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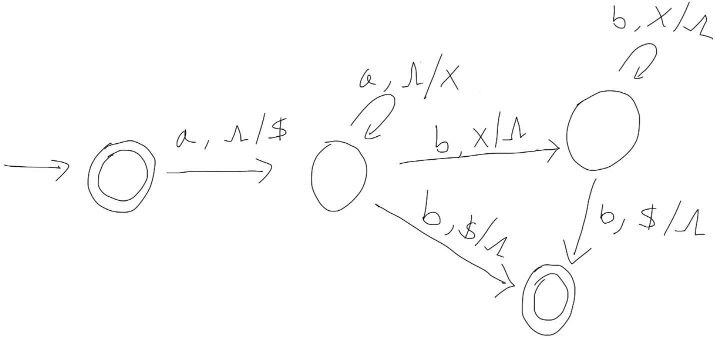

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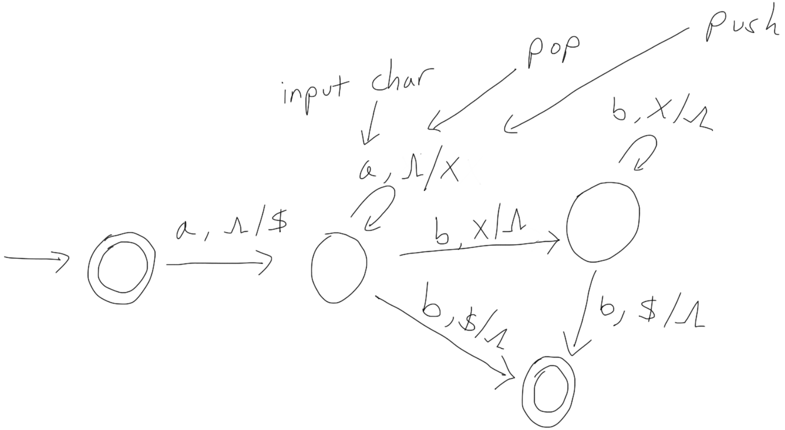

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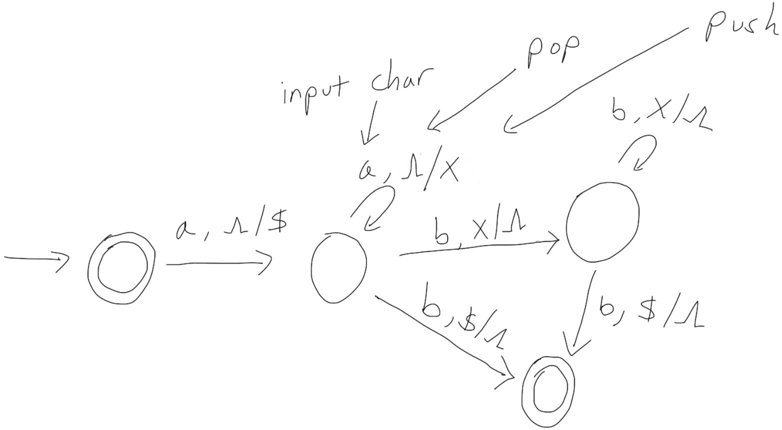

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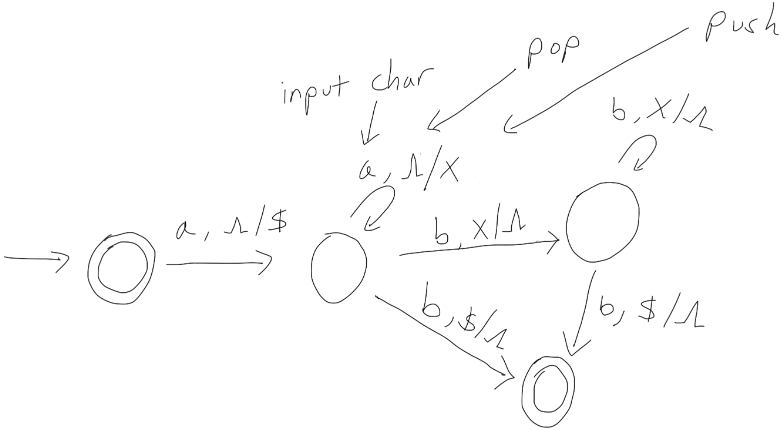

Γ is the stack alphabet. In this case, $\Gamma = \{\$, X\}$
For transition labeled $a, X/XX$:

- You can take that transition only if the next input character is a, AND the top of the stack holds X.
- You remove the X and replace it with XX.

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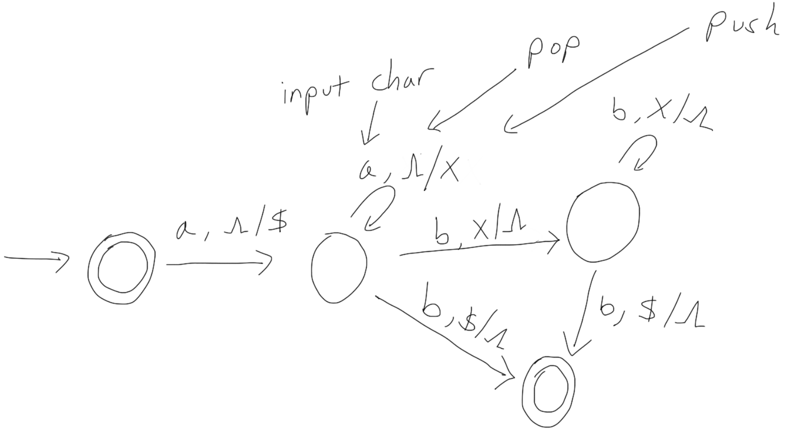

Γ is the stack alphabet. In this case, $\Gamma = \{\$, X\}$
For transition labeled $a, \Lambda/XX$:

- You can take that transition if the next input character is a, *regardless* of what's on the stack (it does not have to be empty).
- You remove nothing from the stack and push XX.

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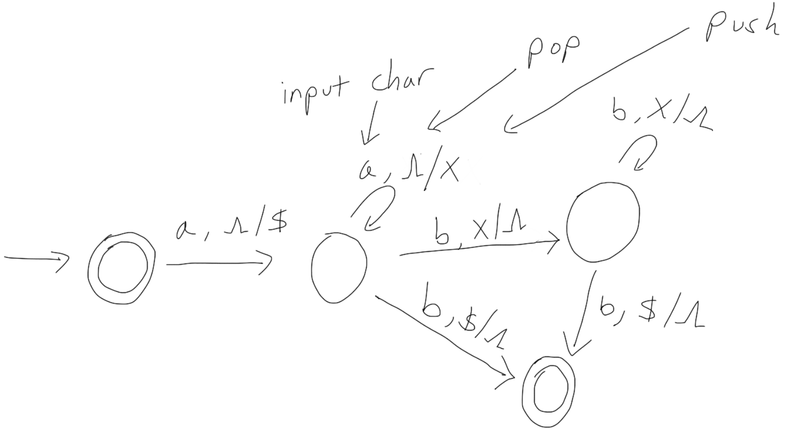

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For transition labeled $b, X/\Lambda$:

- You can take that transition if the next input character is b, AND the top of the stack holds X.
- You pop the X and push nothing.

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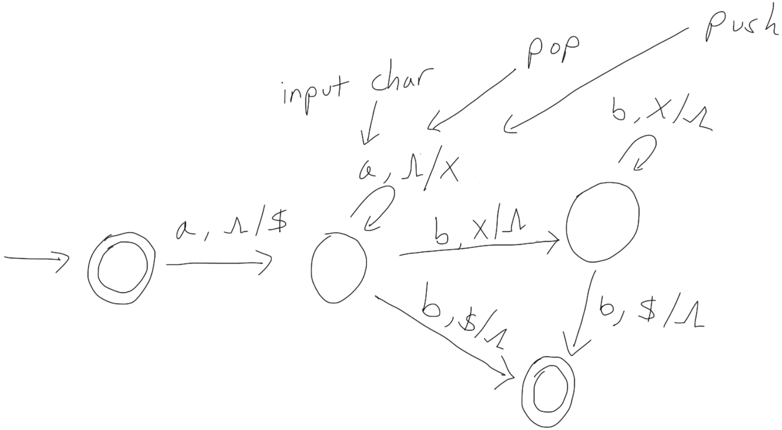

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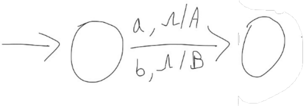
What is the language of this NPDA?

NPDA: Palindromes



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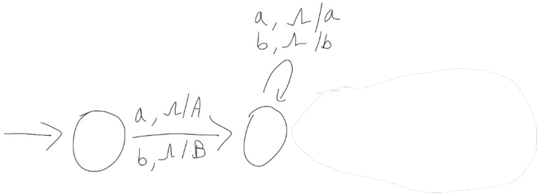

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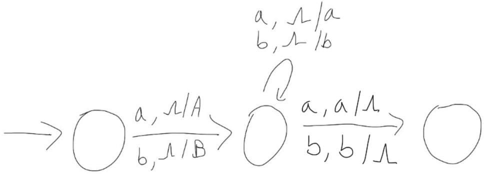

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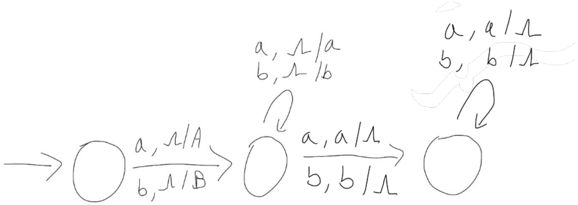

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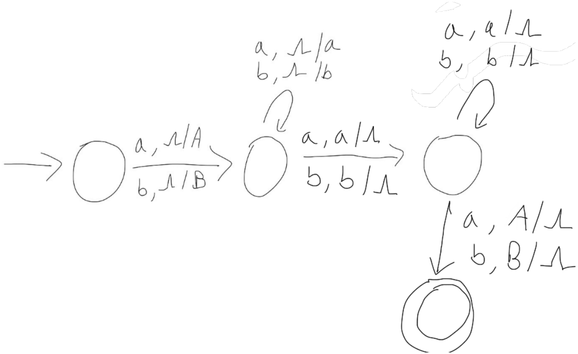

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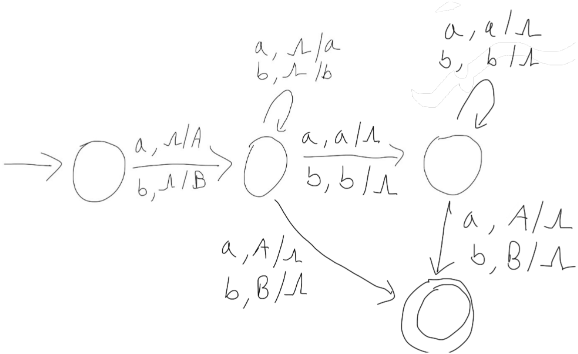

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Since we know that this language is *not* a CFL, we've arrived at a contradiction.
Therefore, M must not exist.

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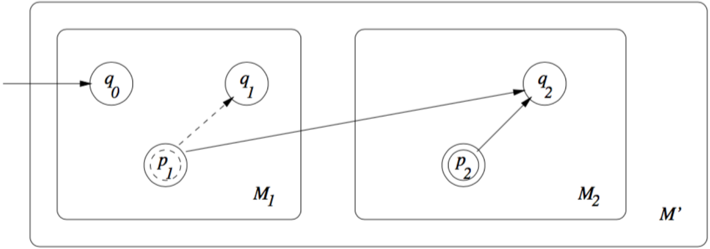

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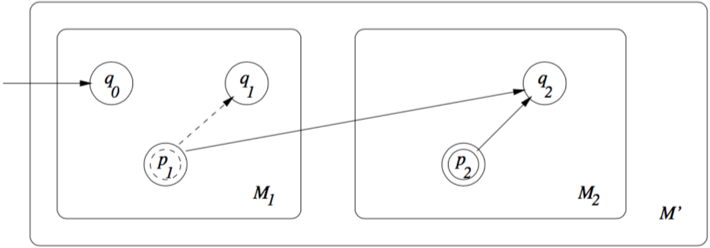

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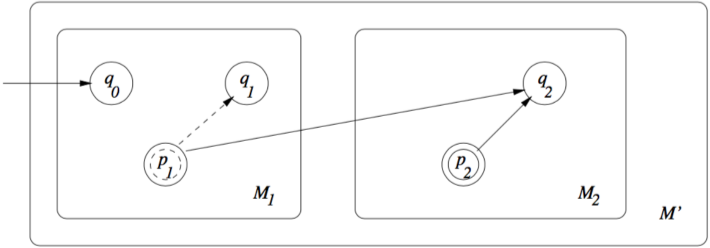

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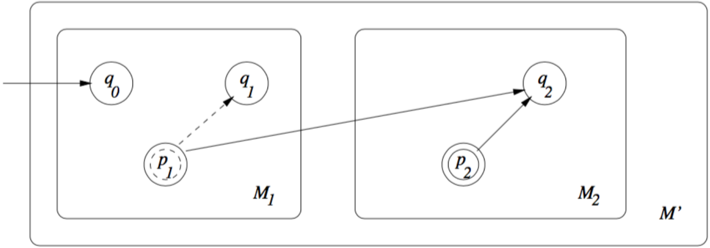

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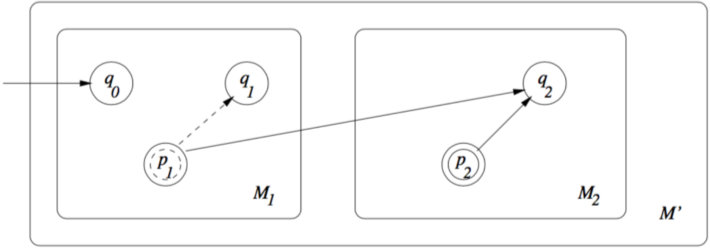

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4. In M_2, replace all b's to c's.

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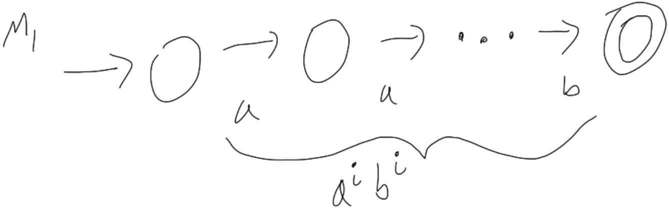

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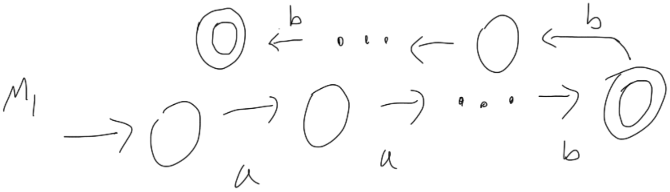

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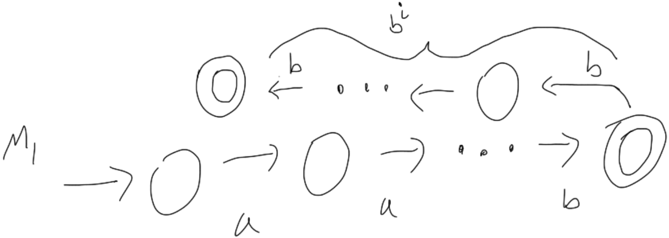

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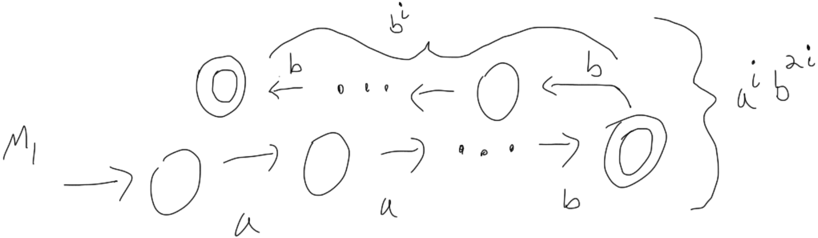

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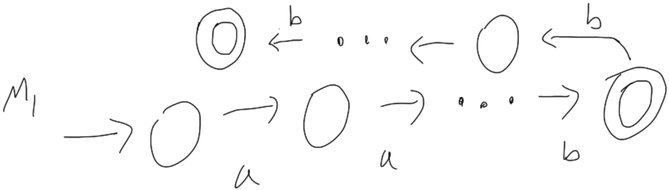

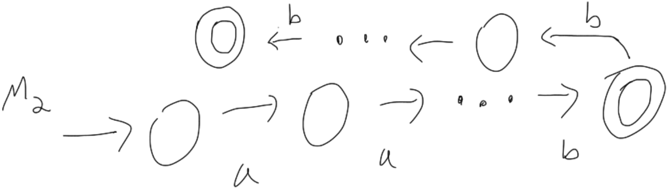

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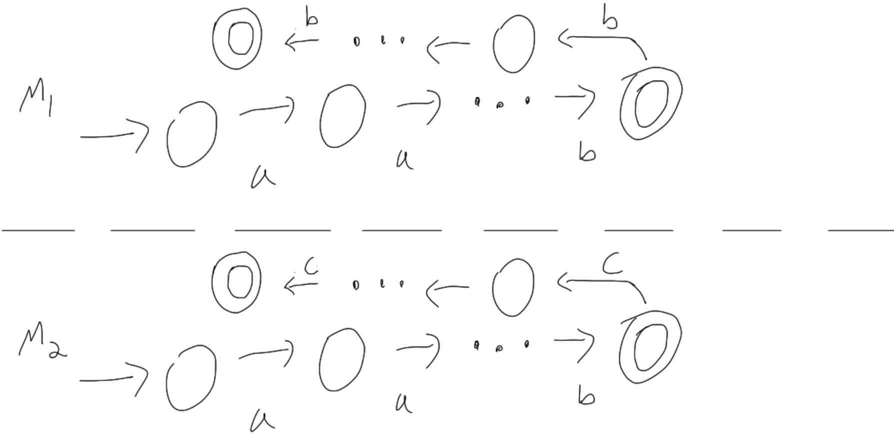

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