Non-deterministic Finite Automata

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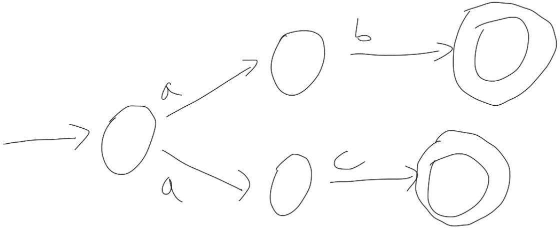

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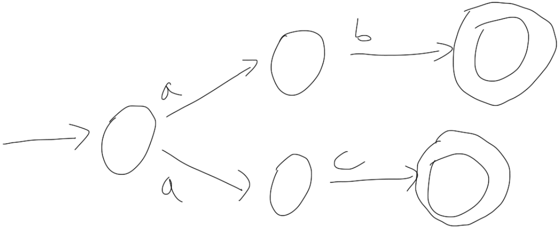

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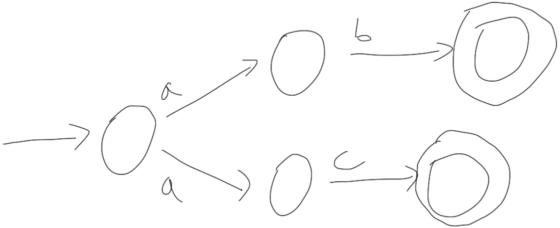

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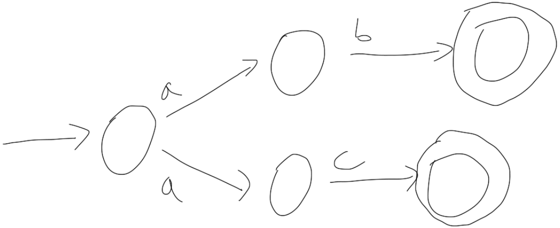

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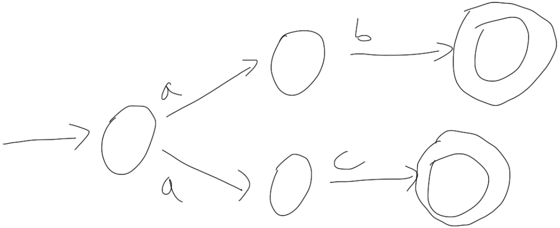

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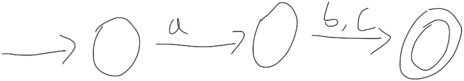

NFA Example

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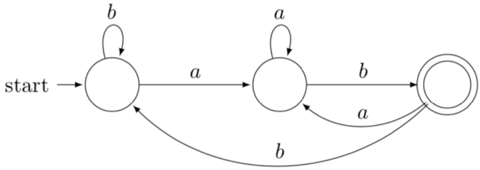

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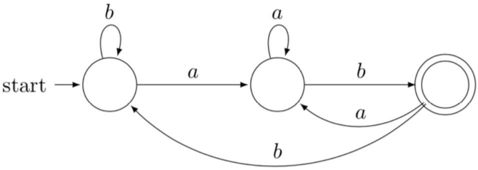

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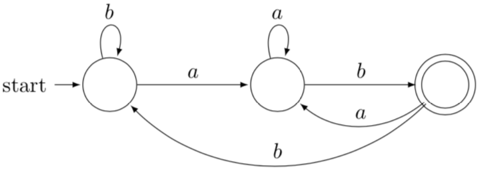

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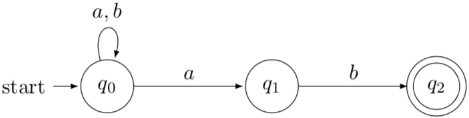


Equivalence between RG and NFA

Lemma 9.4

If $L = \mathcal{L}(G)$ for some regular grammar G, then there exists an NFA M such that $L = \mathcal{L}(M)$.

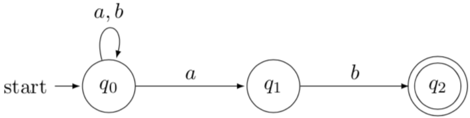
Lemma 9.5

If $L = \mathcal{L}(M)$ for some NFA M, then there exists a regular grammar G such that $L = \mathcal{L}(G)$.

Theorem 9.2

L is regular if and only if there exists an NFA M such that $L = \mathcal{L}(M)$.

Example: RG from NFA



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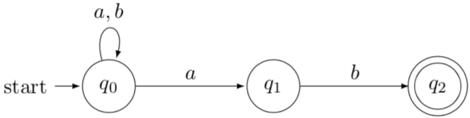

$G = (\{Q_0, Q_1, Q_2\}, \{a, b\}, q_0, P)$, where:
$P = \{Q_0 \to aQ_0, Q_0 \to bQ_0, Q_0 \to aQ_1, Q_1 \to bQ_2, Q_2 \to \Lambda\}$

Formalizing NFA

Given a set of elements, Q, we denote the set of all subsets of Q, which we call the *power set* of Q, by 2^Q.

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NFAs

An NFA, M, is a quintuple, $(Q, \Sigma, q_0, \delta, A)$, where Q, Σ, q_0 and A are defined as in DFAs, and $\delta : Q \times \Sigma \to 2^Q$

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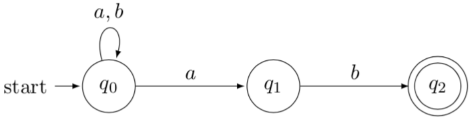
δ^*

For NFA $M = (Q, \Sigma, q_0, \delta, A)$, δ^* is a function that takes a state and a string as input and produces a resulting set of states. That is $\delta^* : Q \times \Sigma^* \to 2^Q$, such that:

- for any $q \in Q$, $\delta^*(q, \Lambda) = \{q\}$, and
- for any $q \in Q$, any $\sigma \in \Sigma$, and any $x \in \Sigma^*$,

$$\delta^*(q, x\sigma) = \bigcup_{p \in \delta^*(q,x)} \delta(p, \sigma).$$

NFA to DFA: Example 1



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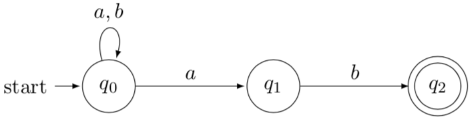

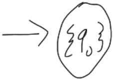

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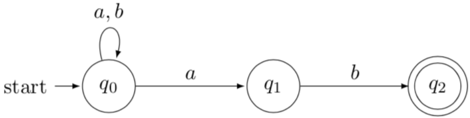

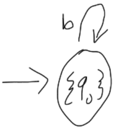

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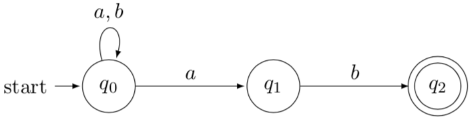

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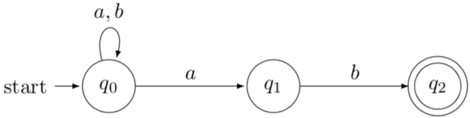

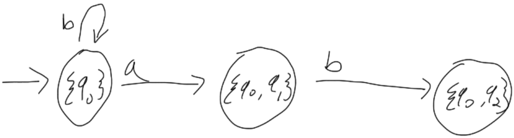

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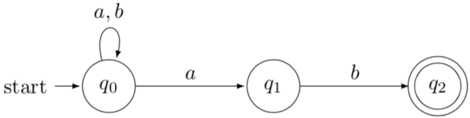

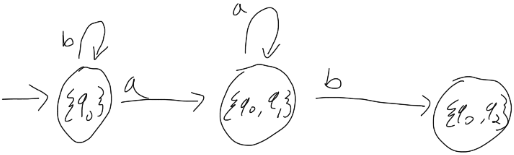

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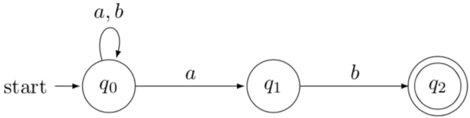

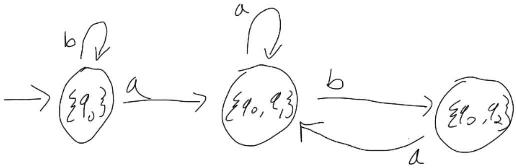

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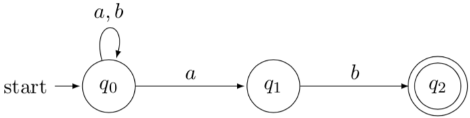

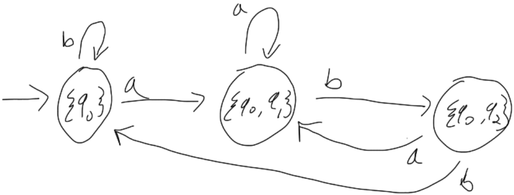

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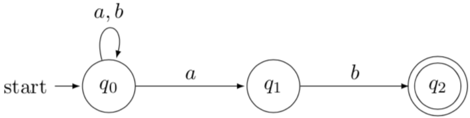

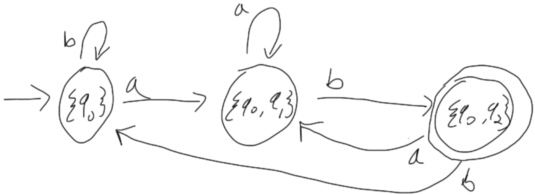

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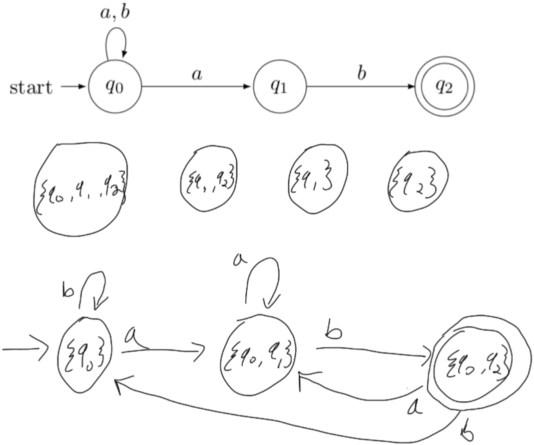

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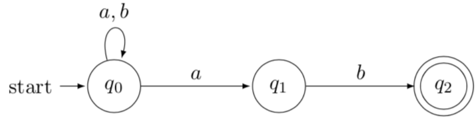
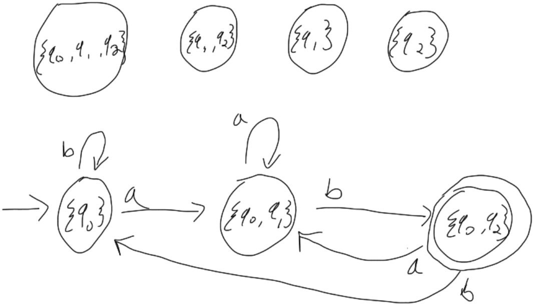
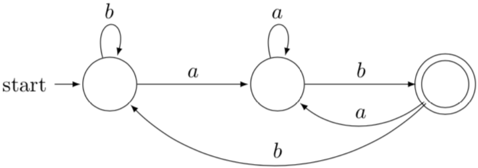
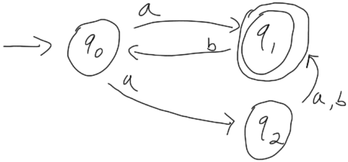






NFA to DFA: Example 2



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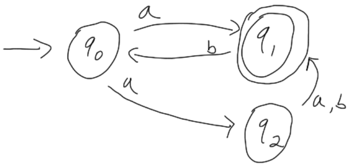

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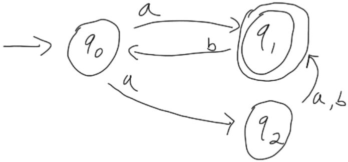

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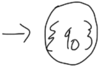

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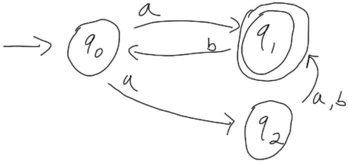

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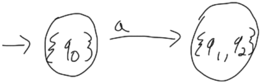

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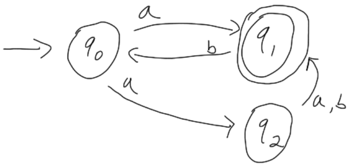

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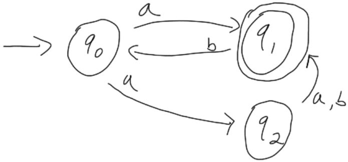

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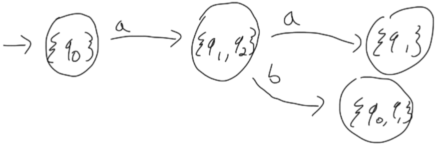

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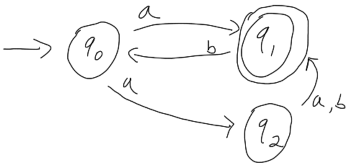

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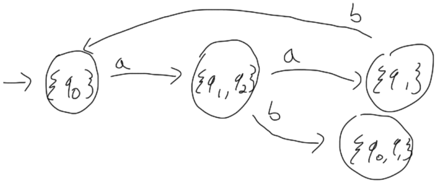

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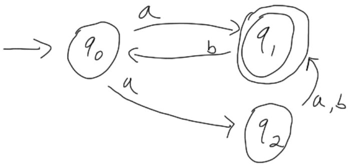

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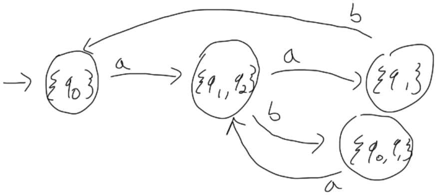

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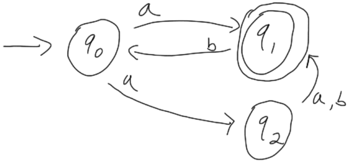

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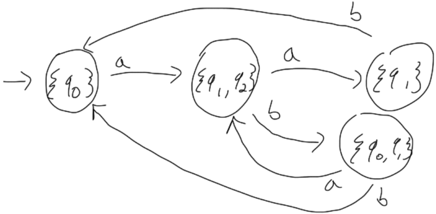

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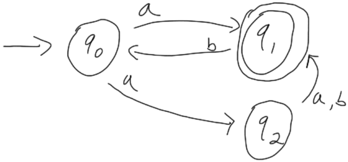

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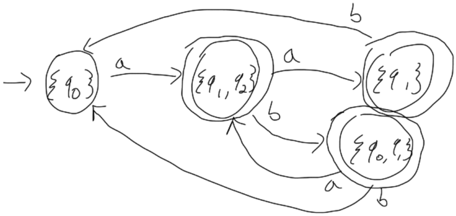
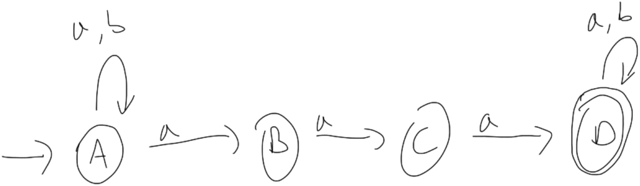


NFA to DFA: Example 3



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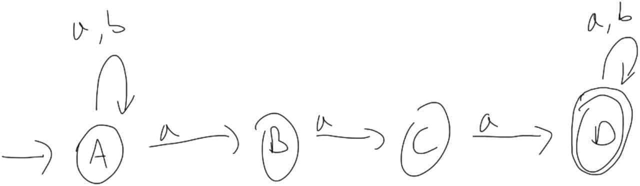

strings containing "aaa"

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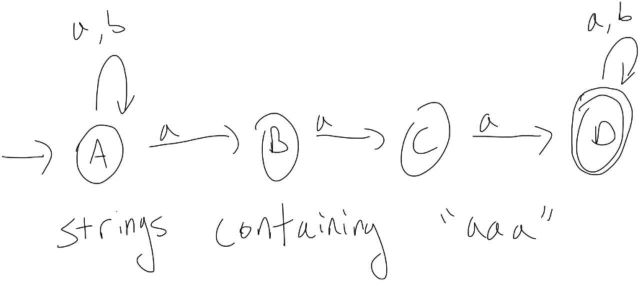

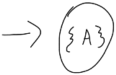

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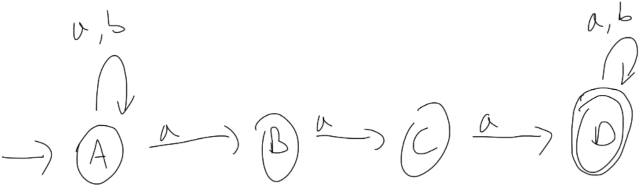

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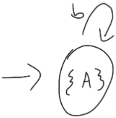

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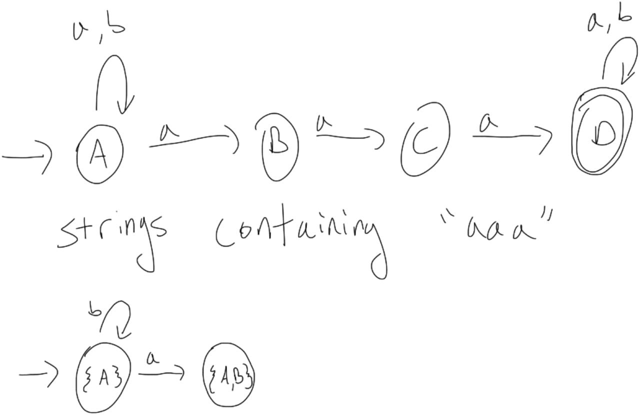

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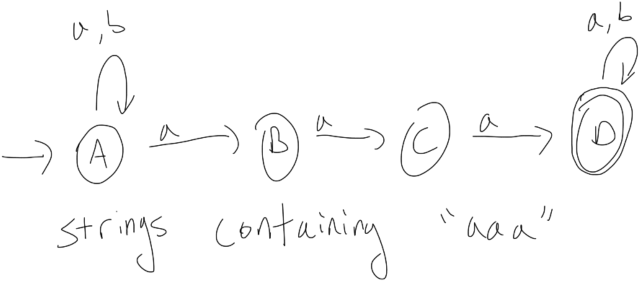

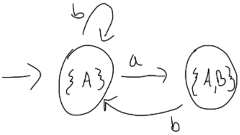

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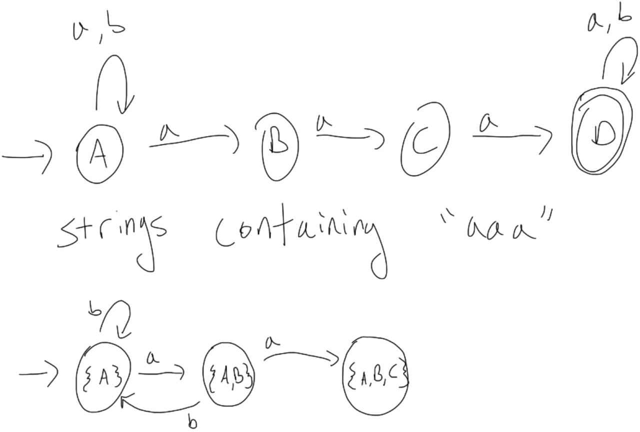
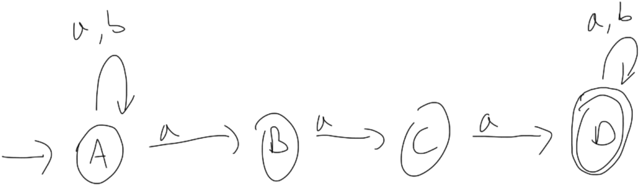
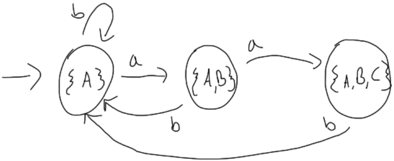


NFA to DFA: Example 3



strings containing "aaa"



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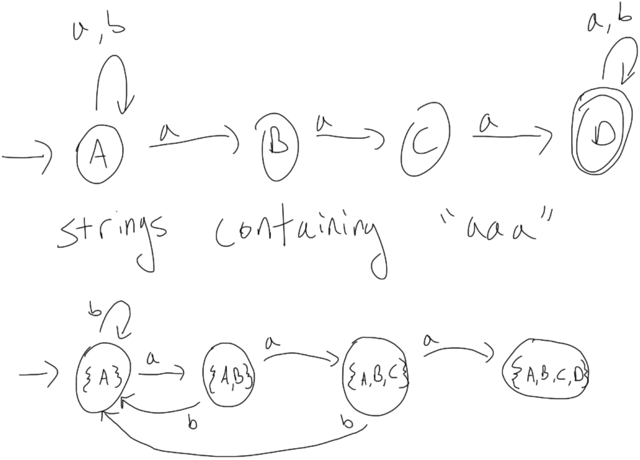

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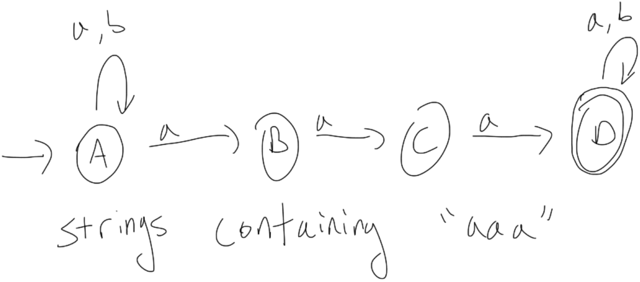

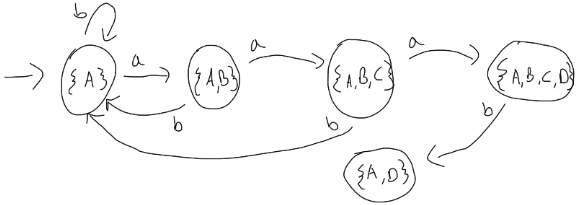

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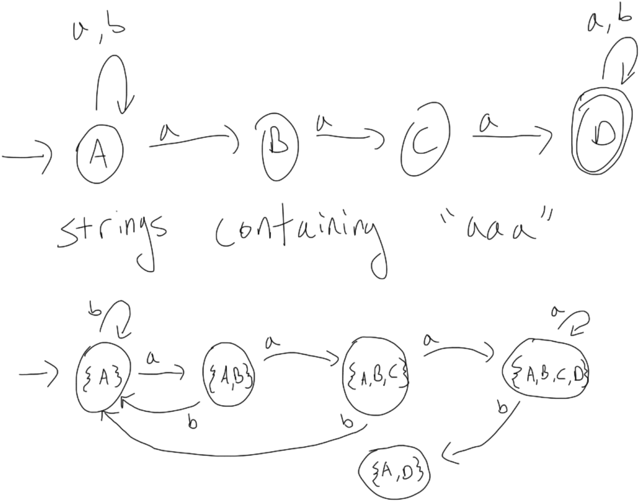

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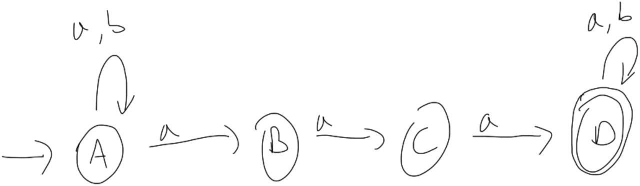

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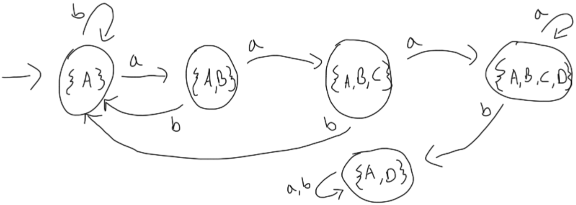

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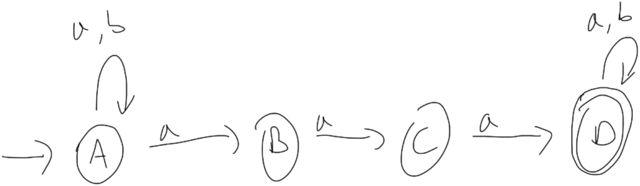

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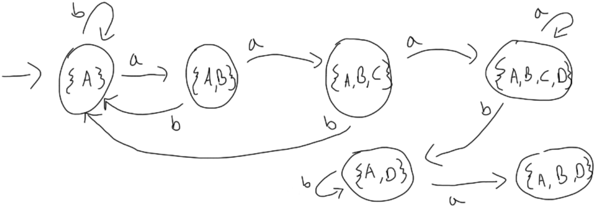

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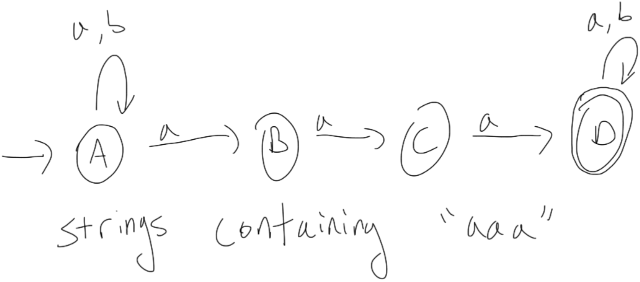

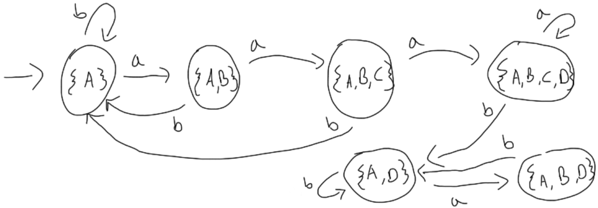

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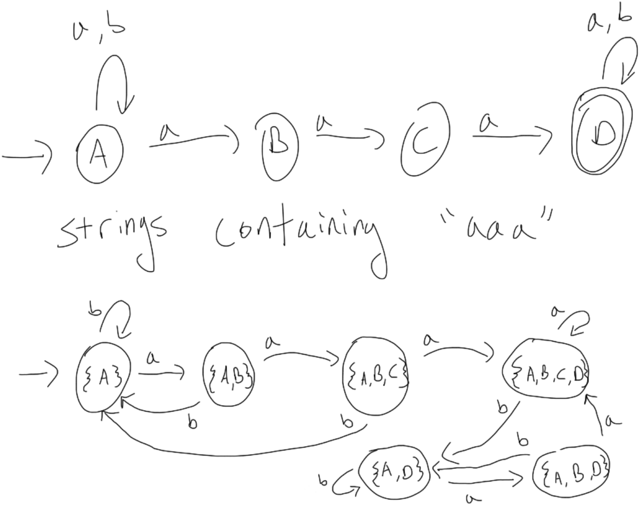

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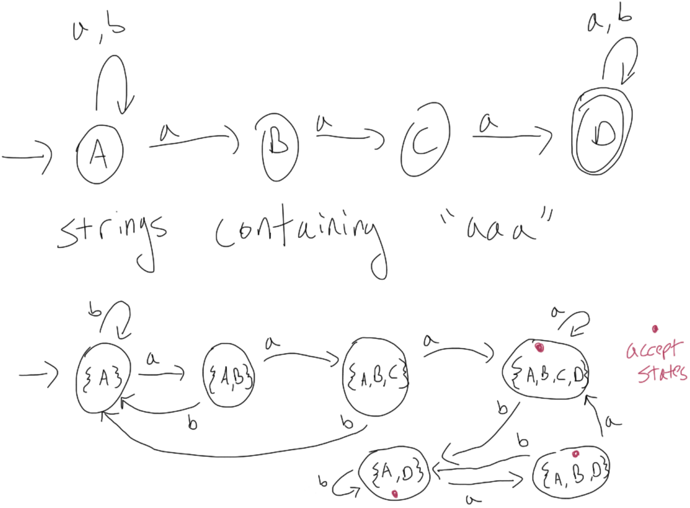

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Choose any such q_i, and repeat.

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$\delta'(T, x) = \bigcup_{q \in T} \delta(q, x)$

Claim: If $w = w_0 \cdots w_{k-1} \in \mathcal{L}(M')$, then $w \in \mathcal{L}(M)$
Let $T_0, \ldots, T_k$ be the sequence of states such that $T_{i+1} = \delta'(T_i, w_i)$.
Claim $\exists (q_0, \ldots, q_k)$, such that $q_i \in Q$, $q_{i+1} \in \delta(q_i, w_i)$, and $q_k \in A$.
We start by choosing q_k and work backwards.
To choose q_k, note that because $T_k \in A'$, $\exists q_k \in T_k$ s.t. $q_k \in A$. Choose any such q_k.
For $i < k$, assume q_{i+1} has already be chosen.
Since $T_{i+1} = \bigcup_{q \in T_i} \delta(q, w_i)$, there exists some $q_i \in T_i$ such that $q_{i+1} \in \delta(q_i, w_i)$.
Choose any such q_i, and repeat.
Since $T_0 = \{q_0\}$, we can choose q_0 as our start state.