CS600 February 26, 2018

Homework 3

Students are welcome to work together, but *every student must write up their own solutions, independently!* I strongly encourage students to use LaTex for writing up their solutions. Please see the course web-page for a template file.

Question 1: We say that a graph $G = (V, E)$ has a *vertex cover* of size k if there exists a set of k vertices, $S \subset V$, $|S| = k$, such that for all edges $(u, v) \in E$, either $u \in S$ or $v \in S$. Let $\mathsf{VC} = \{(G, k) \mid G \text{ is a graph with a vertex cover of size } k\}$. Show that VC is $\mathcal{NP}$-complete by giving a reduction from $3SAT$. That is, show $3SAT \leq_p \mathsf{VC}$. (Hint: if ϕ is a boolean formula with n variables and ℓ clauses, then, for $f(\phi) = (G, k)$, $k = 2\ell + n$.)

Question 2: We say that a graph $G = (V, E)$ has an *independent set* of size k if there exists a set of k vertices, $S \subset V$, $|S| = k$, such that for all edges $(u, v) \in E$, either $u \notin S$ or $v \notin S$. That is, no nodes in S are connected to one another. Let $\mathsf{IndSet} = \{(G, k) \mid G \text{ is a graph with an independent set of size } k\}$. Show that IndSet is $\mathcal{NP}$-complete, by giving a reduction from VC. That is, show $\mathsf{VC} \leq_p \mathsf{IndSet}$.

Question 3: We say that a graph $G = (V, E)$ has a *dominating set* of size k if there exists a set of k vertices, $S \subset V$, $|S| = k$, such that for all vertices $v \in V$, either $v \in S$ or there exists and edge (u, v) such that $u \in S$. That is, every node is either in S, or neighbors a node in S. Let $\mathsf{DomSet} = \{(G, k) \mid G \text{ is a graph with a dominating set of size } k\}$. Show that DomSet is $\mathcal{NP}$-complete, by giving a reduction from VC. That is, show $\mathsf{VC} \leq_p \mathsf{DomSet}$.

Question 4: (We will define Subset Sum on Thursday, but feel free to look it up.) Define the language

$$\text{EVENSPLIT } = \left\{ L \subset \mathcal{N} \ : \ \begin{array}{c} \exists P_1, P_2 \subset L \text{ such that } \sum_{x \in P_1} x = \sum_{y \in P_2} y, \\ |P_1| + |P_2| = |L|, \text{ and } P_1 \cap P_2 = \emptyset. \end{array} \right\}.$$

Without using the Internet, or any other resources, show that this language is $\mathcal{NP}$ complete (hint: use SUBSET-SUM).