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Syntactic structure of a proposition

1. Each of the logical constants is a proposition
2. Logical variables are propositions
3. If α and β are propositions, then so are $(\alpha \wedge \beta), (\alpha \vee \beta)$ and $\neg\alpha$.
4. Nothing else is a proposition.

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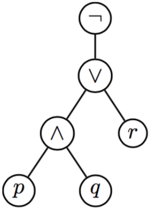
$\neg$ has precedence over $\wedge$ which has precedence over $\vee$:
So, $\neg p \vee q \wedge r$ is the same as $(\neg p) \vee (q \wedge r)$.
(But we'll use parenthesis to avoid confusion.)

Expression Trees and Truth Tables



$\neg((p \wedge q) \vee r)$

Truth Table:

p	q	r	$p \wedge q$	$(p \wedge q) \vee r)$	$\neg((p \wedge q) \vee r)$
False	False	False	False	False	True
False	False	True	False	True	False
False	True	False	False	False	True
False	True	True	False	True	False
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Consider $3x + 2 = 11$. Is this true for every "state" of x? NO! Only when $x = 3$.
Consider $3x + 2 = 11 \equiv 3x = 9$.

Law of negation:

$$\neg\neg\alpha \equiv \alpha$$

Combining a variable with itself:

$\alpha \vee \neg\alpha \equiv \text{TRUE}$	Excluded middle
$\alpha \wedge \neg\alpha \equiv \text{FALSE}$	Contradiction
$\alpha \vee \alpha \equiv \alpha$	Idempotence of $\vee$
$\alpha \wedge \alpha \equiv \alpha$	Idempotence of $\wedge$

Properties of constants:

$$\alpha \vee \text{TRUE} \equiv \text{TRUE}$$
$$\alpha \vee \text{FALSE} \equiv \alpha$$
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Commutativity:

$$\alpha \wedge \beta \equiv \beta \wedge \alpha$$
$$\alpha \vee \beta \equiv \beta \vee \alpha$$

Associativity:

$$\alpha \vee (\beta \vee \gamma) \equiv (\alpha \vee \beta) \vee \gamma$$
$$\alpha \wedge (\beta \wedge \gamma) \equiv (\alpha \wedge \beta) \wedge \gamma$$

Distributivity:

$$\alpha \vee (\beta \wedge \gamma) \equiv (\alpha \vee \beta) \wedge (\alpha \vee \gamma)$$
$$\alpha \wedge (\beta \vee \gamma) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$$

DeMorgan's Laws:

$$\neg(\alpha \wedge \beta) \equiv \neg\alpha \vee \neg\beta$$
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Subsumption:

$$\alpha \wedge (\alpha \vee \beta) \equiv \alpha$$
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Using Equivalence Laws

Given the following equivalence laws:

$\alpha \vee (\alpha \wedge \beta) \equiv \alpha$	(2nd Subsumption)
$\alpha \wedge \alpha \equiv \alpha$	(Idempotence of $\wedge$)
$\alpha \wedge (\beta \vee \gamma) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$	(2nd Distributivity)

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Given the following equivalence laws:

$\alpha \lor (\alpha \land \beta) \equiv \alpha$	(2nd Subsumption)
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T	T	F
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- If I also tell you that p is False, what do you know about q?

This is called the *implication* function:
$p \rightarrow q$

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- If I also tell you that q is True, what do you know about p?

This is called the *bi-conditional* function:
$p \leftrightarrow q$

Three New Equivalence Laws

Conditional Law:	$p \to q \equiv \neg p \vee q$
Biconditional Law:	$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
Contrapositive Law:	$p \to q \equiv \neg q \to \neg p$

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
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Biconditional Law:	$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
Contrapositive Law:	$p \to q \equiv \neg q \to \neg p$

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
F	F	T	T	T	T
F	T	F	T	F	F
T	F	F	F	T	F
T	T				

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$$
\begin{aligned}
p \leftrightarrow q &\equiv (p \to q) \land (q \to p) \\
&\equiv (\neg p \lor q) \land (\neg q \lor p) \\
&\equiv (\neg p \land \neg q) \lor (\neg p \land p) \lor (q \land \neg q) \lor (q \land p)
\end{aligned}
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Example: $q \rightarrow (p \rightarrow q)$ is a tautology.

q	p	$p \rightarrow q$	$q \rightarrow (p \rightarrow q)$
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F	T	F	T
T	F	T	T
T	T	T	T