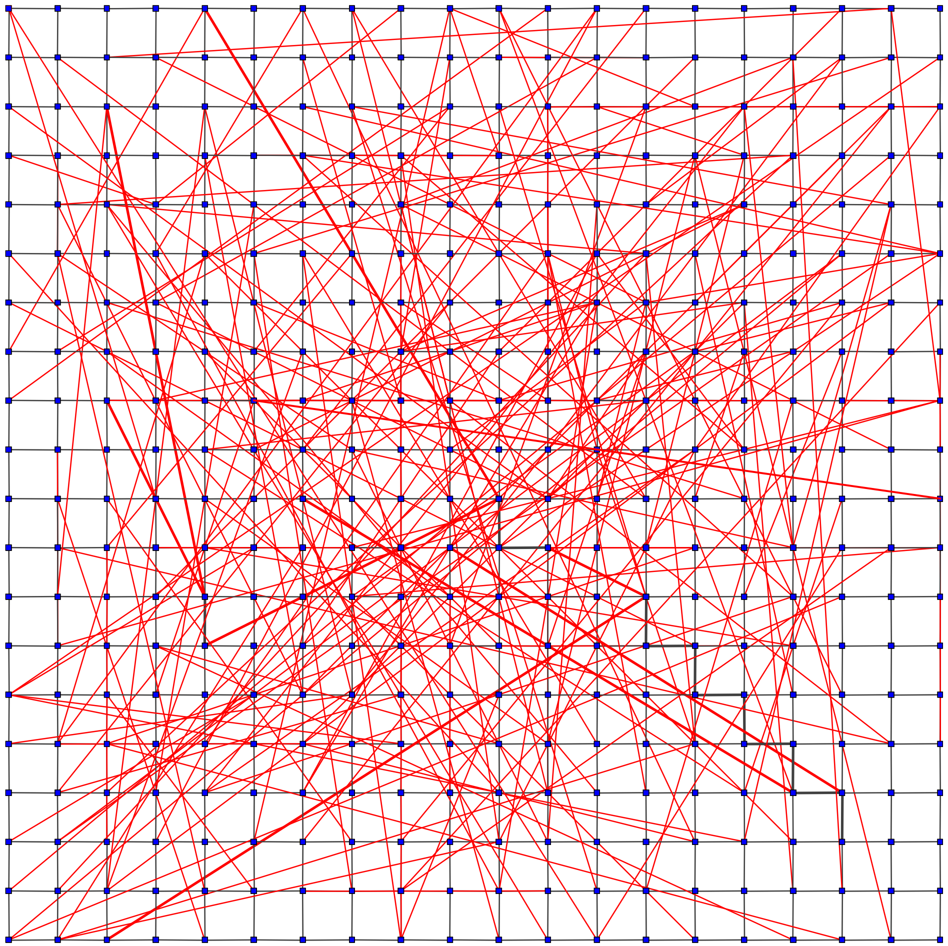
- n people on a ring/ torus

Strong Ties



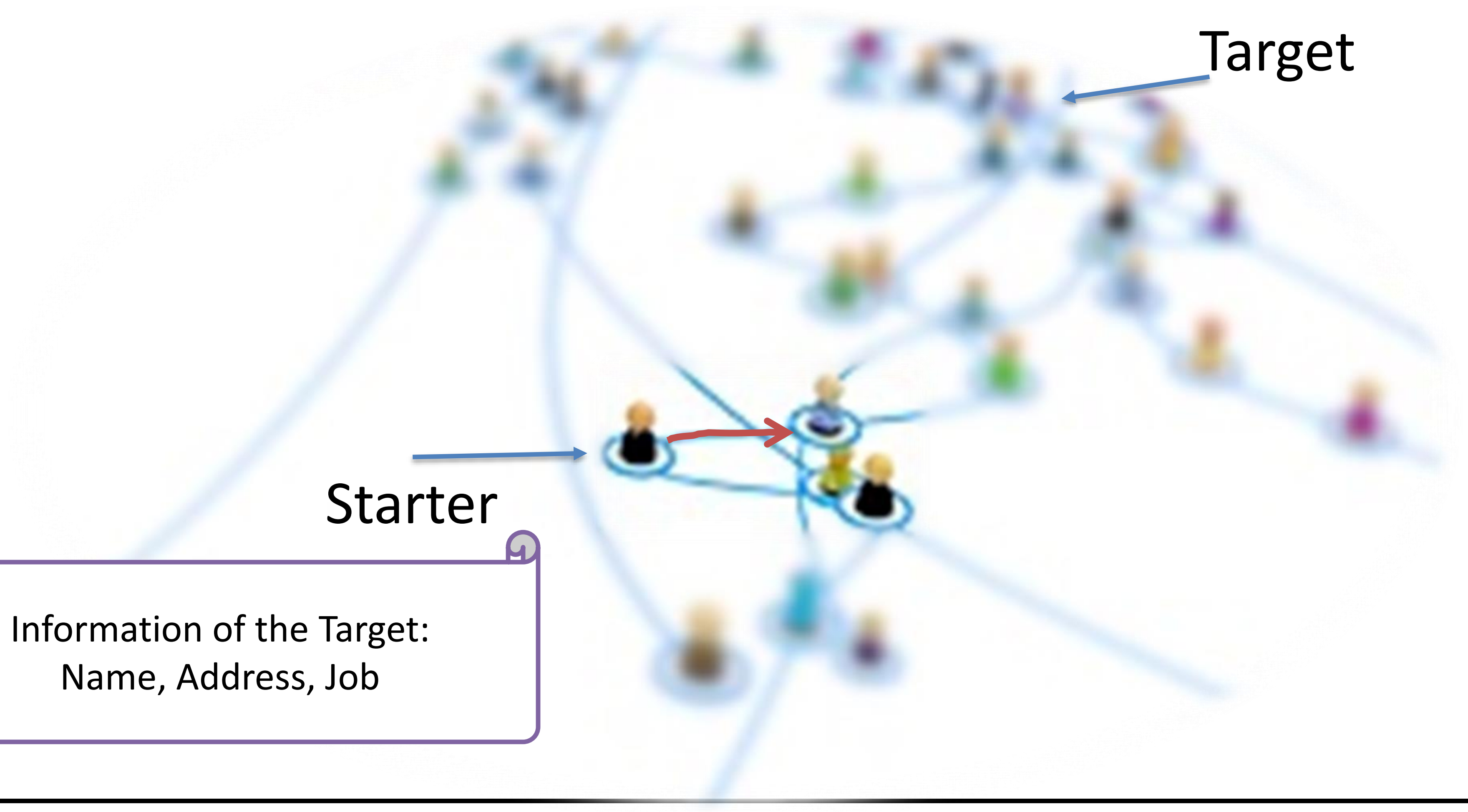
- n people on a ring/ torus
- Strong ties within distance q

Weak Ties



- n people on a ring/ torus
- Strong ties within distance q
- Weak ties: $p_{uv} = p$

Algorithmically Small World



Information of the Target:
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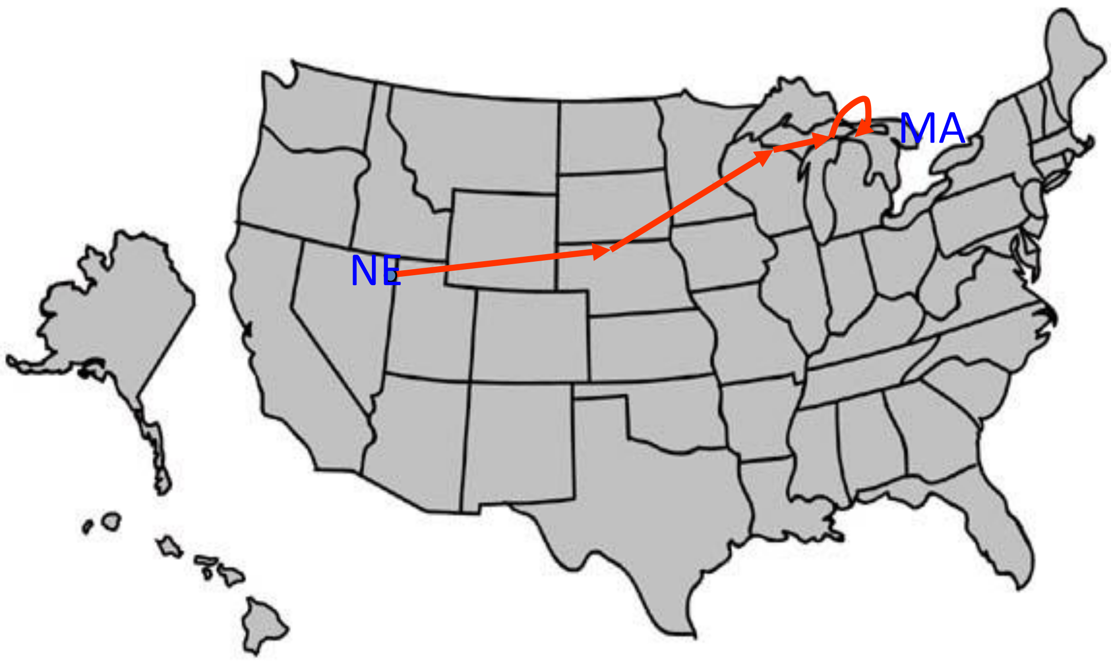
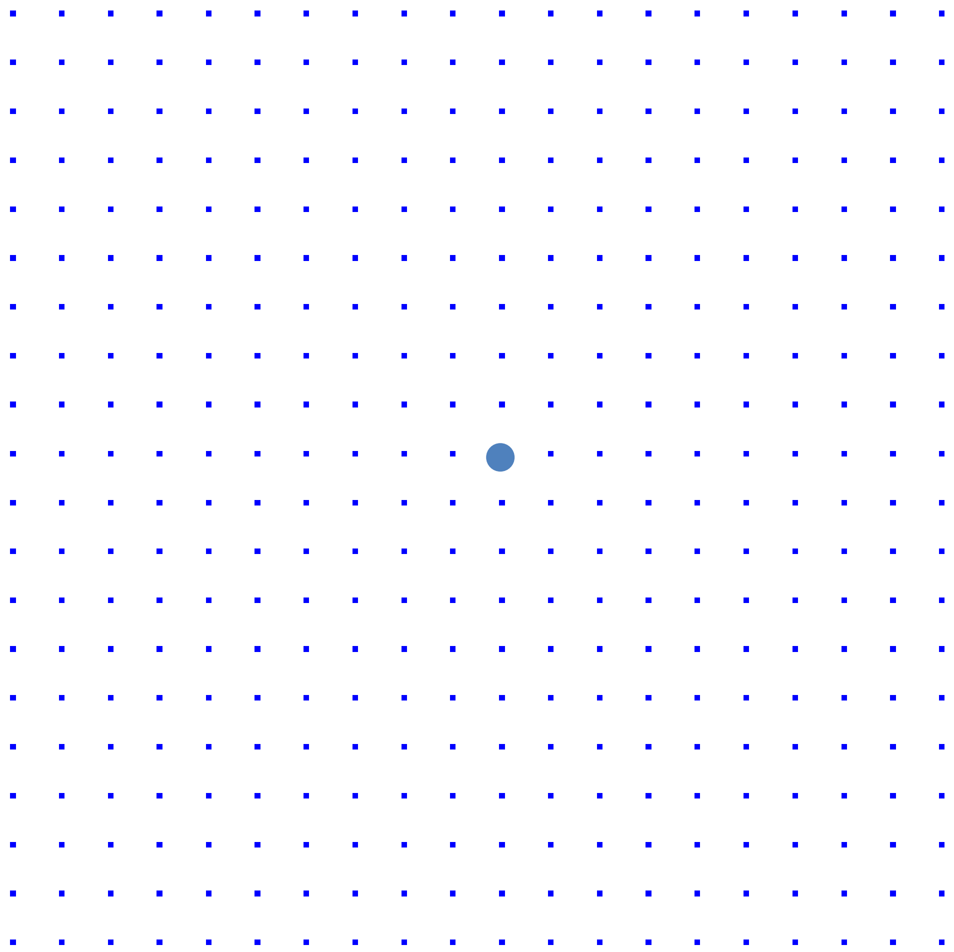
Small World Model 2.0

- Six degrees of separation--- very short paths between arbitrary pairs of nodes
- Decentralized routing--- Individuals with local information are very adept at finding these paths

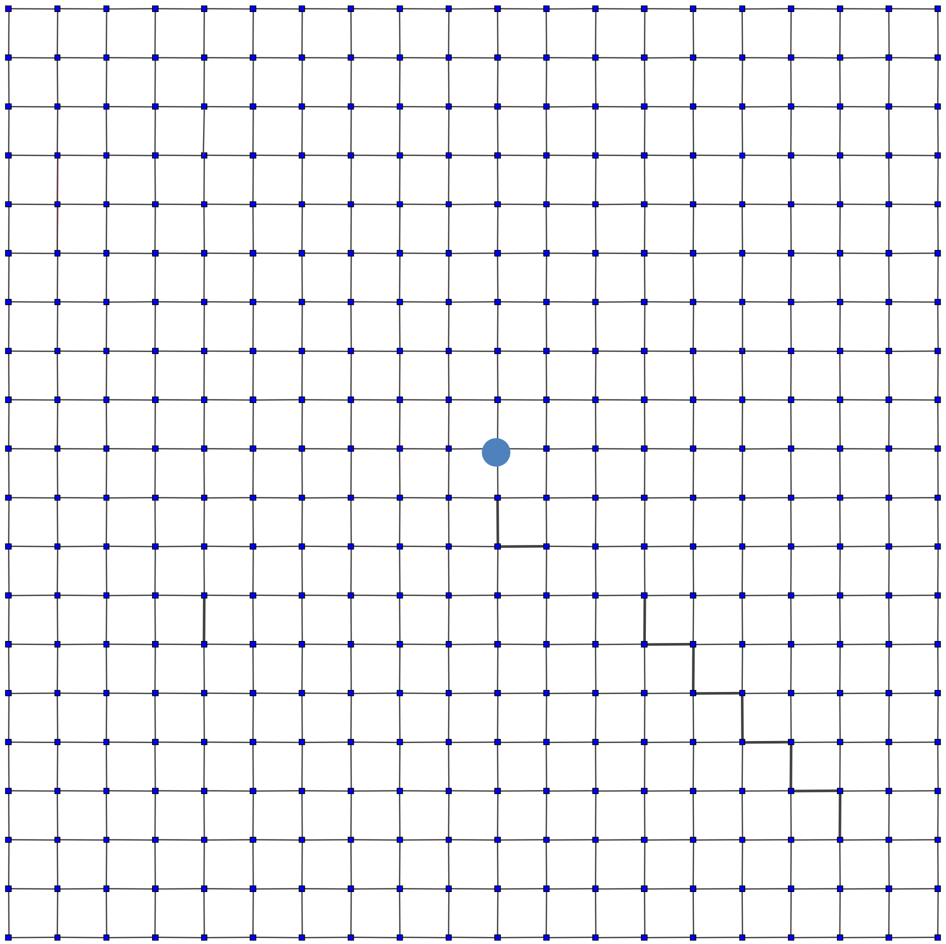


Kleinberg's Small World Model[2000]



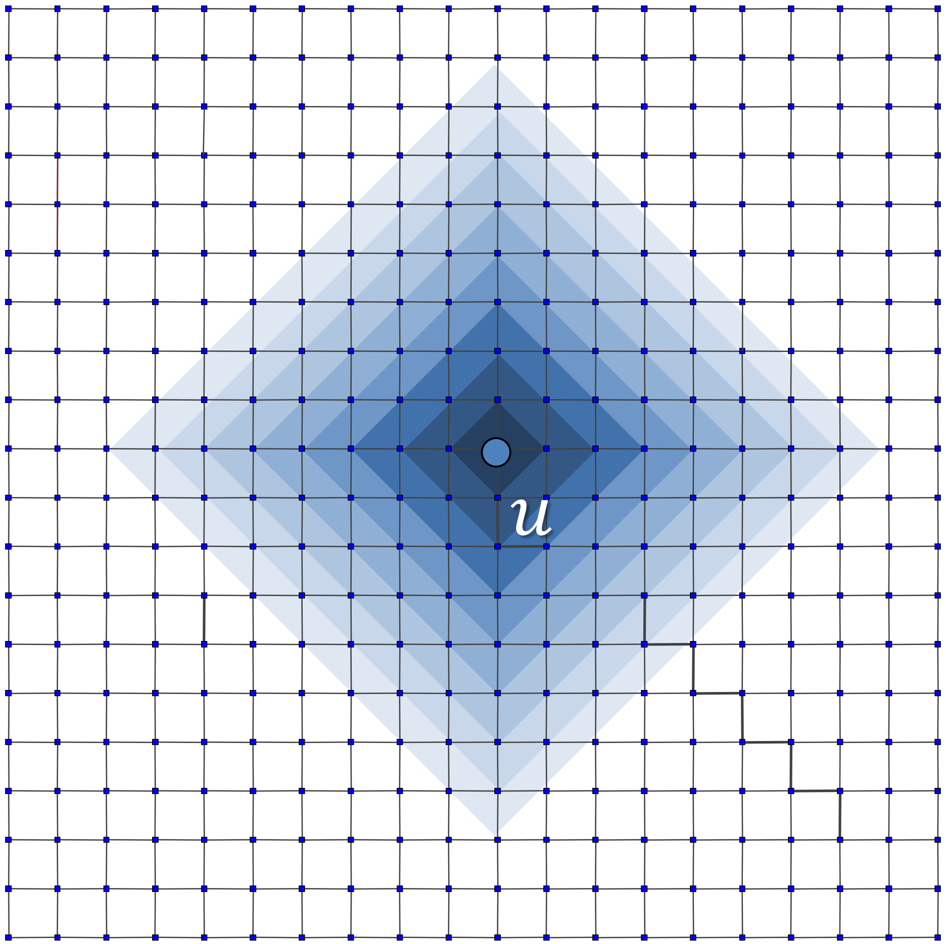
- n people on a k-dimensional grid

Strong Ties



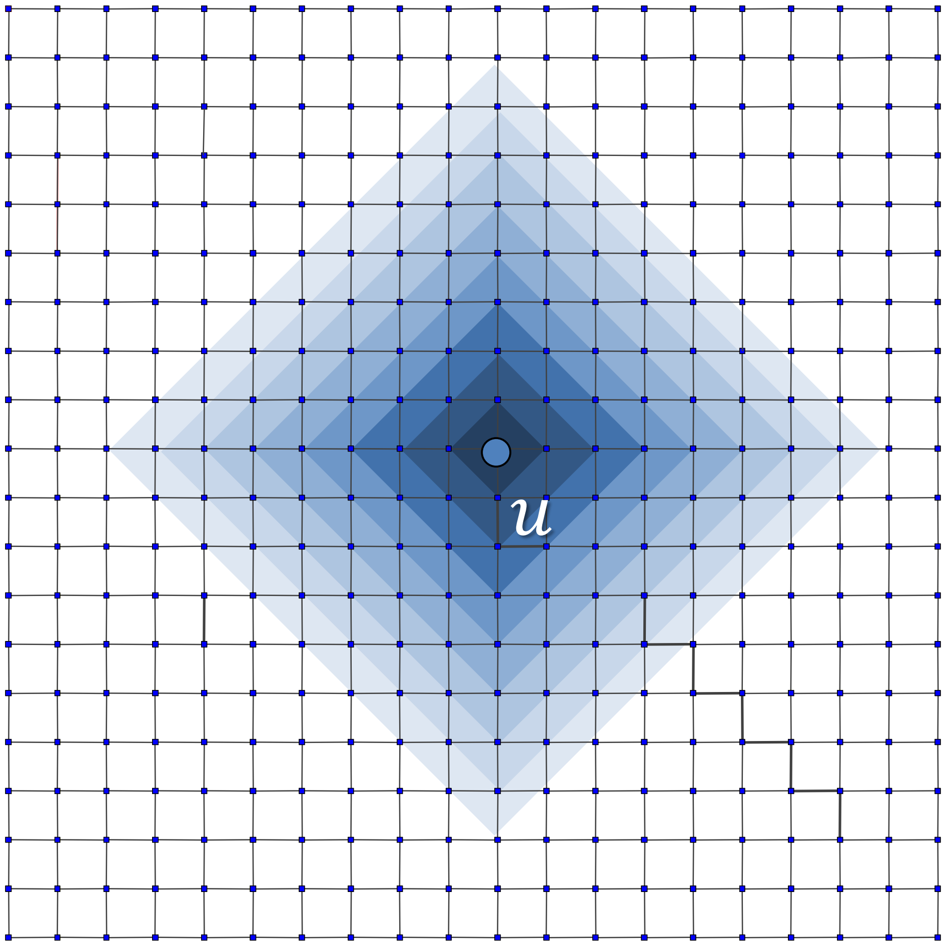
- n people on a k-dimensional grid
- Strong ties within distance q

Weak Ties



- n people on a k-dimensional grid
- Strong ties within distance q
- Weak ties: $p_{uv} \sim \frac{1}{d(u,v)^{\gamma}}$

Weak Ties



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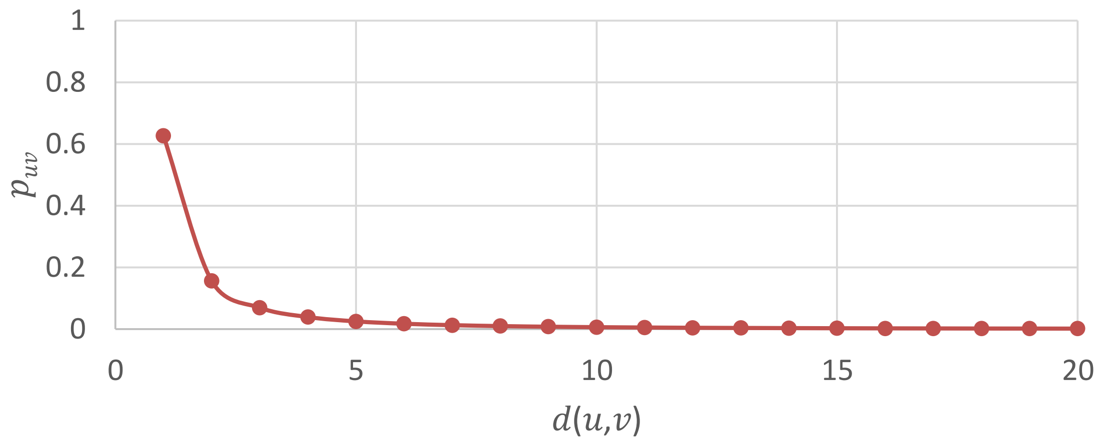
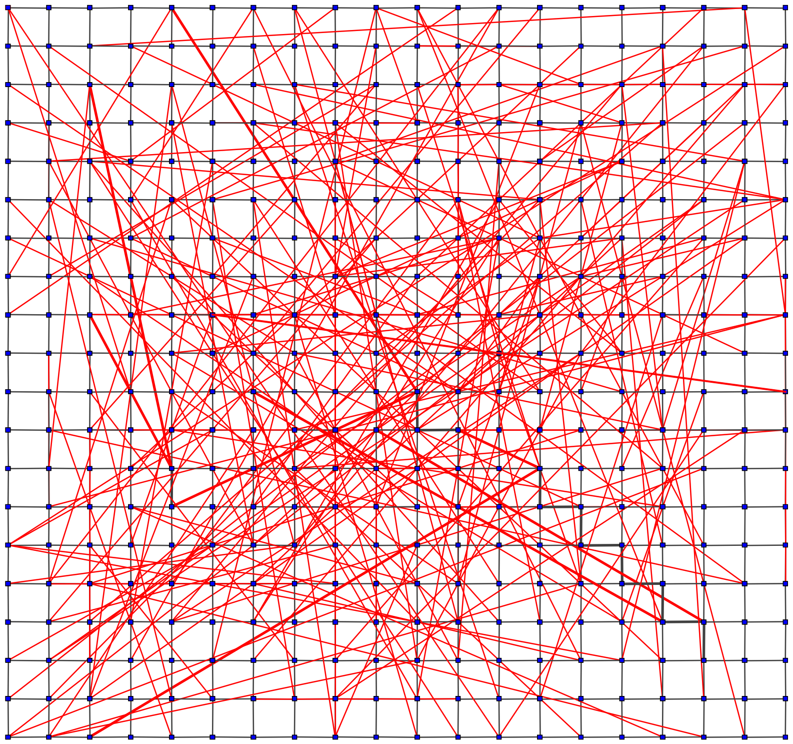
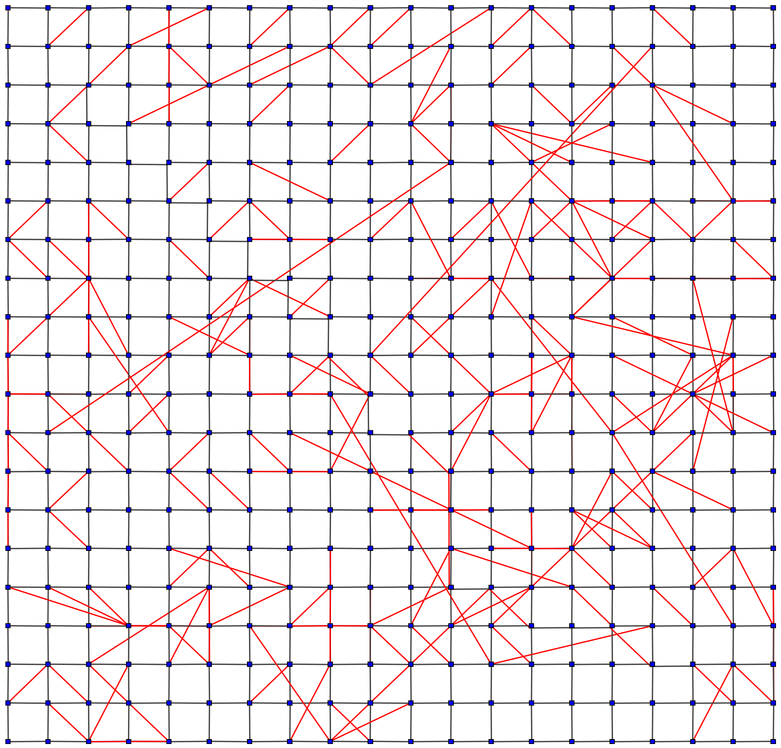


Weak Ties with Different γ

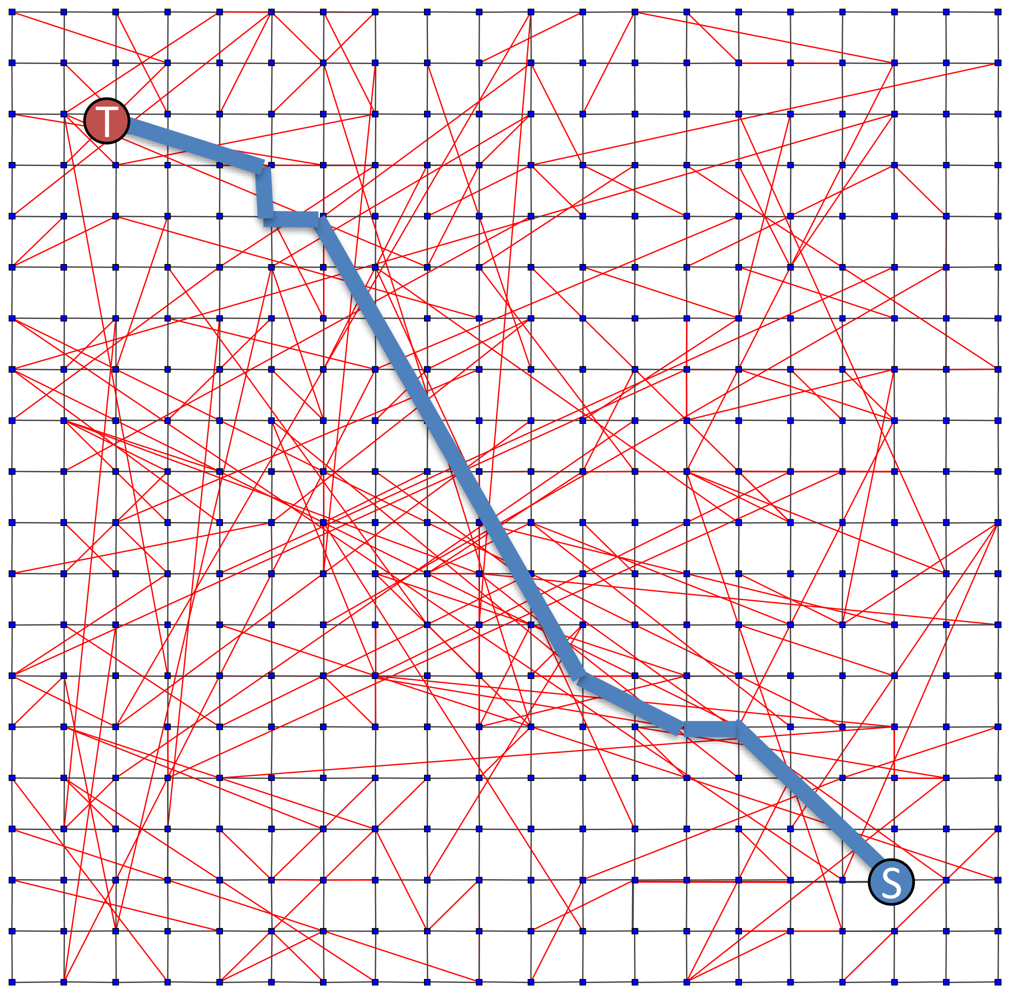
Small γ



Large γ



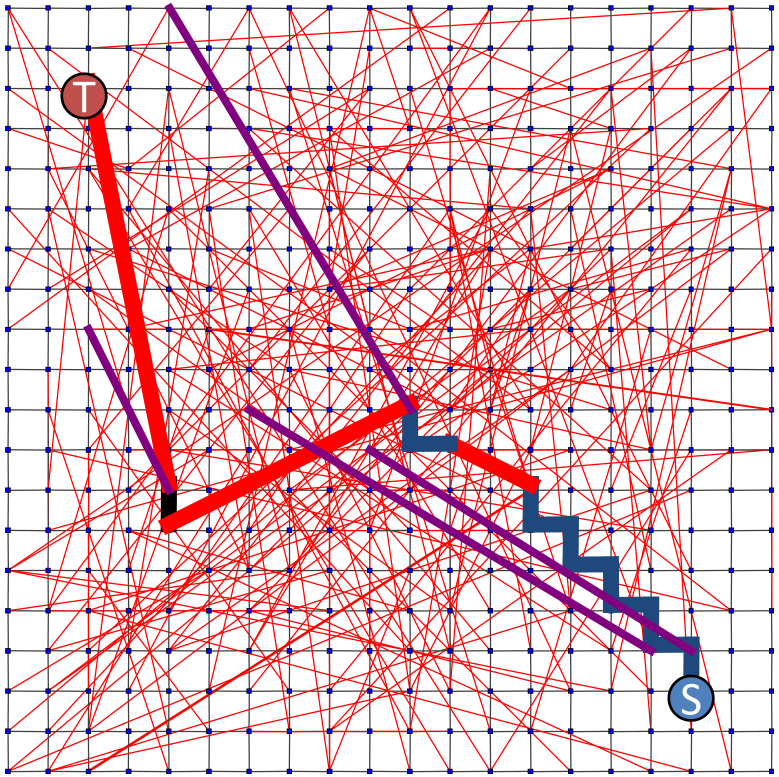
Decentralized Routing on Kleinberg's Model



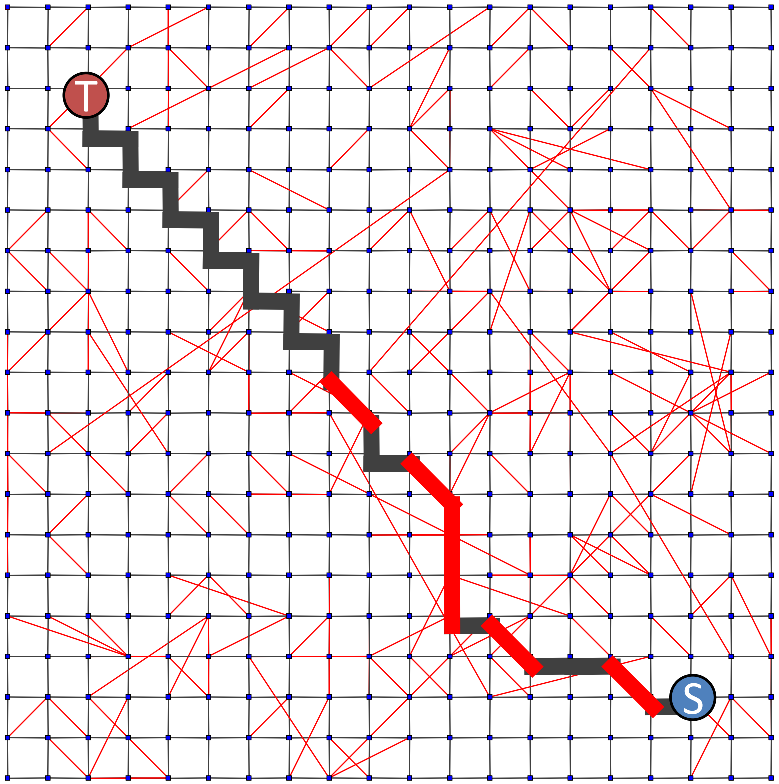
When $\gamma = 2$

Weak Ties with Different γ

When $\gamma < 2$



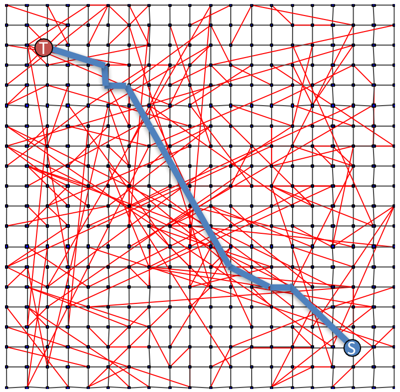
When $\gamma > 2$

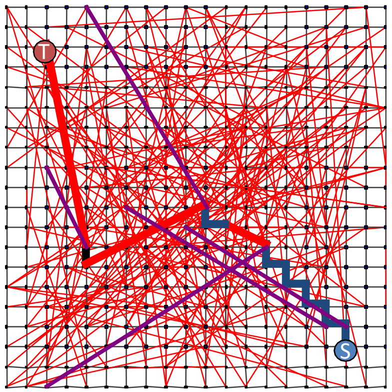


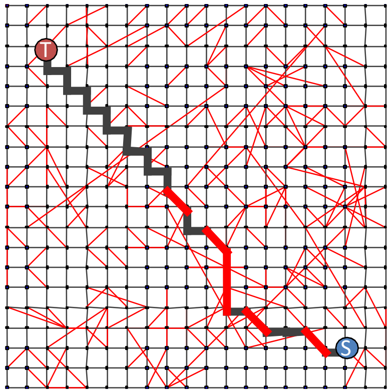
Threshold Property

If $\gamma = 2$ and $p, q \geq 1$, there is a decentralized algorithm A, so that the delivery time of A is $O(\log^2 n)$.

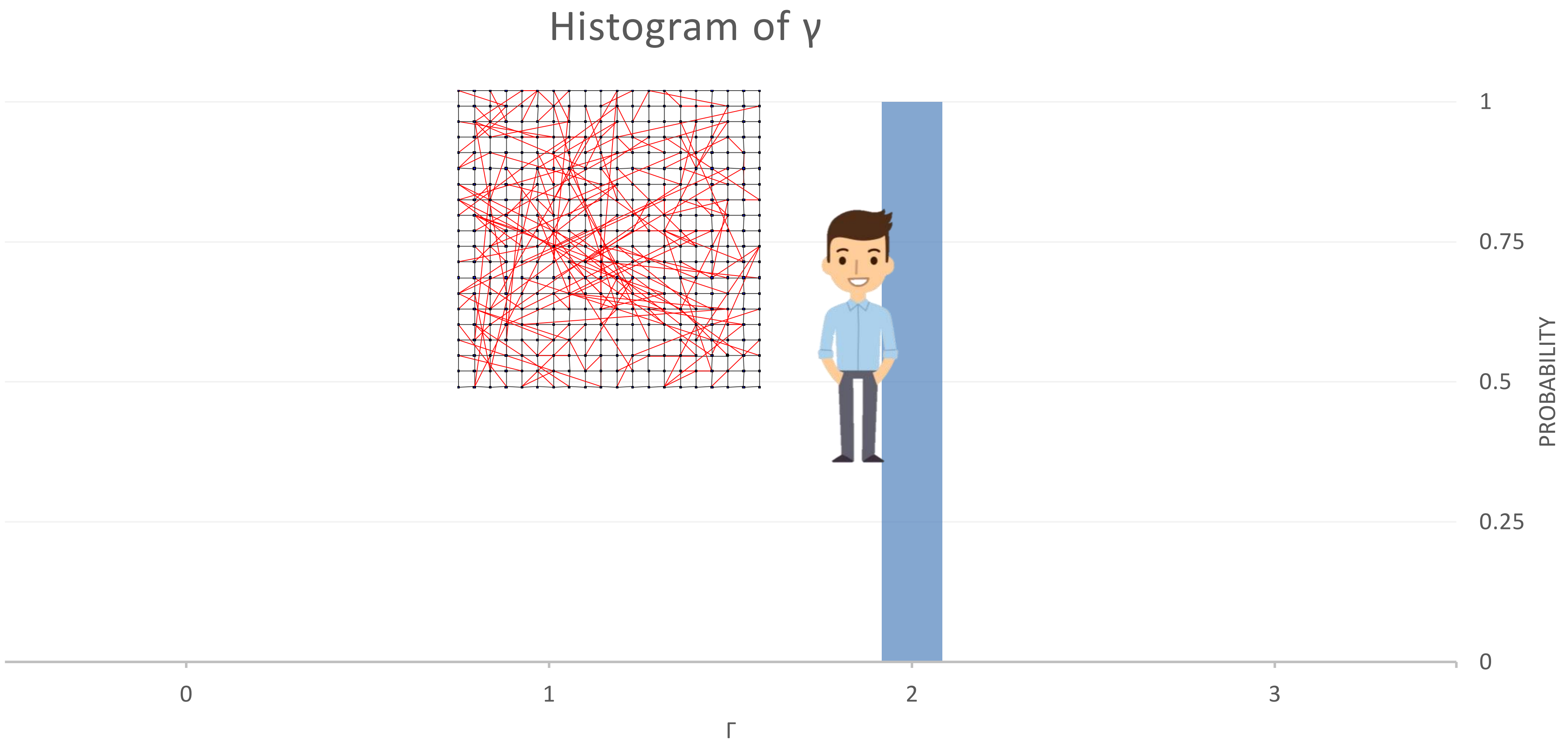
If $\gamma \neq 2$, there is a constant $\xi > 0$, so that the delivery time of any decentralized algorithm is $\Omega(n^{\xi})$.



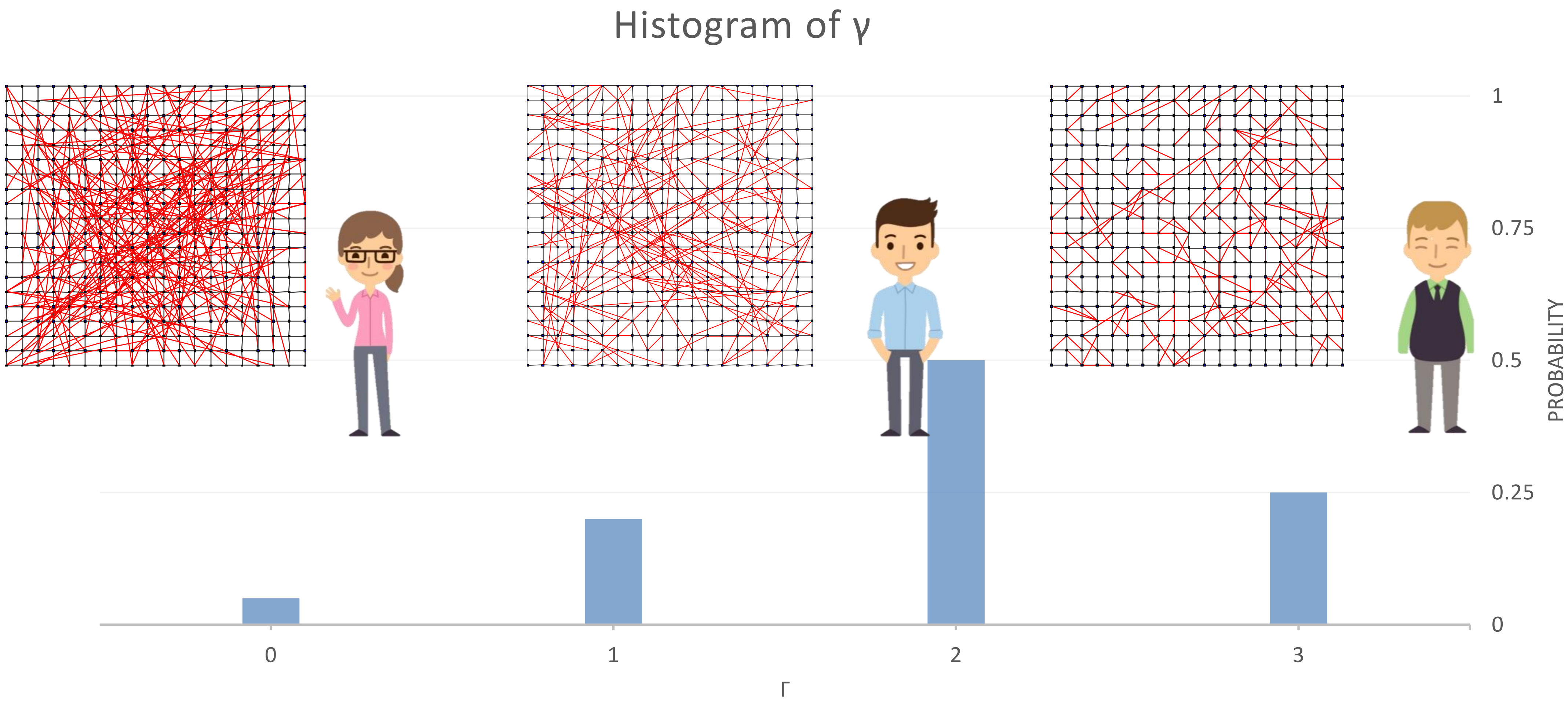




Threshold Property

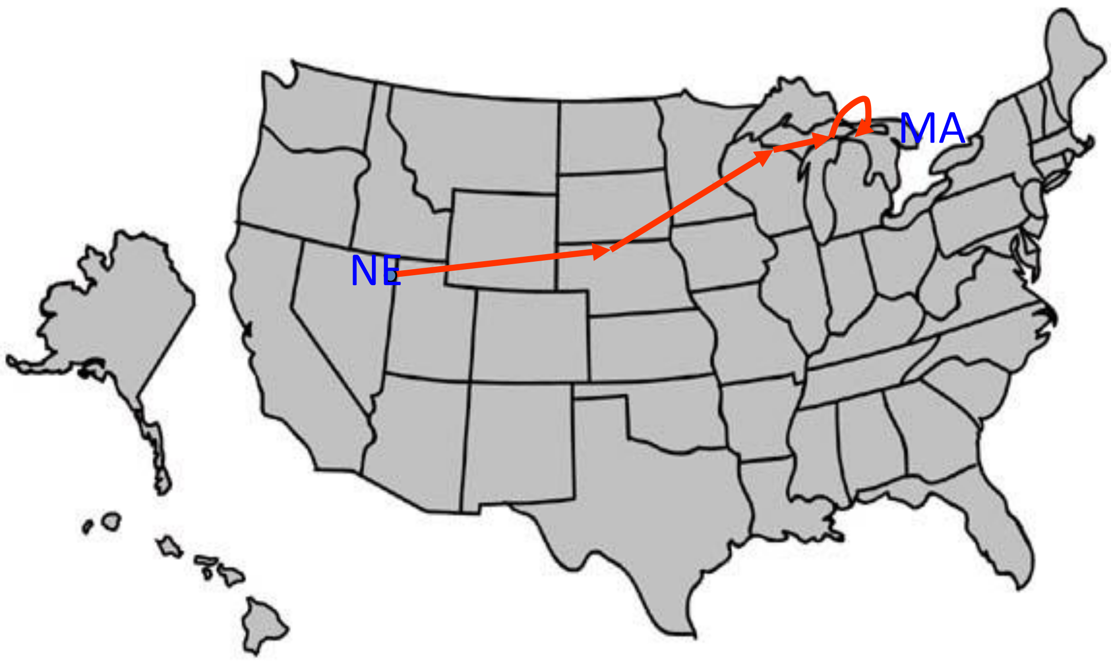


Diversity



Small World Model 2.0.1

- Six degrees of separation--- very short paths between arbitrary pairs of nodes
- Decentralized routing--- Individuals with local information are very adept at finding these paths



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- **Nonhomogeneous Kleinberg’s Small World Model**
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  - Theorem
  - Proof Outline
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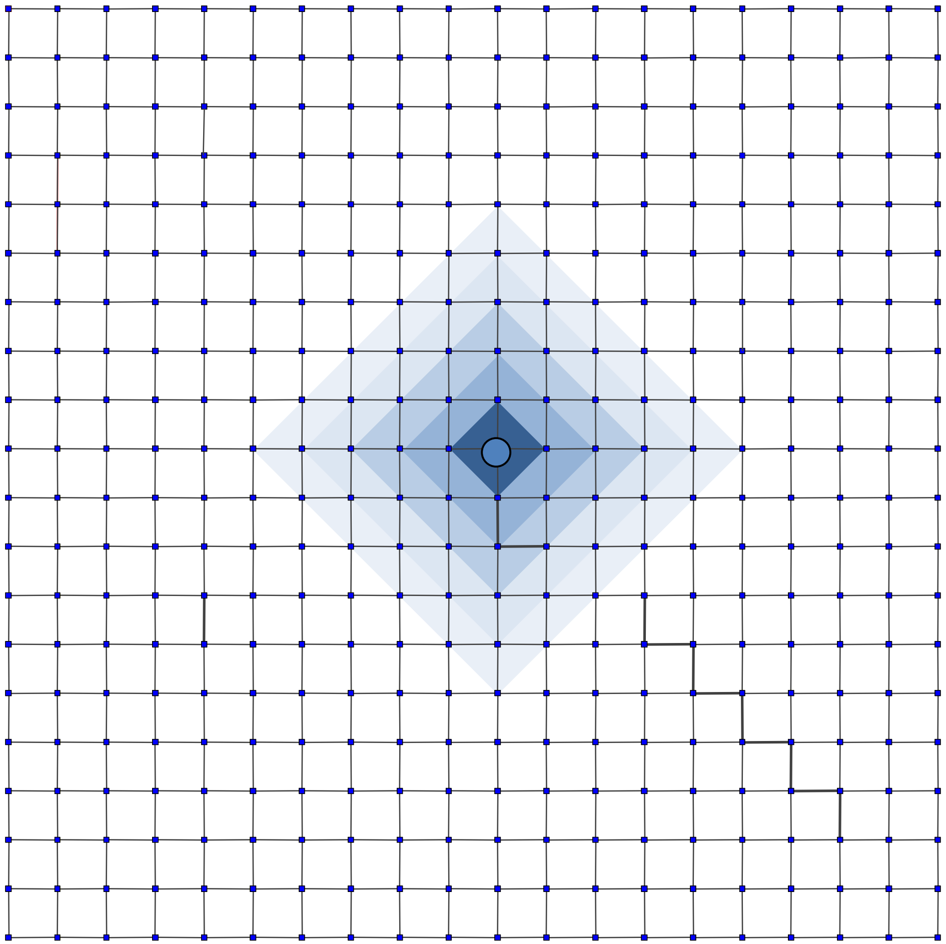
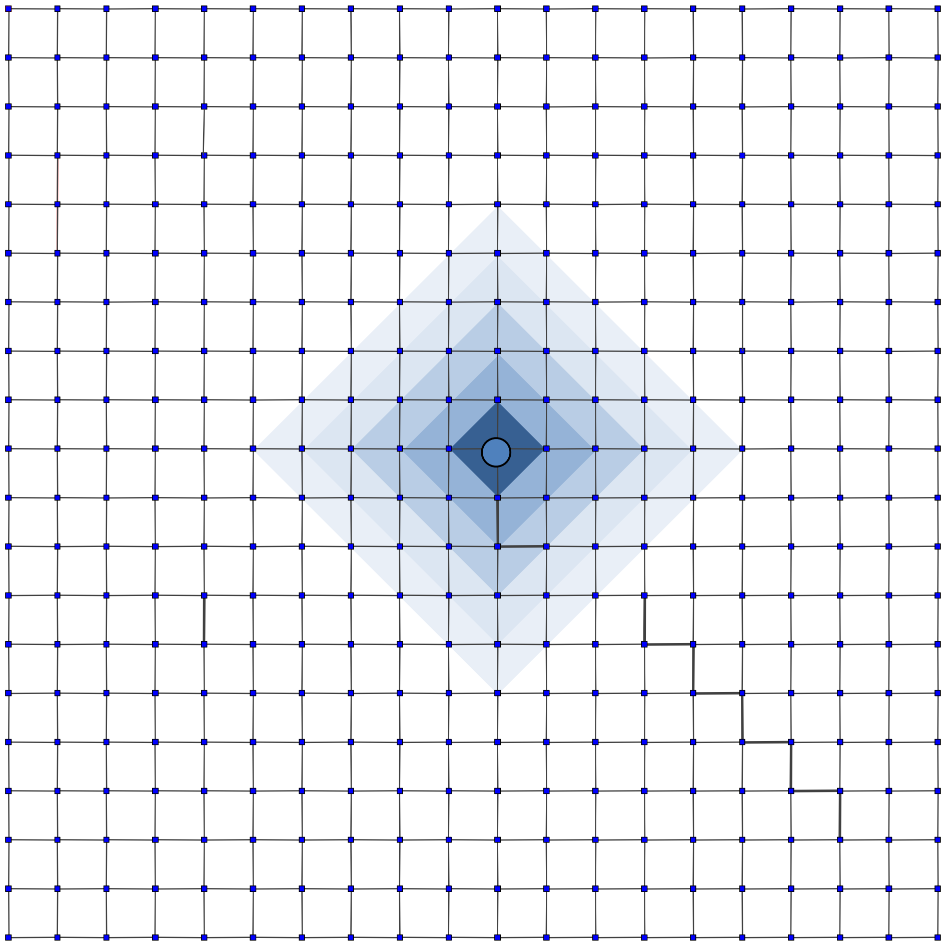
Recall: Kleinberg's Small World Model



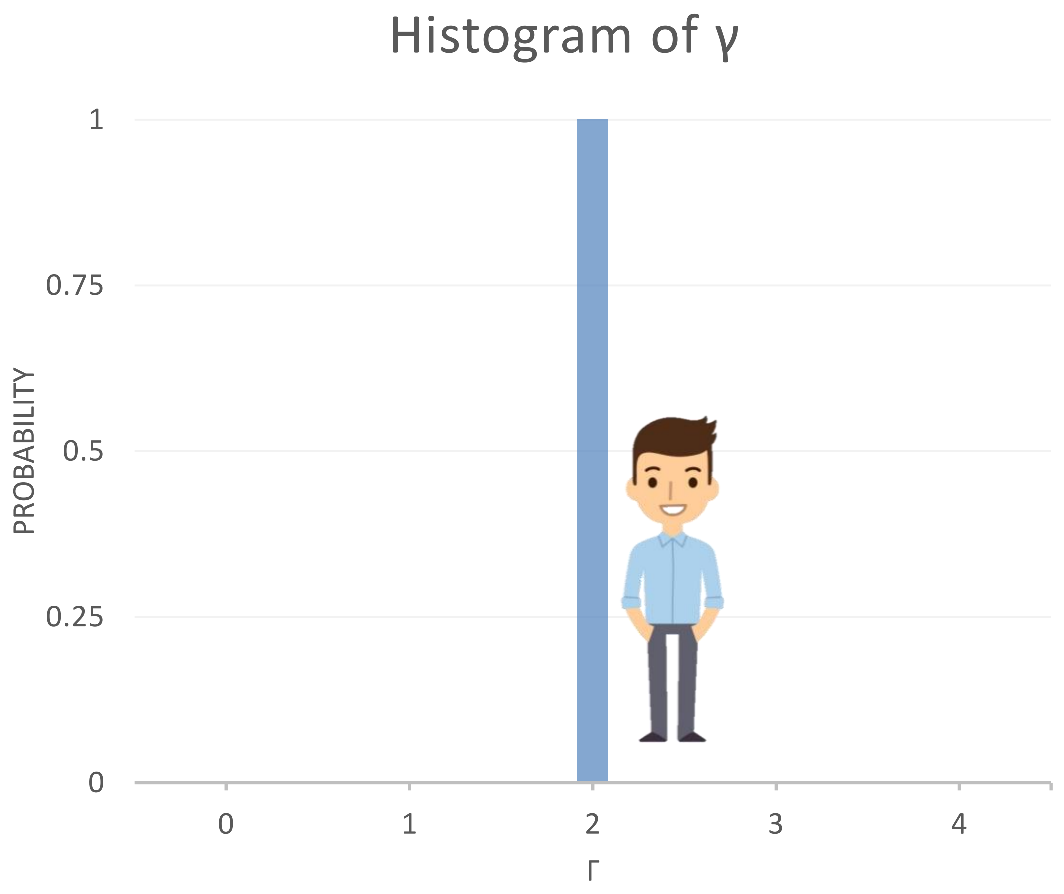
- n people on a k-dimensional grid
- Strong ties within distance q
- Weak ties: $p_{uv} \sim d(u, v)^{-\gamma}$

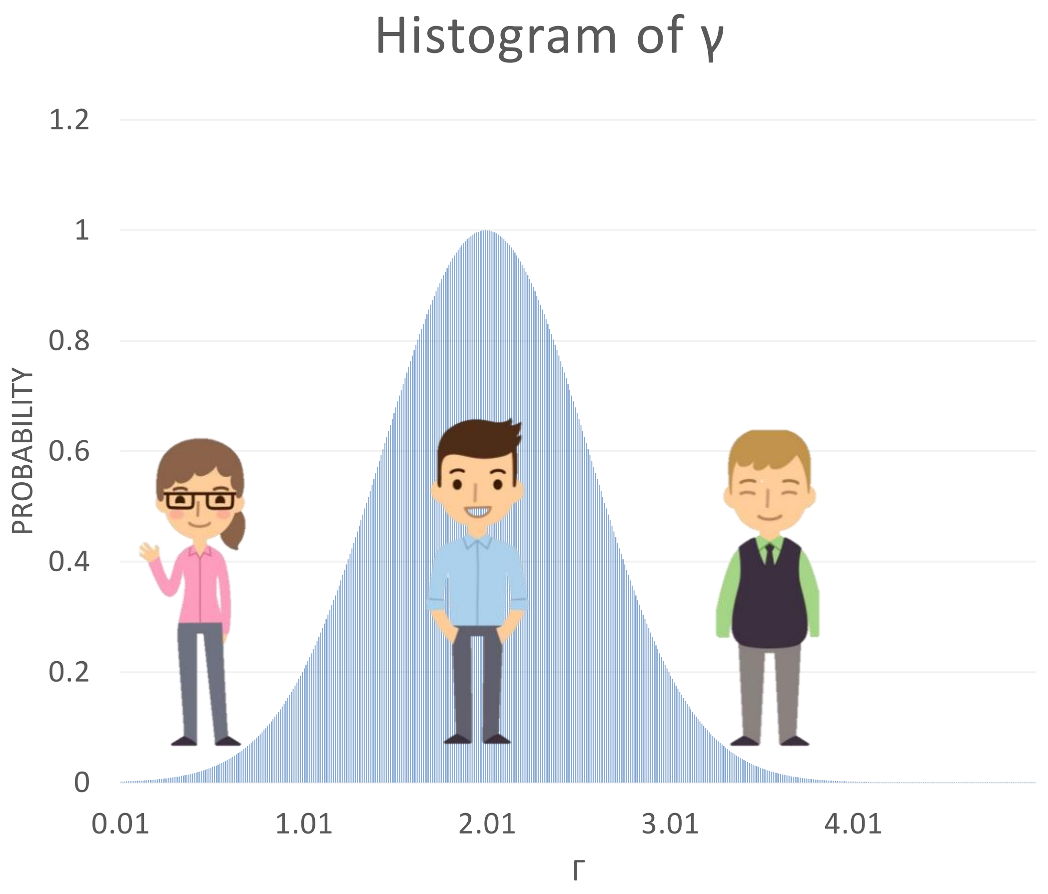
Nonhomogeneous Kleinberg's $HetK_{p,q,D}(n)$



- n people on a k-dimensional grid
- Strong ties within distance q
- Weak ties: u has γ_u from D, and p ties sample from $p_{uv} \sim d_{uv}^{-\gamma_u}$.

A More Natural Histogram





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  - **Theorems**
  - Proof Outline
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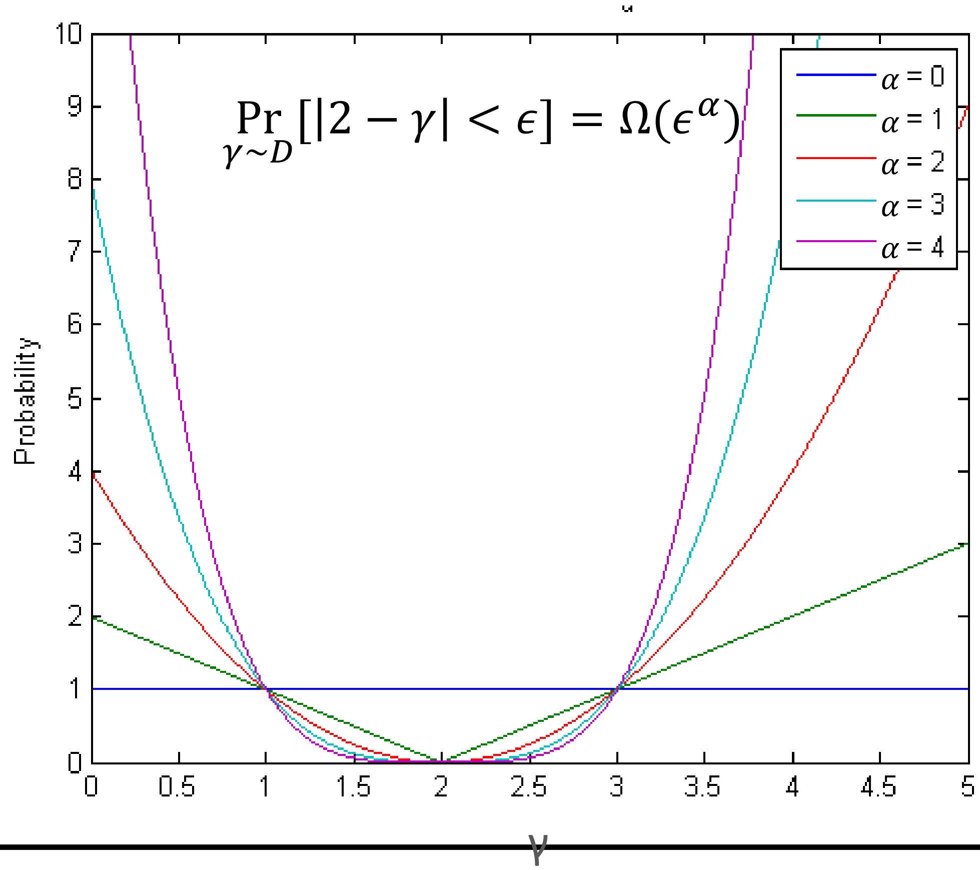
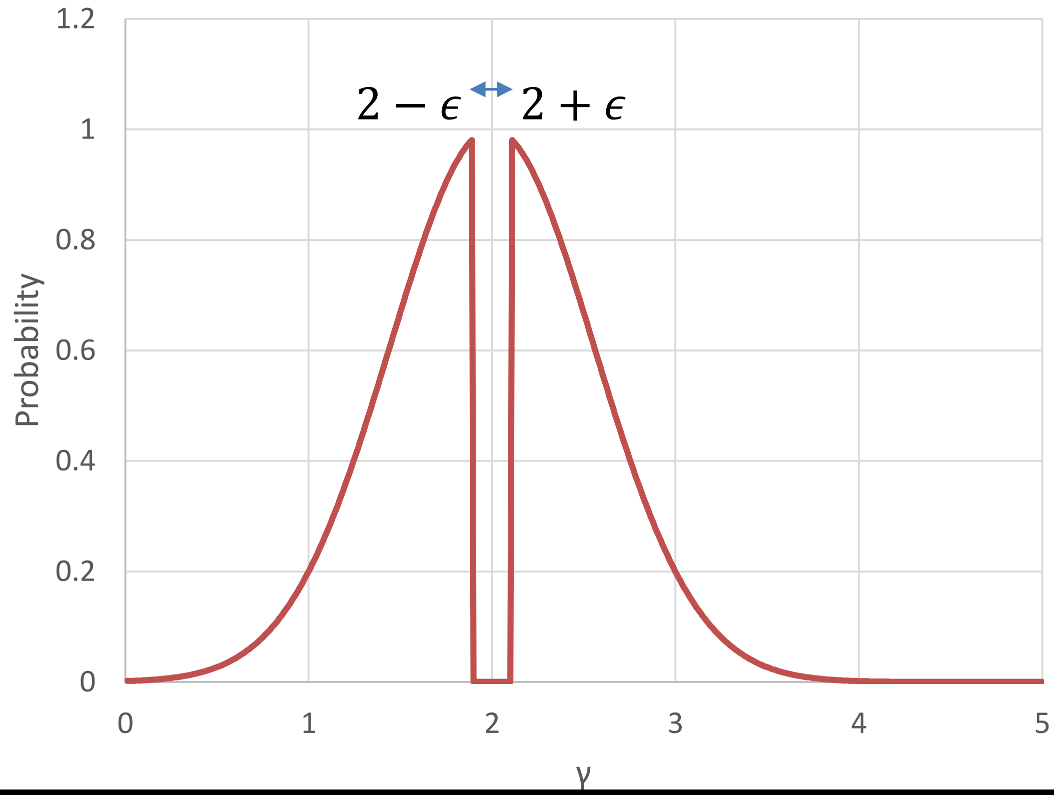
Theorems

Upper bounds



Lower bounds



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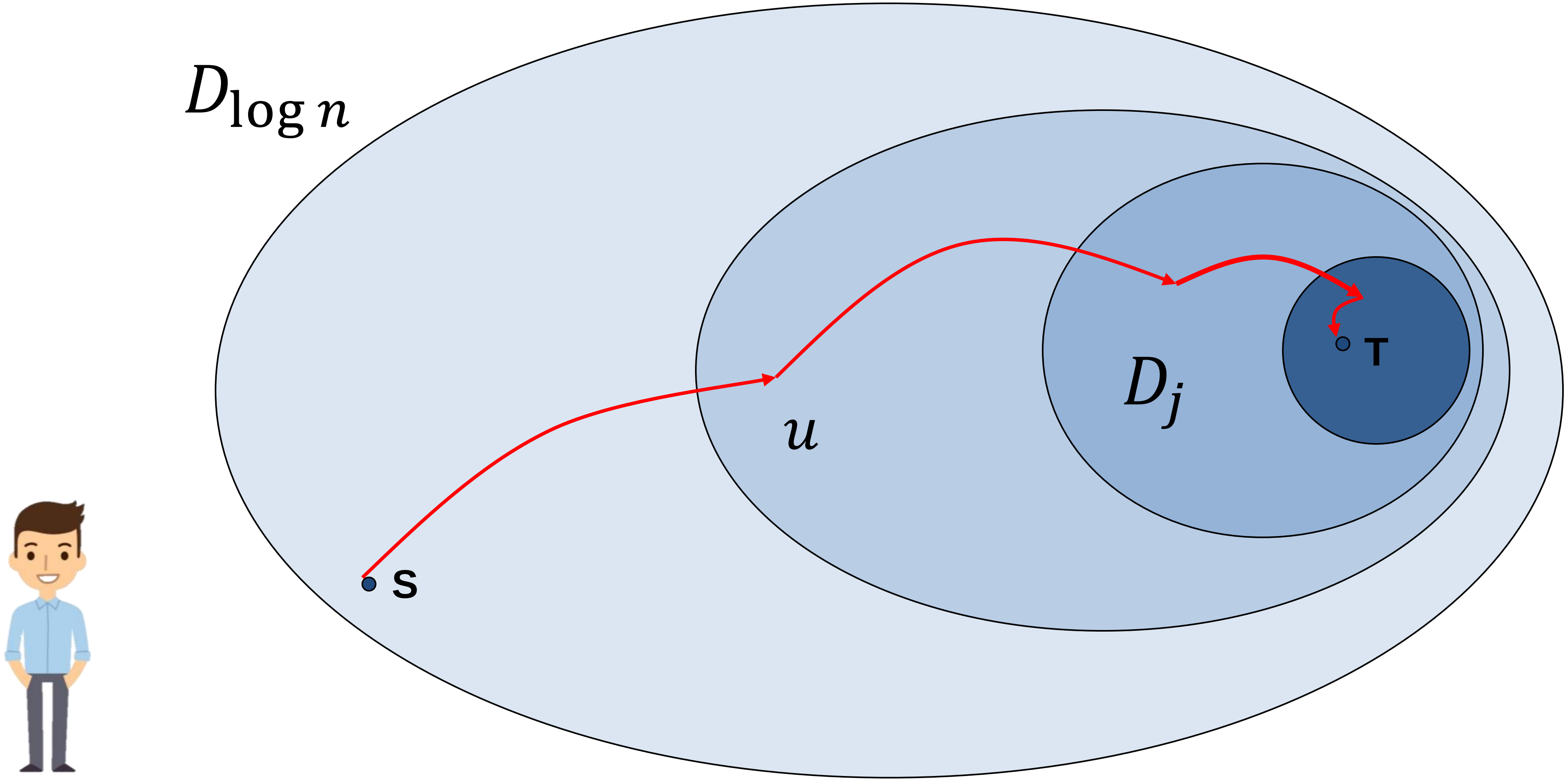
  - **Proof Outline (upper bound)**
- k-Complex Contagions Model

When $\gamma = 2$



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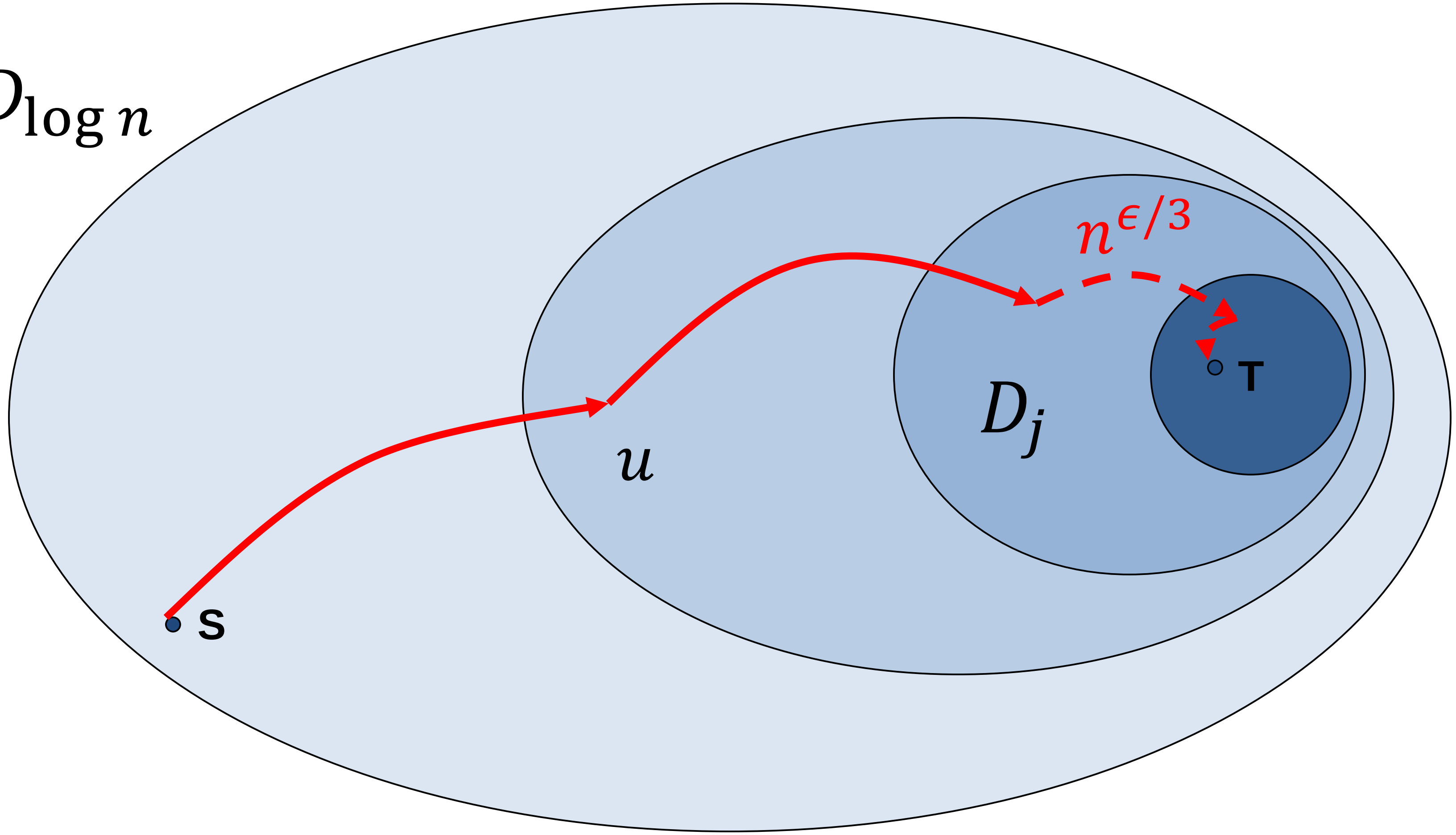
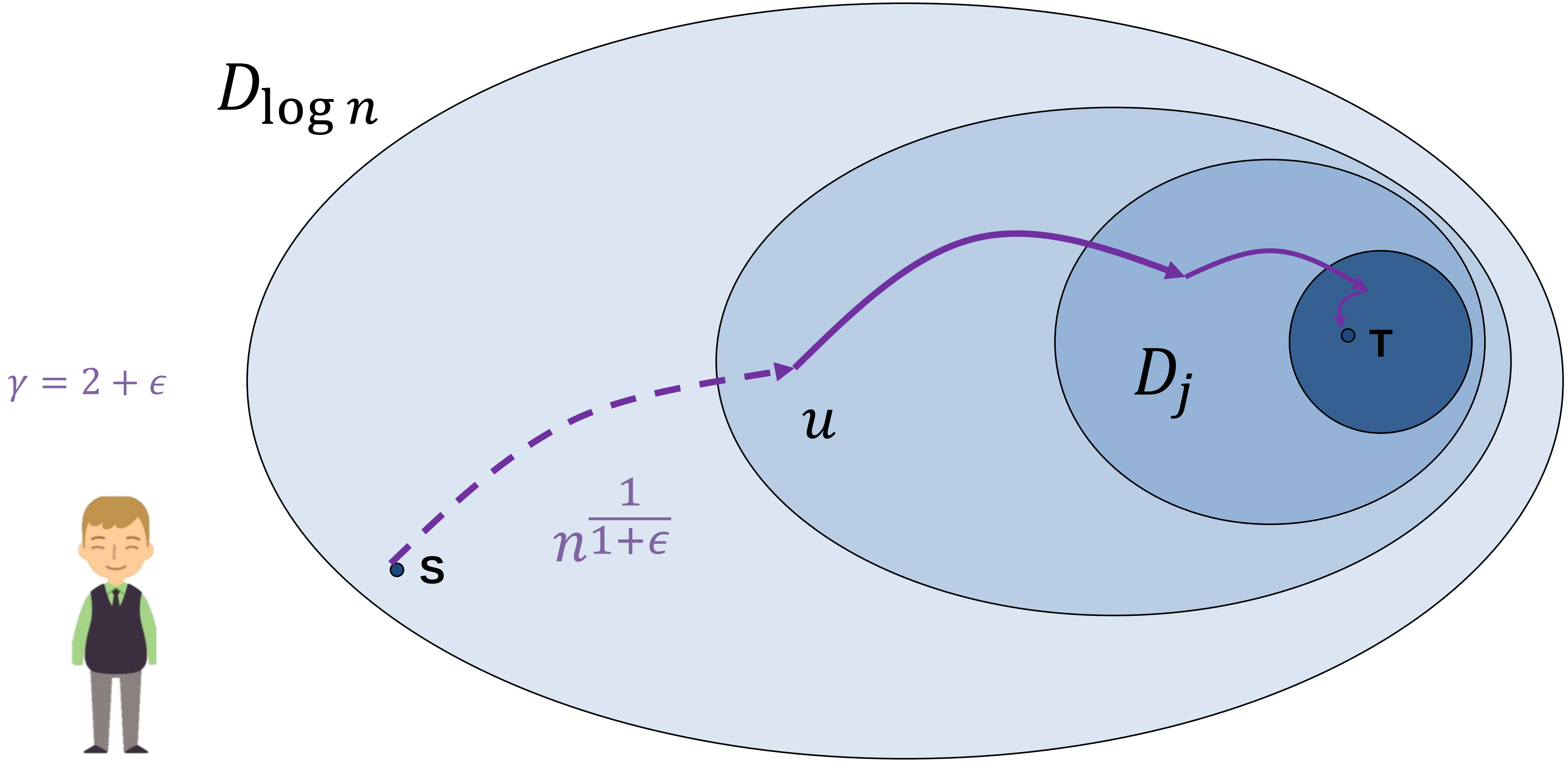
When $\gamma < 2$, weak ties are too random

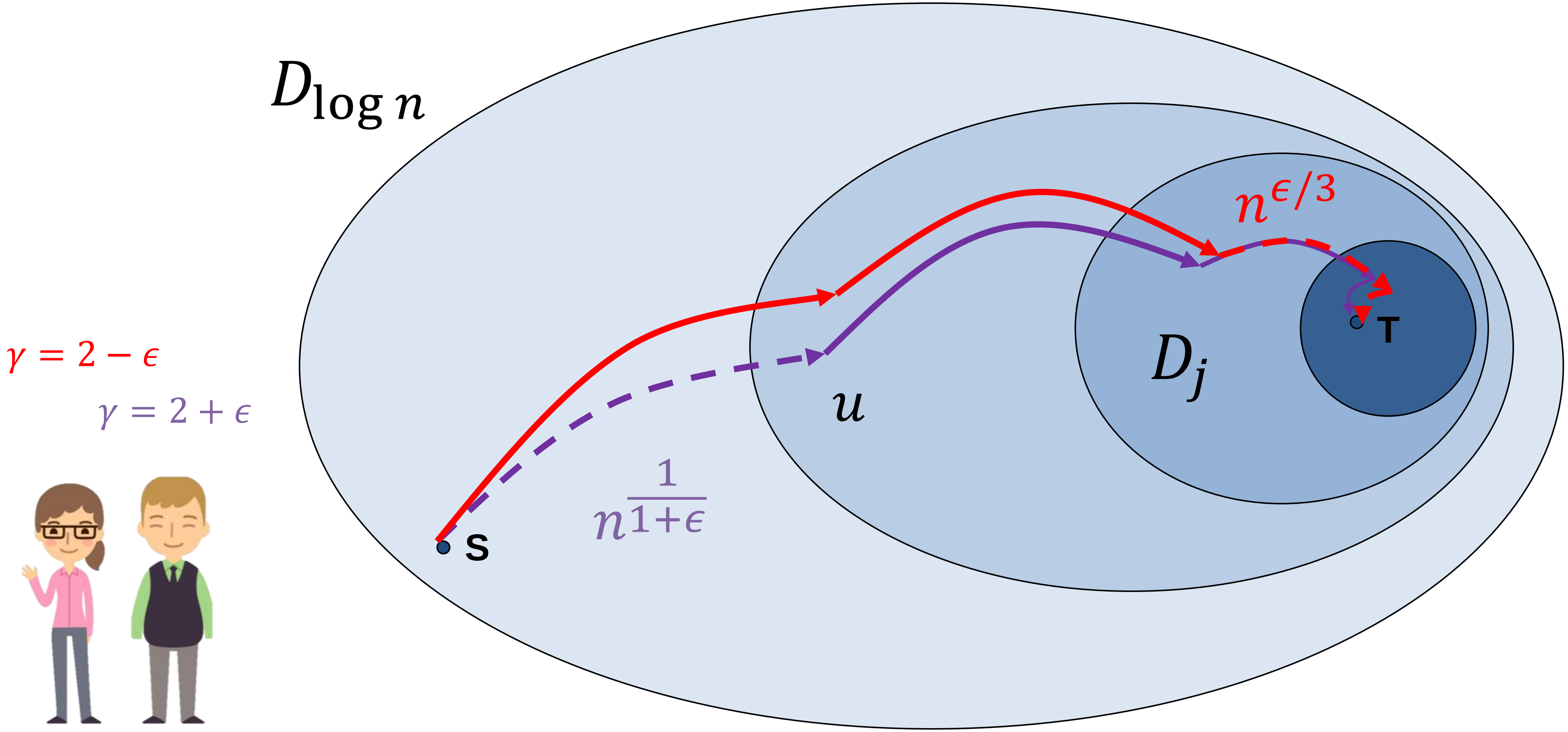
$\gamma = 2 - \epsilon$



When $\gamma > 2$, weak ties are too short



Mixture of Both



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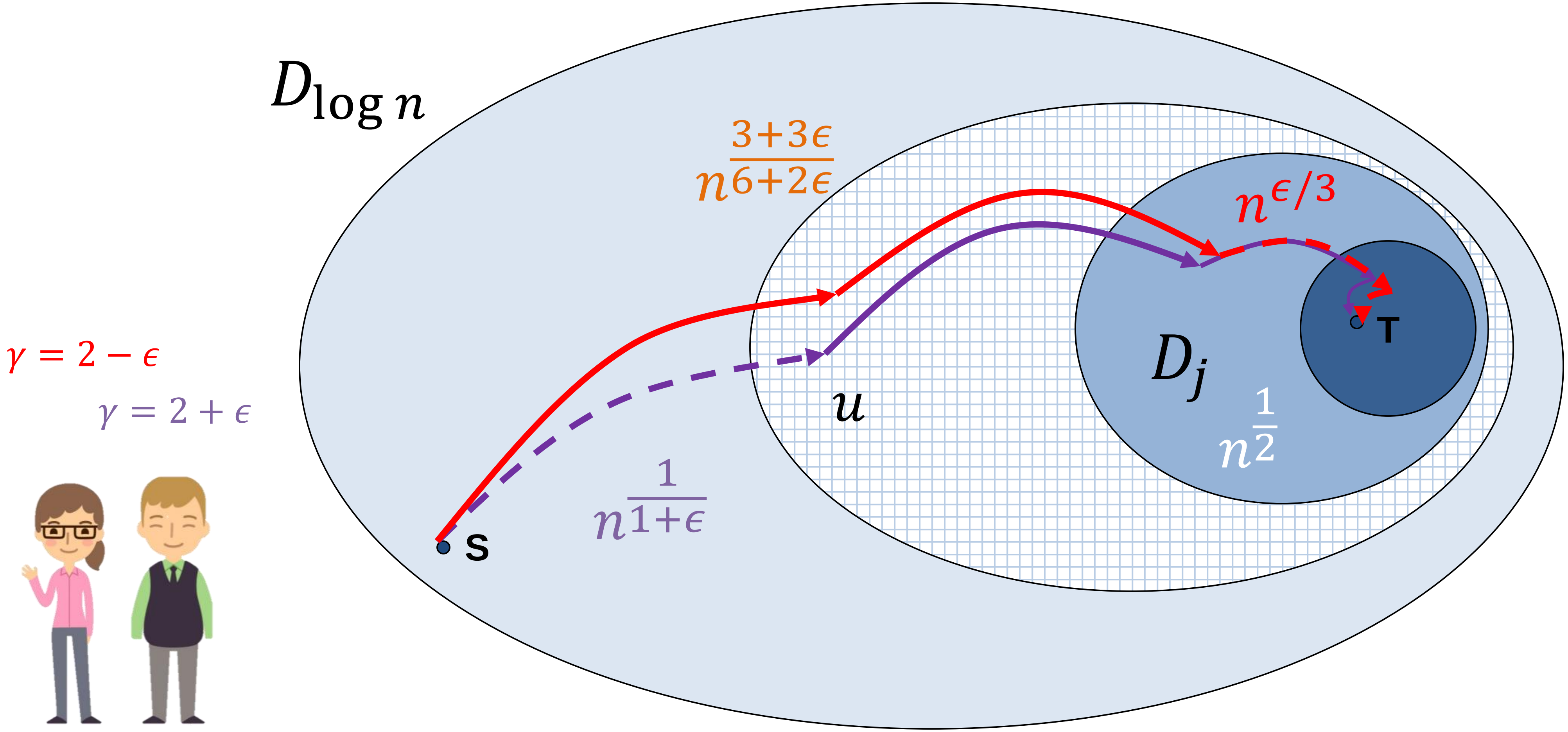
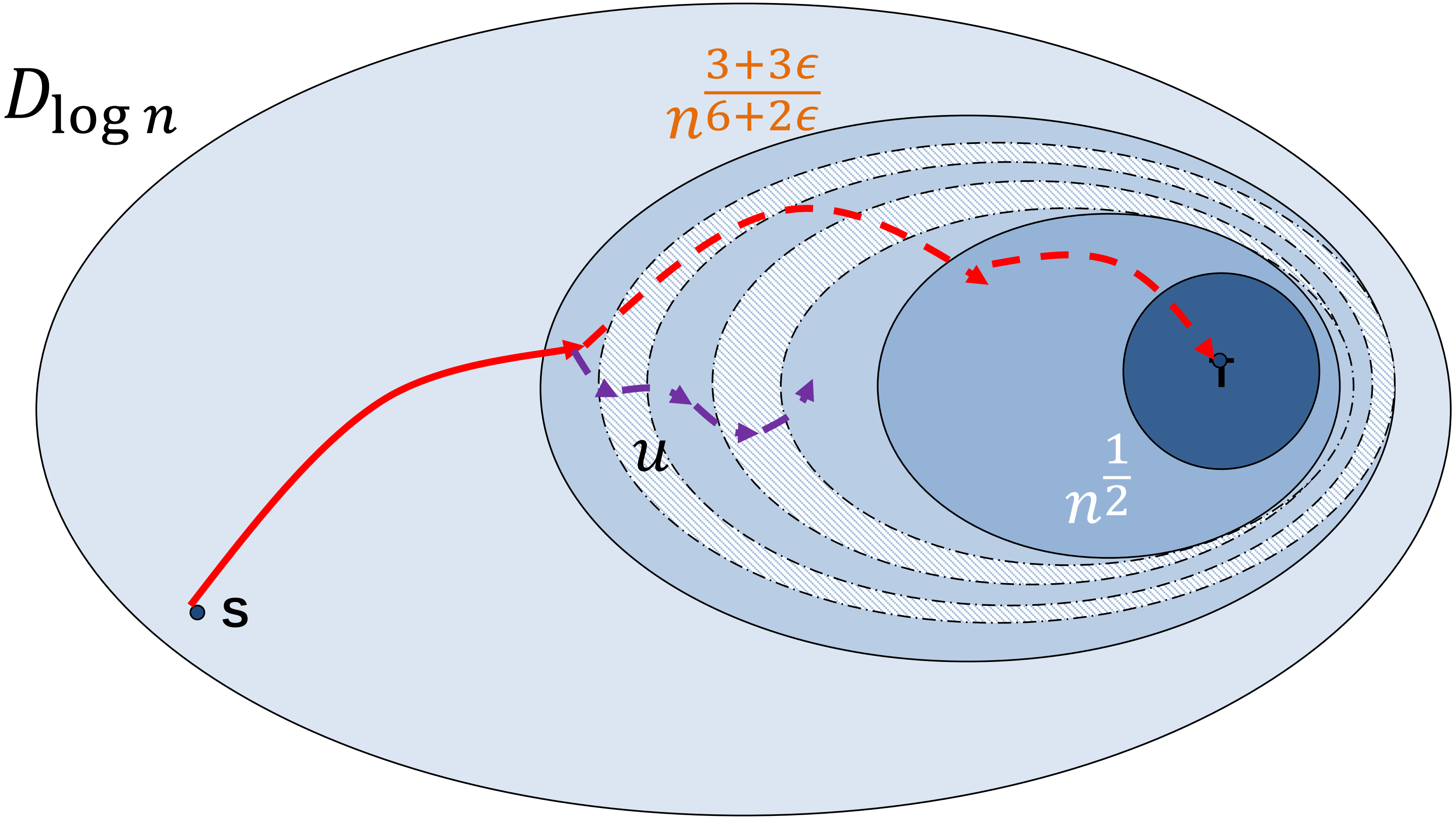


Mixture of Both



$\gamma = 2 - \epsilon$

$\gamma = 2 + \epsilon$

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Thanks for your listening

Upper Bound — Non-negligible Mass Near 2

- Fixed a distribution D with constant α ≥ 0 where $F_D(2+\epsilon) - F_D(2-\epsilon) = \Omega(\epsilon^{\alpha})$ for any integer k > 0 and η > 0, there exists ξ = 3+α+k, such that a k-complex contagion $CC(HetK_{p,q,D(n)}, k, I)$ starting from a k-seed cluster I and where $p > k, q^2/2$ ≥ k takes at most $O(\log^{\xi} n)$ time to spread to the whole network with probability at least $1 - n - \eta$ over the randomness of $HetK_{p,q,D(n)}$.

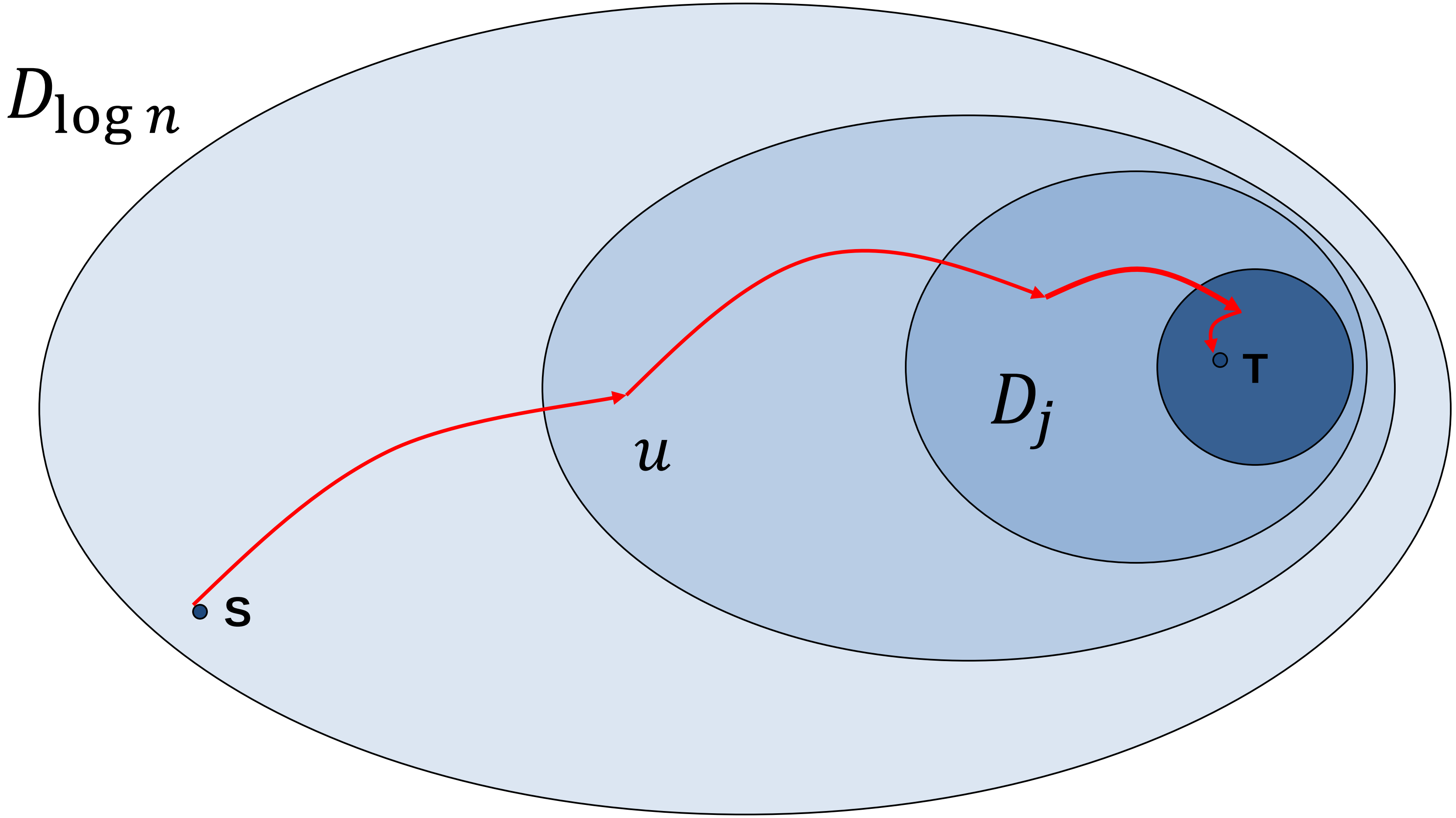
Upper Bound — Fixed k

- Given a distribution D and an integer k > o, such that $\Pr_{\gamma \leftarrow D}[\gamma \in [2, \beta k)] > 0$ where $\beta_k = 2(k+1)$, for all $\eta > 0$ there exists ξ > o depending on D and k such that, the speed of a k-complex contagion $CC(HetK_{p,q,D(n)}, k, I)$ starting from a k-seed cluster I and $p > k, q2/2 \geq k$ is at most $O(\log^{\xi} n)$ with probability at least $1 - n^{-\eta}$.

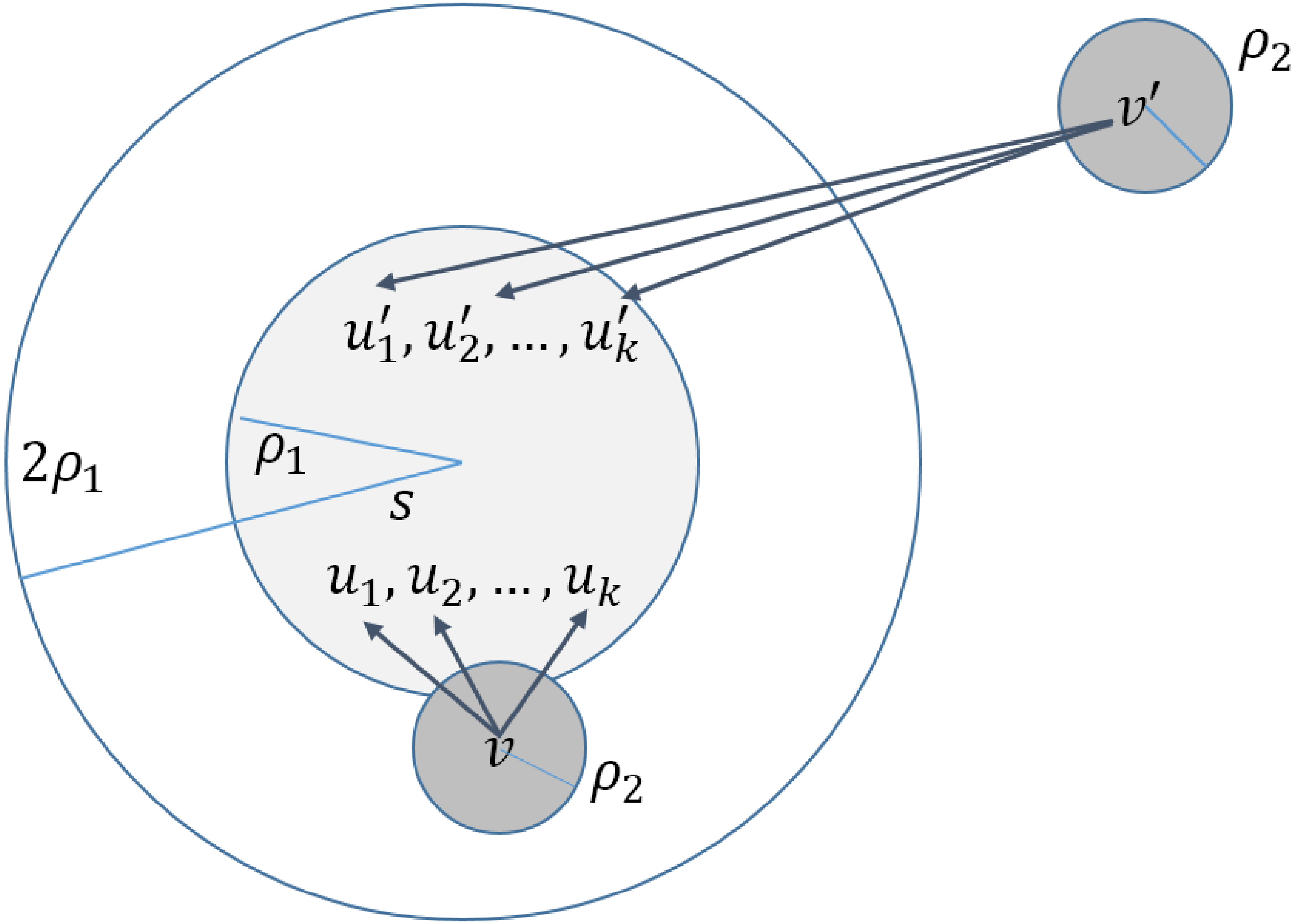
Lower Bound

- Given distribution D, constant integers $k, p, q > 0$, and ε > o such that $F_D(2+\epsilon) - F_D(2-\epsilon) = 0$, then there exist constants $\xi, \eta > 0$ depending on D and k, such that the time it takes a k-contagion starting at seed-cluster I, $CC(HetK_{p,q,D(n)}, k, I)$, to infect all nodes is at least n^{ξ} with probability at least $1 - O(n^{-\eta})$ over the randomness of $HetK_{p,q,D(n)}$.

Idea of Myopic Routing Upper Bound



Idea of Complex Contagion Lower Bound



- Number of nodes within region D_j

$$2^{2j}$$

- Probability of node u connecting to a node $v \in D_j$

$$\frac{1}{K_{2+\epsilon} d_{uv}^{2+\epsilon_u}}$$

- Probability for node u entering region D_j

$$\Omega\left(\frac{\epsilon}{2^{j\epsilon}}\right) \text{if } \epsilon > 0 \text{ and } \Omega\left(\frac{|\epsilon|}{2^{(\log n - j)\epsilon}}\right) \text{if } \epsilon < 0$$

- Probability entering region D_j

$$\Omega\left(\int_0^{\epsilon_0} \frac{\epsilon}{2^{j\epsilon}}\ \epsilon^{\alpha-1} d\epsilon\right)$$

or

$$\Omega\left(\int_0^{\epsilon_0} \frac{\epsilon}{2^{(\log n - j)\epsilon}}\ \epsilon^{\alpha-1} d\epsilon\right)$$

Proof Sketch for lower bound

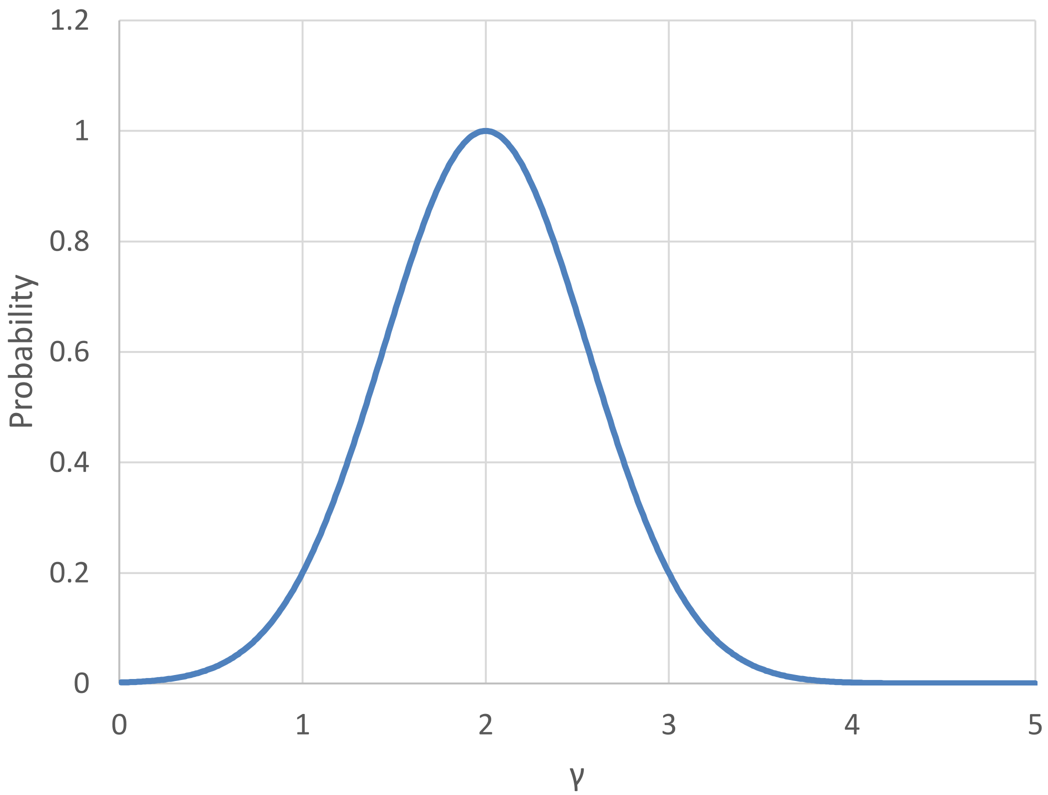
- $\gamma > 2$ the weak ties will be too short (concentrated edges)
- $\gamma < 2$ the weak ties will be too random (diffuse edges)

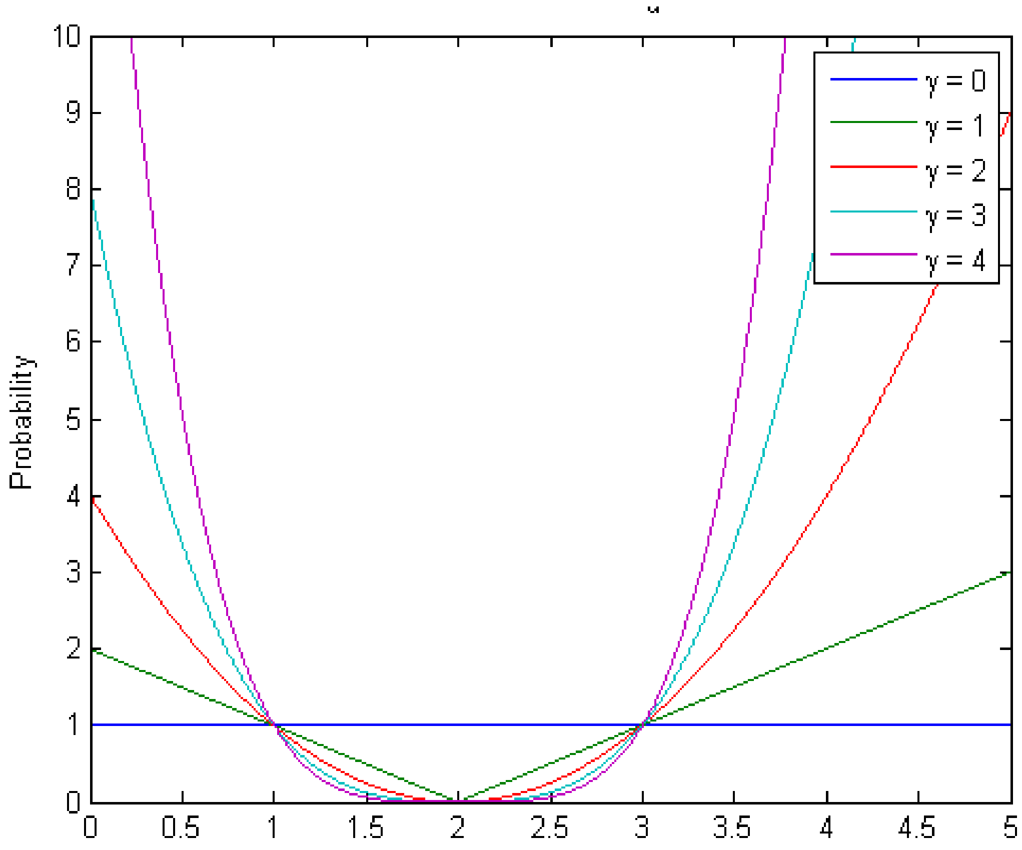
A Very Brief Summary — History

- Kleinberg's small world model models social networks with both strong and weak ties, and the distribution of weak-ties, parameterized by γ.
  - He showed how value of γ influences the efficacy of myopic routing on the network.
  - Recent work on social influence by k-complex contagion models discovered that the value of γ also impacts the spreading rate

A Very Brief Summary — Our Work

- A natural generalization of Kleinberg's small world model to allow node heterogeneity is proposed, and
  - We show this model enables myopic routing and k-complex contagions on a large range of the parameter space.
  - Moreover, we show that our generalization is supported by real-world data.


1.2
1
0.8
0.6
0.4
0.2
0
Probability
0
1
2
3
4
5
γ


10
9
8
7
6
5
4
3
2
1
0
Probability
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
γ = 0
γ = 1
γ = 2
γ = 3
γ = 4