Limitations on regular languages

Pumping Lemma

If L is a regular language, then there exists an integer $p \geq 1$ such that for any $w \in L$ with $|w| \geq p$, w can be divided into 3 strings, $w = xyz$ such that:

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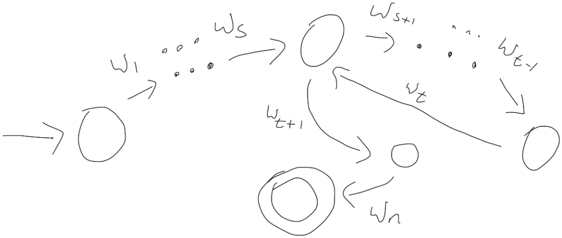

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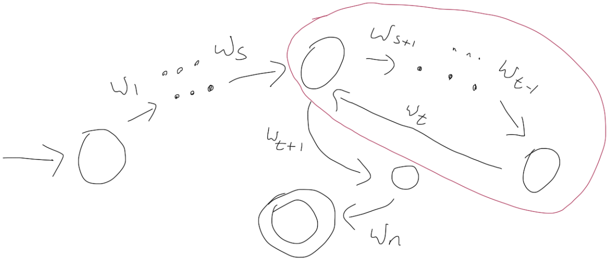

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To see that property 3 is satisfied: recall, t is the first repetition. If $t > p$, we must have more than p states.

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Suppose property 3 is not violated.

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Suppose property 3 is not violated.
If property 2 is violated, we're done.
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Examples of strings in the language: $((()))()()$ and $()(()())$

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Wrong answer:
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Assume $p \geq 1$ (towards $\forall$ introduction)
Wrong answer:
Let $w = \underbrace{()()()\cdots()()()}_{p \text{ times}}$
Claim: $\forall x, y, z$, w violates (at least) one of the 3 properties.

Limitations on regular languages

Pumping Lemma

If L is a regular language, then there exists an integer $p \geq 1$ such that for any $w \in L$ with $|w| \geq p$, w can be divided into 3 strings, $w = xyz$ such that:

1. $\forall i \geq 0, xy^i z \in L$
2. $|y| > 0$
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Theorem

The language L of "balanced parenthesis" over $\Sigma = \{\text{‘(’}, \text{‘)’}\}$ is not regular.

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