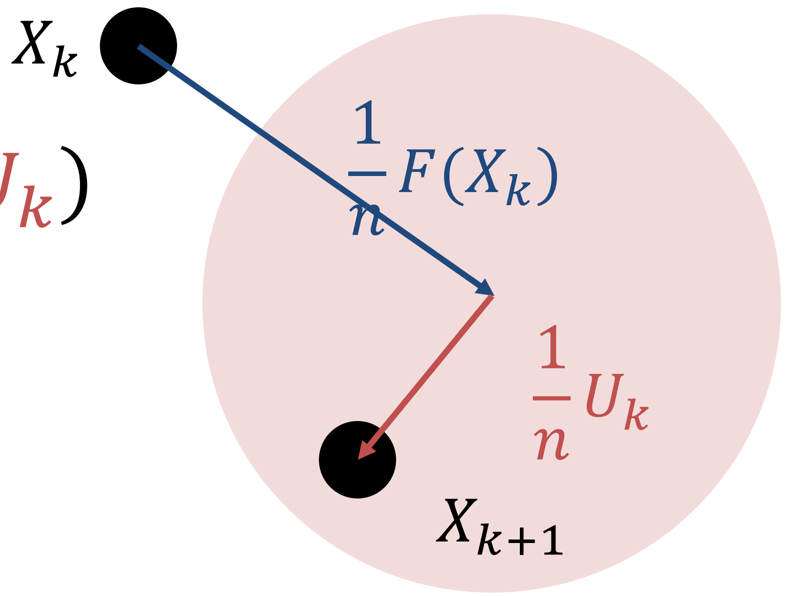
Reinforced random walk with F

A discrete time stochastic process $\{X_k : k = 0, 1, \ldots\}$ in $\mathbb{R}^d$ that admits the following representation,

$$X_{k+1} - X_k = \frac{1}{n}\left(F(X_k) + U_k\right)$$



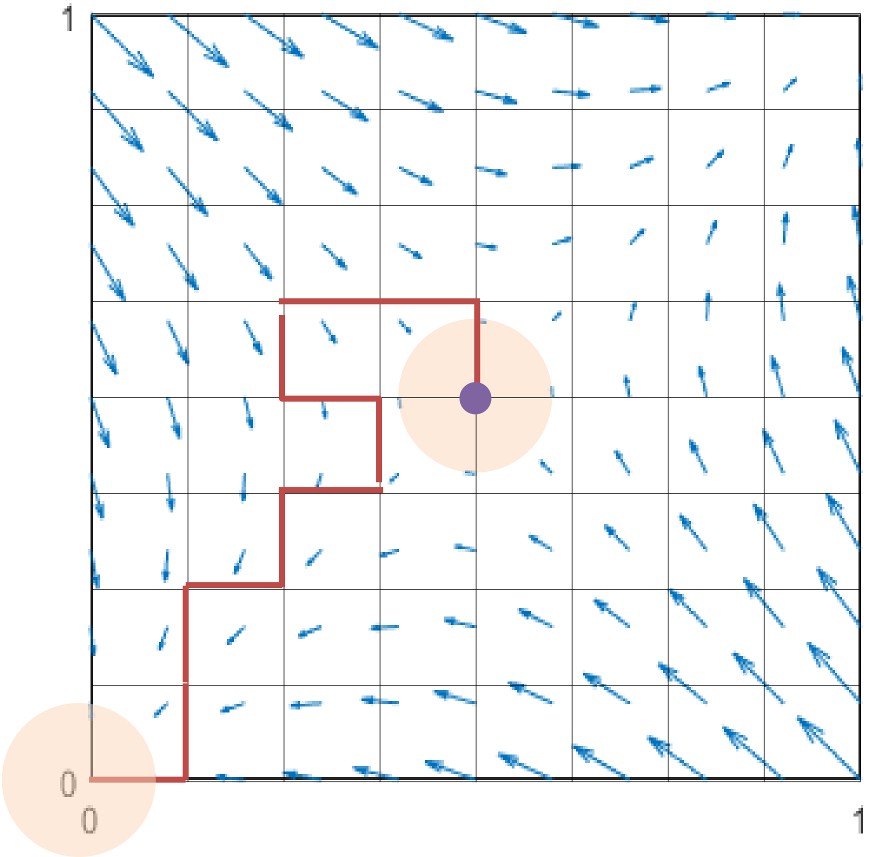
- Agent based models with n agents
  - Evolutionary games
  - Dynamics on social networks
- Heuristic local search algorithms with uniform step size $1/n$

Gradient-like dynamics

Converges to an attracting fixed-point region in $O(n \log n)$ steps.

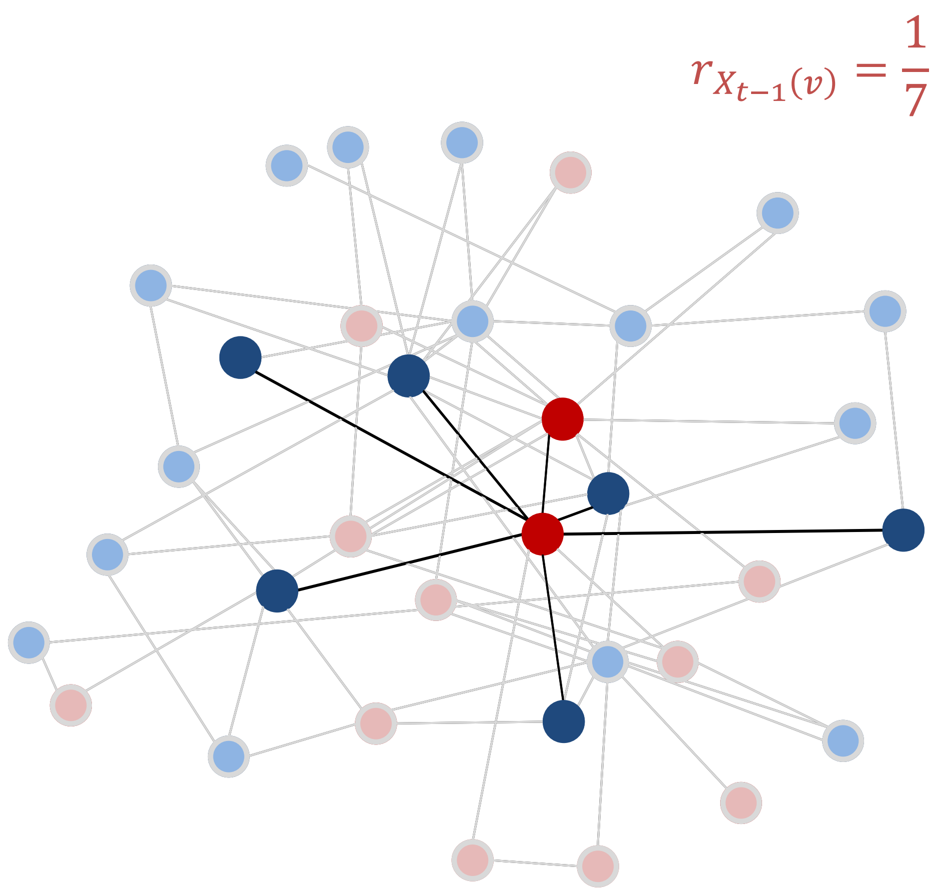
If

- Noise, U_k
  - Martingale difference
  - bounded
  - Noisy
- Expected difference, $F \in \mathcal{C}^2$
  - Fixed points are hyperbolic
  - Potential function



Node Dynamic ND(G, f_{ND}, X_0)[SY18]

- Fixed a (weighted) graph $G = (V, E)$ opinion set $\{0,1\}$, **an update function $\boldsymbol{f_{ND}}$**
- Given an initial configuration $X_0 : V \mapsto \{0,1\}$
- At round t,
  - A node v is picked uniformly at random
  - $\boldsymbol{X_t(v) = 1}$ ***w.p.*** $\boldsymbol{f_{ND}\left(r_{X_{t-1}(v)}\right)}$ **;**
    $= 0$ otherwise



Our Dichotomy Theorem

- Given a smooth rich-get-richer function $f_{ND} \in \mathcal{C}^2$, and a planted community graph $G = K(n,p)$. The maximum expected consensus time of $\mathrm{ND}(G, f_{ND}, X_{\mathbf{0}})$ has two cases: