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Subsets: $\{1, 3\} \subseteq \{1, 2, 3, 4\}$. Also, $\{1, 3\} \subset \{1, 2, 3, 4\}$

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Elements in the relation R indicate which students are enrolled in which classes.
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Example: $R : \mathcal{N} \rightarrow \mathcal{N}$, where $R(x) = x^2$

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Taking $\mathcal{U} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
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Let R denote the set of simple mathematical expressions.

- If ρ is a variable, then $\rho \in R$.
- If $\rho_1, \rho_2 \in R$ then $(\rho_1 + \rho_2) \in R$ and $(\rho_1 * \rho_2) \in R$.
- The first two rules define every element of R.