Students are welcome to work together, but every student must write up their own solutions, independently! I strongly encourage students to use LaTeX for writing up their solutions. Please see the course web-page for a template file.

**Question 1:**

1. Let $L'$ be an NP-complete language. Prove that any $L \in \text{coNP}$ is Cook-Turing reducible to $L'$.

2. Prove that if there is a Karp-reduction from a $\text{coNP}$-complete language to a language in $\text{NP}$, then $\text{coNP} = \text{NP}$. Click for a hint.

**Question 2:** Show that SUBSET-SUM is self-reducible.

**Question 3:** The language

$$3\text{-COLOR} = \left\{ G \mid \text{The vertices of graph } G \text{ can each be labeled Red, Green or Blue such that no adjacent vertices have the same label.} \right\}.$$

Show that this language is self-reducible.

**Question 4:** In the proof of the space-hierarchy theorem, we defined a language $L$ by describing a machine $M_L$ that uses space $O(G(n))$. We then assumed that some $M_{L'}$ decides the same language while using space $o(G(n))$, and arrived at a contradiction. In demonstrating the contradiction, we looked at the outcome of running $M_L(w)$, where $w = (\langle M_L \rangle, 1^k)$ for sufficiently large $k$.

1. Do we arrive at a contradiction if we use some other $w'$ as input? Say, $w' = (\langle M' \rangle, 1^k)$ for some arbitrary machine $M'$. Why or why not?

2. What happens if we run $M_L(\langle M_L \rangle, 1^k)$?