CS600	March 1, 2018

Homework 2

Students are welcome to work together, but *every student must write up their own solutions, independently!* I strongly encourage students to use LaTex for writing up their solutions. Please see the course web-page for a template file. When problems require you to draw a state machine, feel free to include a hand-drawn picture with your typed up solutions.

The total homework is worth 10 points.

Question 1: Let $L_{\mathsf{reg}} = \{\langle M\rangle \mid \text{ The } M \text{ recognizes a regular language. } \}$. Show L_{reg} is not decidable by showing $L_{\mathsf{halt}} \leq_m L_{\mathsf{reg}}$. (Don't look online! Hint: although $\{0^n1^n\}$ is not regular, note that some regular languages *contain* this language!)

Question 2: Let $L = \{(\langle M_1\rangle, \langle M_2\rangle, w) \mid M_1(w) \text{ and } M_2(w) \text{ both halt, with opposite output}\}$. Show that L is not decidable by giving a mapping reduction from some language we have already shown to be undecidable.

Question 3: If $A \leq_m B$, and B is a regular language, does that imply that A is a regular language?

Question 4: Prove that if L is recognizable, and $L \leq_m \bar{L}$, then L is decidable.

Question 5: (Optional.) Consider the proof in the lecture notes on automata, Section 3.2, showing that $\{a^ib^i\} \cup \{a^ib^{2i}\}$ cannot be decided by a deterministic PDA. Recall that in that proof, we assumed the contrary, and then described a machine that decides $L = \{a^ib^ic^i\}$. Prove that if $x \in L$, that the machine we described does indeed accept x.

Question 6: (Optional.) Let $L = \{\langle M\rangle \mid M \text{ decides a language containing the string ``GMU''}\}$. Show that L is undecidable.

Question 7: (Optional.) Let $L = \{\langle M\rangle \mid M \text{ decides a language containing exactly 3 strings}\}$. Show that L is undecidable.