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Even if the message is sent in the clear, Alice might still like a proof that Bob sent it.

We still assume Alice and Bob share a key. Intuitively, we want a way to “mark” or “tag” a message, using the secret key. The recipient of the message should be able to verify that the tag was generated using the shared key: it could only have come from Bob.
Message Authentication

Message Authentication Codes

A MAC consists of three p.p.t. algorithms, \((\text{Gen}, \text{Mac}, \text{Vrfy})\).

- **\(\text{Gen}(1^n)\):** outputs a key \(k\), \(|k| > n\).

- **\(\text{Mac}(k, m)\):** outputs a tag, \(t\). This might be randomized, so we write it as \(t \leftarrow \text{Mac}(k, m)\). Also, \(m\) might be variable length, or we might only support some fixed length for each \(n\): \(m \in \{0, 1\}^{\ell(n)}\)

- **\(\text{Vrfy}(k, m, t)\):** is a deterministic algorithm that outputs a bit \(b\). \(b = 1\) means \(t\) is a valid tag for \(m\) using \(k\), and \(b = 0\) means \(t\) is invalid and \(m\) should be rejected.
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Just as with encryption, we have a correctness requirement:

For every \( n \), every \( k \) output by Gen and every \( m \) (of appropriate size), \( \text{Vrfy}(k, m, \text{Mac}(k, m)) = 1 \).
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**Canonical Verification**: Unlike with encryption schemes, deterministic MAC algorithms *can* be secure. For such algorithms, we have a simple way of verifying.
**Vrfy\((k, m, t)\)**: Compute \(t' = \text{Mac}(k, m)\). If \(t' = t\) output 1. Else, output 0.
Unforgeability

We define a new security game, Mac-forgé_{A, Π}, for capturing that a MAC scheme, Π = (Gen, Mac, Vrfy) is unforgeable. That is, without the key k, an adversary A cannot create a valid tag.
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We define a new security game, $\text{Mac-forg}_{A,\Pi}$, for capturing that a MAC scheme, $\Pi = (\text{Gen}, \text{Mac}, \text{Vrfy})$ is unforgeable. That is, without the key $k$, an adversary $A$ cannot create a valid tag.

Let $Q$ be the set of messages queried.

$A$ wins if

1) $\text{Vrfy}(k, m^*, t^*) = 1$
2) $m^* \notin Q$
Unforgeability

Definition

A MAC scheme \( \Pi = (\text{Gen}, \text{Mac}, \text{Vrfy}) \) is existentially unforgeable under an adaptive chosen-message attack, (or, secure) if for all p.p.t. adversaries \( \mathcal{A} \), there is some negligible function \( \text{negl} \) such that:

\[
\Pr[\text{Mac-forge}_{\mathcal{A},\Pi}(n) = 1] \leq \text{negl}(n)
\]
A Construction

A fixed length MAC, \( \Pi \)

Let \( F \) be a PRF. We define a fixed length MAC for messages of length \( n \):

- \( \text{Gen}(1^n) : k \leftarrow \{0, 1\}^n \)
- \( \text{Mac}(k, m) : \) If \( |m| \neq n \), abort. Otherwise, output \( t = F_k(m) \).
- \( \text{Vrfy}(k, m, t) : \) If \( |m| \neq n \), output 0. Otherwise, compute \( t' = F_k(m) \). If \( t' = t \) output 1, otherwise, output 0.

Claim: If \( F \) is a PRF, then the construction above is a secure fixed-length MAC.

Claim: If there exists \( A \) with non-negligible advantage in the \( \text{Mac-forg}\) game, there exists \( A_r \) that wins in the \( \text{PrivK-prf} \) game with probability \( \frac{1}{2} + \frac{1}{p(n)} \).
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Claim: If $F$ is a PRF, then the construction above is a secure fixed-length MAC.

Claim: If there exists $\mathcal{A}$ with non-negligible advantage in the Mac-forge$_{\mathcal{A},\Pi}$ game, there exists $\mathcal{A}_r$ that wins in the PrivK$_{\mathcal{A},F}^{\text{prf}}$ game with probability $\frac{1}{2} + \frac{1}{p(n)}$. 