

Homework 1

Students are welcome to work together, but *every student must write up their own solutions, independently!* I strongly encourage students to use LaTeX for writing up their solutions. Please see the course web-page for a template file.

Each question is worth 10 points.

Question 1:

1. Consider the following game. I roll a six-sided die and look at the outcome, v , but without revealing it to you. You make a guess g , after which I reveal some hint by choosing a random value $h \neq g$ and revealing whether or not $v = h$. I then offer you a chance to change your guess. Clearly if I revealed that $v = h$, you would prefer to change your guess to h . What if $v \neq h$: is there any advantage in changing your guess? To answer the question, let V be a random variable denoting the value of the die, let G_1 be a random variable denoting your original guess, let G_2 be a random variable denoting your final guess, if you change it after receiving a hint, and let H be a random variable denoting the value of the hint. Evaluate the following probabilities:

- (a) $\Pr[H \neq V]$
- (b) $\Pr[G_1 = V \wedge H \neq V]$
- (c) $\Pr[G_1 = V \mid H \neq V]$
- (d) $\Pr[G_1 \neq V \mid H \neq V]$
- (e) $\Pr[G_2 = V \mid H \neq V]$
- (f) $\Pr[G_2 \neq V \mid H \neq V]$

2. Consider the following game. I roll a six-sided die and look at the outcome, v , but without revealing it to you. You make a guess, g , after which I reveal some hint, h , which is chosen randomly, subject to the constraint that $h \neq g \wedge h \neq v$. (Note, this is *not* the same method of choosing h as in the previous part!) I offer you a chance to change your guess. Is there any advantage to changing your guess? Justify your answer by calculating the following probabilities

- (a) $\Pr[H = h]$
- (b) $\Pr[G_1 = V \mid H = h]$ (hint: use the fact that this is equal to $\frac{\Pr[G_1=V \wedge H=h]}{\Pr[H=h]}$)
- (c) $\Pr[G_2 = V \mid H = h]$ (hint: as above).

Question 2: Exercise 2.4 in the book

Question 3: Exercise 2.6 in the book.

Question 4: Exercise 2.9, parts (a) and (b), in the book. Hint for part (b): it may not be the most “natural” message space. But it has size greater than 26.