CS600	April 6, 2018

Homework 4

Students are welcome to work together, but *every student must write up their own solutions, independently!* I strongly encourage students to use LaTex for writing up their solutions. Please see the course web-page for a template file.

Question 1:

1. Let L' be an NP-complete language. Prove that any $L \in \mathsf{co}\mathcal{NP}$ is Cook-Turing reducible to L'.
2. Prove that if there is a Karp-reduction from a $\mathsf{co}\mathcal{NP}$-complete language to a language in $\mathcal{NP}$, then $\mathsf{co}\mathcal{NP} = \mathcal{NP}$. Click for a hint.

Question 2: Show that SUBSET-SUM is self-reducible.

Question 3: The language

$$\text{3-COLOR} = \left\{ G \mid \begin{array}{l} \text{The vertices of graph } G \text{ can each be labeled Red, Green or Blue} \\ \text{such that no adjacent vertices have the same label.} \end{array} \right\}.$$

Show that this language is self-reducible.

Question 4: In the proof of the space-hierarchy theorem, we defined a language L by describing a machine M_L that uses space $O(G(n))$. We then assumed that some $M_{L'}$ decides the same language while using space $o(G(n))$, and arrived at a contradiction. In demonstrating the contradiction, we looked at the outcome of running $M_L(w)$, where $w = (\langle M_{L'} \rangle, 1^k)$ for sufficiently large k.

1. Do we arrive at a contradiction if we use some other w' as input? Say, $w' = (\langle M' \rangle, 1^k)$ for some arbitrary machine M'. Why or why not?
2. What happens if we run $M_L(\langle M_L \rangle, 1^k)$?