Limitations on regular languages

Pumping Lemma

If $L$ is a regular language, then there exists an integer $p \geq 1$ such that for any $w \in L$ with $|w| > p$, $w$ can be divided into 3 strings, $w = xyz$ such that:

1. $\forall i \geq 0, xy^iz \in L$
2. $|y| > 0$
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Since \( L \) is regular, \( \exists \) DFA \( M \) that recognizes \( L \).
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Let $p$ to be the number of states in $M$. For $w = w_1, \ldots, w_n$, let $q_0, \ldots, q_n$ be the sequence of states that lead from start to accept on string $w$. 

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Let $x = w_1 \cdots w_s$, $y = w_{s+1} \cdots w_t$, and $z = w_{t+1} \cdots w_n$. 
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[Diagram of DFA with states and transitions highlighted]
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Let $x = w_1 \cdots w_s$, $y = w_{s+1} \cdots w_t$, and $z = w_{t+1} \cdots w_n$.
To see that property 3 is satisfied: recall, $t$ is the first repetition. If $t > p$, we must have more than $p$ states.
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The language $L = \{a^nb^n | n \geq 0\}$ is not regular.
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Our lemma says:

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Assume $p \geq 1$ (towards $\forall$ introduction)
Let $w = a^p b^p$

Claim: $\forall x, y, z, w$ violates (at least) one of the 3 properties.
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Assume $p \geq 1$ (towards $\forall$ introduction)
Let $w = a^pb^p$

Claim: $\forall x, y, z, w$ violates (at least) one of the 3 properties.
If property 3 is violated, we’re done.
Suppose property 3 is not violated.
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Assume $p \geq 1$ (towards $\forall$ introduction)

Let $w = a^pb^p$

Claim: $\forall x, y, z$, $w$ violates (at least) one of the 3 properties.

If property 3 is violated, we’re done.

Suppose property 3 is not violated.

If property 2 is violated, we’re done.

Suppose property 2 is not violated.
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Let $w = a^p b^p$

Claim: $\forall x, y, z, w$ violates (at least) one of the 3 properties.

If property 3 is violated, we’re done.
Suppose property 3 is not violated.
If property 2 is violated, we’re done.
Suppose property 2 is not violated.
Then $y = aa^*$. $\forall i \geq 0 : xy^i z \notin L$. 
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Theorem

The language \( L \) of “balanced parenthesis” over \( \Sigma = \{‘(’, ‘)’\} \) is not regular.
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The language \( L \) of “balanced parenthesis” over \( \Sigma = \{', ')'\} \) is not regular.

Our lemma says:
\[ \exists p : \forall w \in L : |w| > p : \exists x, y, z \text{ s.t. all 3 properties are satsified} \]

We want to show:
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Assume $p \geq 1$ (towards $\forall$ introduction)

Let $w = (((((\ldots (\ldots ))\ldots ))\ldots ))$

$p$ times $p$ times
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If $L$ is a regular language, then there exists an integer $p \geq 1$ such that for any $w \in L$ with $|w| > p$, $w$ can be divided into 3 strings, $w = xyz$ such that:

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Assume $p \geq 1$ (towards $\forall$ introduction)

Let $w = (((\cdots (((()))\cdots)))$ $p \text{ times}$ $p \text{ times}$

Claim: $\forall x, y, z, w$ violates (at least) one of the 3 properties.
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If $L$ is a regular language, then there exists an integer $p \geq 1$ such that for any $w \in L$ with $|w| > p$, $w$ can be divided into 3 strings, $w = xyz$ such that:

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Assume $p \geq 1$ (towards $\forall$ introduction)

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Claim: $\forall x, y, z, w$ violates (at least) one of the 3 properties.
Assume properties 2 and 3 are satisfied.
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Wrong answer:
Let $w = \underbrace{(())(\cdots())()}_p$
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The language $L$ of “balanced parenthesis” over $\Sigma = \{('(', ')')\}$ is not regular.

We want to show:
$\forall p : \exists w \in L : |w| > p : \forall x, y, z$ s.t. NOT all 3 properties are satisfied
Assume $p \geq 1$ (towards $\forall$ introduction)
Wrong answer:
Let $w = (())() \cdots ()()$

$p$ times
Claim: $\forall x, y, z, w$ violates (at least) one of the 3 properties.
Assume properties 2 and 3 are satisfied.
Limitations on regular languages

**Pumping Lemma**

If $L$ is a regular language, then there exists an integer $p \geq 1$ such that for any $w \in L$ with $|w| > p$, $w$ can be divided into 3 strings, $w = xyz$ such that:

1. $\forall i \geq 0, xy^i z \in L$
2. $|y| > 0$
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\( \neg \forall x, y, z : \forall i \geq 0 : xy^i z \notin L \).