2x2 Graph Games

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Abstract

To apply game theory to social science we need to ask whether and how a particular abstract game formulation or experimental game or the latter's results have the potential to inform us, or actually do inform us, about real phenomena treated in some social science discipline. Graph games offer a different - and for some situations a more realistic - model of the dynamics of interactive decision-making than games defined with trees or matrices. This paper defines graph games, considers how they relate to real-world situations, and presents a taxonomy of the two-person, two-choice case using ordinal payoffs.

1. Introduction

To represent situations realistically, models of human affairs need to take into account the passage of time. It's conventional wisdom that time flies, time heals, even that time is money. Whether or not it's *all* in the timing, time is of the essence, from being the early bird to being the one who laughs last. To a participant in a situation - nation or individual; corporation or political party; litigant or conspirator; friend or enemy; parent, child, spouse or workmate - the sequencing of actions or even the mere passage of time without action can have crucial consequences.

Timing issues have not escaped the notice of economists and game theorists. They have had much to say about the temporal aspects of hiring, shopping, investing, and innovation, which they have abstracted into games characterized by continuity (finely divided time), preemption (who acts at an earlier time), stopping (at what time), and deadlines (time boundaries). (See, for example, Shmaya and Solan, 2004, Hamadene and Zhang, 2010 and Ferguson, 2008.) In doing so, they have not confined themselves to the two classic formats for the presentation of games, the tree and the matrix. Indeed these formats fall short in representing a significant class of situations in which time plays a key role.

To see this, first note that matrix-defined games envision both players acting simultaneously, and that in a tree game, at each decision point one particular player, and that one only, has the possibility of acting. In contrast to both, now consider a situation, or more precisely a particular moment in a two-party situation to be modeled, and imagine the reasonable possibility that, at that moment, either of the two parties has the capacity to take action, but that an act by either of them changes the situation, rendering the other's action no longer possible or, though possible, no longer having its previous potential effect(s). Thus action at that point may be by one party or by the other or by neither... but not by both as in the matrix, and not confined to a particular one as in the tree. Such a moment, then, does not directly fit the structure of a tree or a matrix. Therefore neither of those familiar two-person game formats directly provides a suitable model for the range of temporal sequencing possibilities in any situation containing that moment. (We say "directly" because like tree games, the graph can be converted to strategic form, which is a matrix.)

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Undeterred by these shortcomings of the simple traditional formats, game theorists have progressed significantly in dealing with time, introducing an impressive array of formal concepts and an arsenal of mathematics to supplement or replace the two formats, leading to broader classes of games, for which there has been considerable success in establishing the presence of equilibria and game values. What's lost, perhaps inevitably, is the original conception of games as simple and visualizable constructs, manipulable to study human behavior as opposed to optimizing (Hamburger, 1973; Guyer, Fox and Hamburger, 1973; Rapoport, Guyer and Gordon, 1976). In an attempt to preserve intuition to some extent, in this paper we move up a notch from trees to graphs. We will use strictly ordinal graph games (GGs) as the basis of a new taxonomy and seek plausible situations, especially ones involving time, for which they provide models, while leaving room for conversion to normal form and/or invoking more sophisticated methods.

To fix ideas we begin the next section with a particular game, Chicken. We look at both its matrix and the range of situations to which nontechnical writers apply the name, typically uncapitalized. Then we'll home in further on a particular current situation that many call chicken: a potential government shutdown that looms as we write. We introduce a graph game for that situation that we believe provides a better model than the matrix, thereby indicating that the approach has merit. That in turn motivates systematically exploring the representation by creating a taxonomy.

In Section 3 come definitions. In particular the graphs of interest are defined to be directed and connected and to have labels, a start state and possibly cycles, but no self-loops. That done, the way is prepared for a taxonomy of 2x2 graph games, first those with two states in Section 4 and then, more interestingly, those with three in Section 5. The last two symmetric 3-state 2x2 graph games will be shown to be chickenesque. Graph games with four or more states cannot be 2x2.

2. Chicken, Crises and Shutdowns

Chicken gained fame in America with the movie *Rebel without a Cause* (1955), where it was called chickie. Highschoolers sought admiration, or at least acceptance, by "simultaneously driving ... cars off the edge of [a] cliff, jumping out at the last possible moment. The boy who jumps out first is 'chicken' and loses." (Poundstone,1992, 197-201) Others have envisioned cars driven toward each other, and swerving as chickening out.

The movie prospered at a time when people were increasingly preoccupied with the USA-USSR nuclear competition and the associated strategic notion of brinkmanship, which morphed into the strategy called mutually assured destruction (MAD). At that time the US and USSR were vying for influence in the Cold War, hoping that hot war would not occur. Bertrand Russell (1959) famously and witheringly linked the policy to the game, which "as played by irresponsible boys, ... is considered decadent and immoral." The ordinal payoff matrix here seems to capture aspects of both situations, though the default outcome is (1,1) in *Rebel* and (3,3) in MAD.

3,3	2,4
4,2	1,1

Formal models like matrix-defined Chicken, being abstractions, leave out features deliberately. That's true for graph-defined games too. Conversely real situations afford decision-makers options not available in the simplest formal models. As extra options in the 1962 Cuban missile crisis, for example, one may look at the USA option to set warships in motion toward Cuba and the USSR option to deploy a blockade in those same waters. Incorporating such successive moves by different players suggests a tree (extensive form) that in normal form implies a matrix more complex than 2x2 - or else a 2x2 matrix with rules for wandering around in it (Zagare, 1984). That crisis was played out in a pre-hotline world and triggered the installation of the Washington-Moscow hotline, intended to be a restructuring of the game. Despite the hotline and the later demise of the Soviet Union, the game - or at least its name - endures in headlines proclaiming nuclear chicken with new opponents (Ignatius, 2010).

Like the chicken of the poultry world, the game called chicken in the real world is a variegated species. The price of the success of the game metaphor has been that academics no longer have exclusive definitional rights. Ordinary language has come to incorporate the notions of strategic thinking, and the 2x2 game, once called the fruit fly or *e coli* of the social science of strategic interaction, has become a journalistic shorthand for everyday complex social encounters. This is especially the case with chicken, in the current polarized political environment, with metaphorical usage of the word proliferating in the news. To gain some objectivity on the frequency and range of this phenomenon, we did some web searching. The "game of chicken" nets a lot of economic news from realms as diverse as information technology, drugs and professional sports, but we chose to narrow the search target to "political chicken," which yields a raft of confrontations over economic affairs. The use of quotes requires the exact phrase, cutting down sharply on documents with unrelated mentions of chicken and political. Of 51,600 hits, Google advises that those beyond the 728th are "very similar to the 728 already displayed." Of these, we examined 50: five groups of ten each, specifically hit-pages 1, 2, 10, 20 and hits 719-728.

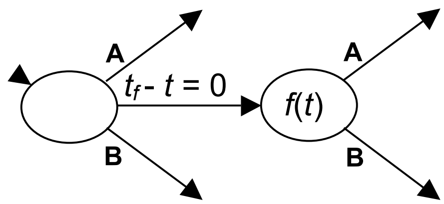
Most of the articles report or analyze real-world situations that refer to chicken somewhat as it is understood in variable-sum game theory, though 12 of them are jocular, just happen to juxtapose the two words or are otherwise irrelevant. The 14 on U.S. national politics span several topics: the current budget bill, a 2010 extension of unemployment benefits, regulatory reform of the financial industry in 2010, supplementary spending on the Iraq war in 2007 and more. The confrontation between Wisconsin's governor and its senators who left the state to avoid a vote on a bill restricting unions was called chicken 13 times. Ten more references were scattered among cities and other states, ranging over court appointments, energy rate hikes, the funding of pension and retiree health care, and other matters. The only other country mentioned is Canada, with one hit, in which the author opposes no-confidence votes. That article speaks of *brinkmanship*, the term made classic by Secretary of State Dulles and mentioned above, and another two also refer to ideas of concern here, one mentioning players with "one eye on the *clock*," and the other calling it crucial "*who goes first* with what" (emphases added).

The action available to each player in these situations is typically to flinch from a sub-optimal initial state, thereby providing the other with a best possible outcome. Note that this description characterizes three cells (excluding the upper left) in the matrix of Chicken as well as the corresponding three cells of Prisoner's Dilemma, as the games are ordinally defined by Rapoport and Guyer (1966). The two game definitions differ in how bad the outcome is for whoever flinches in comparison to what happens if neither does so. Authors of news articles do not concern themselves with that distinction, but they do take up situations in which time is of the essence, whether as the result of a deadline or because the situation is understood to deteriorate if not resolved. On February 25, for example, the New York Times reported that "The *time frame* leaves little opportunity for the Senate to alter the measure and send it back to the House. Senate Republicans said Democrats had few good options *with the clock ticking*..." [emphasis added]

It's also worth noting that a situation need not be symmetric for a reporter, analyst or blogger to call it chicken. In one episode of the legislative battle over unions, the Wall Street Journal reported (Maher and Merrick, 2011) that "Playing a game of *political chicken* ... Democratic senators who fled Wisconsin to stymie restrictions on public-employee unions said they planned to come back from exile soon, betting that even though their return will allow the bill to pass, the curbs are so unpopular they'll taint the state's Republican governor and legislators." The Democrats never had any prospect of blocking the vote and they knew they'd have to return. All they could hope to do was to show some solidarity through personal sacrifice and to gain in the public debate.

To fix ideas and make a case for using graph games, we now develop a graph game model of the U.S. budget crisis of 2011. In a budget confrontation in the U.S. the available actions include enacting a budget or not, passing a continuing budget resolution or not, and raising the debt ceiling or not. Critical points in time come at the end of the fiscal year, whenever a continuing budget resolution expires (like the one covering March 4-18, 2011 and called chicken by philly.com) and when the debt passes its ceiling. At least two other substantial news descriptions featuring parts or all of this interactive decision structure have been called chicken in widely read sources: The New York Times in the current case (Hulse, 2011) and Wikipedia on the budget battle of 1995-96 (wikipedia.org).

The Times article quotes a Bush deputy Treasury secretary as calling the current budget crisis "kind of a game of chicken in which you get points for making the other branch dodge." And in case you missed the one from 15 years ago or have forgotten the fine points, here is the gist of the Wikipedia entry: Clinton vetoed the spending bill. No budget passed by September 30, the end of the fiscal year. Speaker Gingrich threatened to refuse to raise the debt limit. The result was a game of chicken, shaking of investor confidence and higher interest rates. The new fiscal year began with the government running on a continuing resolution set to expire on November 13 at midnight, when non-essential government services would be shut down in order to prevent default. Major players met but no deal emerged. On November 14-19, much of the government became inoperative until a temporary spending bill passed. The underlying disagreement was not resolved and a second shutdown, from Dec 16, lasted three weeks.



The diagram here is a start toward representing a budget crisis as a graph game. It is only a start, because the payoffs to the outcomes at the arrowheads, including $f(t)$, need to be specified. The left-hand state represents the starting situation, a phase during which there is bargaining and/or confrontation. A deadline is approaching but overt consequences are not yet accruing. The fact that there is a deadline for leaving this state is shown by the label on its horizontal out-transition expressing the condition for spontaneous activation of that transition. Breaking it down, let t be clock time, initialized to zero when the situation is first understood to exist. The time t_f is the moment the government is to shut down, absent an agreement, so $t_f - t$ is the time remaining before the deadline is reached. When $t_f - t = 0$, if neither player has flinched, that is, departed the start state, the transition to the right-hand state occurs.

At this other state consequences do begin to occur and accumulate, a phenomenon that may be thought of, especially by economists, as a mapping from time to money, $f(t)$, a decreasing function of time, because economic damage is occurring or potential bits of GDP are going unrealized. In this state there is no deadline, so an exit will come only as the result of players' decisions. We therefore look next at the four slanting arrows, each representing an option (at one of the states) for one of the players, A or B, to yield and thereby terminate the game. Since they are capable of communicating, the players may manage to make a deal and thereby, in effect, orchestrate a coordinated departure. This happened in the right-hand state in 1995 but "the underlying disagreement was not resolved" so the exit from that state was, in effect, a joint move back to the starting node for a second round (or instance or playing) of the same type of game, presumably with reduced payoffs, according to $f(t)$.

Continuous games might be put to use here, by envisioning that players pick a future time at which to concede if the other hasn't yet done so, thereby expressing precisely their tolerance for the passing of time. Of course, since there are infinitely many times this leads to infinite sets of options, hardly consistent with the two-choice games that are the topic of this session. Reflecting on the graph approach exemplified in the foregoing diagram and discussion, specifically states and movement among them, we believe such models will often capture the structure of a real-world situation more effectively than simply points on a one-dimensional continuum, but that the latter needs to be incorporated into the former at key points, something we have not undertaken.

Before leaving the world of budget crises, let's think briefly about the players. The Wikipedia article names Clinton and Gingrich in the 1995 showdown. Today the buck stops with Obama, but it is less clear that the Republican House caucus will act as a single player. And what makes them tick? We have not shown payoffs, but in any case those are a matter of political analysis of what motivates the players, perhaps a mix of principled policy preferences with strategy and prospects for the next election. The car-collision outcome in the variant of *Rebel without a Cause* and the nuclear holocaust outcome in a missile crisis correspond here to damage to the economy along with loss of income for government workers. How concerned are this game's players, whoever they are, with that? If that's not regarded by both decision-makers as the worst possible outcome, this isn't matrix chicken.

3. Graph Game Definitions

The graphs here will all be directed graphs, in the mathematical sense. A directed graph has vertices and directed edges (or arrows), but for GGs we'll call them states and transitions, respectively, those terms being more evocative of their real-world counterparts: states of affairs and transitions between them. Each state has a label that specifies an ordinal payoff for each player. That is, a player can state which of two states is preferable, but not by how much and there are no ties. Each transition goes from one state to another and bears the label of a player who may move the game from one state (its from-state) to another (its to-state). We omit explicit transitions from a state to itself, though so long as both/all players refrain from activating currently available out-transitions (outward transitions from the current state), the state does not change. The graph must be connected, there being no point in unreachable states.

We do not constrain out-transitions at a node. If a player has n out-transitions from a state, that player may opt for any of n+1 things, including inaction, and n may exceed 1. Further, it is permitted to have a state (or more than one) from which two players (or more) each have an outward transition (or more than one), since things may be that way in the real world. Thus at state S, Player A may have available a transition to T and Player B may have one to U. Either can activate the relevant transition so long as the other has not yet done so. The "not yet" means that time matters. Suppose that A is the first to decide to act and activates the transition from state S to state T. Now it is no longer possible for Player B to go anywhere from S because the state of the game is no longer S but is now T. This seems realistic to us, hence a point in favor of this formulation, insofar as one can envision situations in which different people have actions available to them that can change the situation.

GGs begin at a uniquely designated start state, but can end in various ways. If play arrives at a state with no outward transitions, no more decisions can be made so realistically and also in a game-theoretic sense that occurrence of the game is finished. In contrast, upon arrival at a state with one or more out-transitions, it is possible to continue, but only if some player having one or more out-transitions decides to act. Given the preceding discussion of the role of time, players are not envisioned as acting instantaneously. However, if they remain inactive for long enough, presumably something in a real world situation will change - a deadline will pass, an election will occur, the train will leave the station, the statute of limitations will run out. In an experimental game the experimenter or assistant will need to leave the lab. So we will say that a game can end at these states too. Finally, note that nothing has been said about requiring GGs to be acyclic. That suggests the possibility of a game in which returns may occur to previously visited states. One can envision that players returning to a state might analyze it as previously. If that kept on happening the cycle would be repeated, yielding one déjà vu after another.

So, to sum up the definition, the graph of a graph game is directed, connected and has labels, a start state and possibly cycles, but no self-loops. Each edge or transition is labeled by a player, a from-state and a different to-state. Inaction is an implicit option, so a player with n explicit transitions from a state has n+1 options. No states are unreachable from the initial state. Each state is labeled by a payoff to each player, on a utility scale. For present (taxonomic) purposes, that scale is ordinal with strict ordering.

In a 2x2 game the number of players is 2, here always named A and B. The other "2" in "2x2" is the number of overall game strategies for each player. This confines each player to a maximum of 1 transition. The two choices for such a player (having one transition) are to act or not to act. A player in a 2x2 game cannot have another transition (a second one) available at the same state or at any other state, since then the player's overall strategy would have to include whether or not to use that transition should play reach its from-state, thereby giving rise to at least 3 overall game strategies, a violation of 2x2. We use this constraint here to conform to the topic of this session-meeting and to keep the discussion manageable but a rationale for allowing a third choice is implicit in the following brief discussion of infinite strategy sets.

A serious challenge to the two-strategy specification is that infinite strategy sets can arise from (i) a cycle or (ii) a state that is a from-state for both players. This is true in the first case - cycles - because decisions must be specified contingently on potential game histories of unbounded length. In the second case, if both players choose to act, each must further choose whether to *hurry*, i.e., act as fast as possible, or *wait* and if the latter, for how long to wait before acting in the event that the other player hasn't yet. Time, even if quantized, provides infinitely many choices of "how long."

For cycles, one might rein in the analysis by assuming that, upon arrival at a state previously visited, players repeat their earlier choice. In life, people see the behavior of others and may decide to change their own. Still, one can finitize an experimental game by enforcing the foregoing repetitive, learningless behavior and collapsing future play onto a new terminating (no out-transtion) state. Various approaches can be taken to determining the payoffs at such a state. For the other source of infinite strategy sets, a state that's a from-state for both players, one can reduce the option set to a small finite number - 3, for example - by eliminating the parameter for waiting and instead allowing the two transitions *hurry* or *wait* and resolving ties (matched choices) by a coin-flip.

4. Two-State 2x2 Games

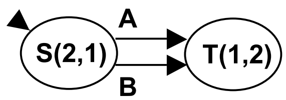
One of our goals is to enumerate the 2x2 graph games, which we'll do in increasing order of the number of states. With just one state, there cannot be a transition (to another state), so neither player has anywhere to go, and the only choice for each is to just sit tight. Such a game cannot be 2x2, so there are no 1-state games of interest here.

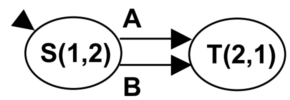
Moving therefore to the case of two states, consider a game with start state S and another state T. There are four possible ways to assign payoff: both players prefer S, or both prefer T, or they have opposite preferences with A preferring either S or T (and B the other). Now let's look at possible patterns of transitions. Since T must be reachable, at least one player must have an S-to-T transition. If both do, those must be the only transitions thereby completing our first pattern of transitions. (Since we are talking about 2-choice games and a transition that one can elect to use or not provides two choices.) In this case T has no out-transitions and so in that sense it is a final state (aka a sink or dead end). The other pattern with two states arises in the case where just one player has an S-to-T transition. In that case, the one who doesn't must have a transition from somewhere else, necessarily T, and it must go to a state other than its from-state, which must mean it goes to S. This latter pattern a is cyclic graph in that play may go from S to T to S, but must then stop if attention is confined to 2-choice games.

Let (p, s→ t) refer to a transition available to player p going from state s to state t. Let (s: u,v) mean that state s has utilities u and v - for players A and B respectively unless stated otherwise. As ordinal payoffs in a two-state game, we will use 2 for a player's utility at the preferred state and 1 at the other. Given the four payoff assignments and two transition patterns laid out in the preceding paragraph, one might anticipate eight (=4x2) 2-state, ordinally defined 2x2 games, but the first two that we'll look at turn out to be equivalent. We then consider two others and dismiss the remaining ones as being of little strategic interest.

Using the foregoing notation, consider the game with states and preferences (S: 2,1) and (T: 1,2) and with each player having a transition from S to T; that is, (A: S→T) and (B: S→T). State T has no out-transitions. Thus the game is one in which...

> Both players have available a game-ending transition to the same state. Executing it gets an outcome better for one and worse for the other than in the start state.





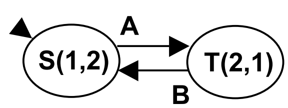
Now consider the game with the same transition pattern but with preferences (S: 1,2) and (T: 2,1). This game is equally well described by the indented sentence above. In other words, since A and B have the same transitions, the interchange of A's and B's payoffs is like reassigning people to opposite roles in the *same game*.

Predator

The foregoing game occurs in the jungle. In the initial state S, a hungry predator (player B in the first description, player A in the second), sees live prey. In the other state, T, the predator is no longer hungry and the prey is no longer alive. Presumably it is the predator who acts to bring about the transition. In human affairs an example is violation of the terms of a loan and some other forms of predatory lending.

Thermostat

Retaining states and preferences - (S: 1,2) and (T: 2,1) - of the second characterization of Predator, now let the transitions be (A: S→T) and (B: T→S). Notice that each player's only transition goes to that player's preferred state and, necessarily, from the other state. This game's only two transitions constitute a cycle. This game captures the essence of a situation in which two people share space but prefer different settings for the thermostat that controls the temperature in that space. Either can change the setting at any time, but the formal game omits transitions out of one's preferred state since there is no point in changing the setting when you are the most recent to have done so.



Après vous

With the same cyclic transition structure but with preferences opposite to those in Thermostat - so that each player does more poorly by choosing to change the state - one gets a game in which no one ever has an incentive to move, provided one takes the view that any altruistic tendencies are already incorporated into the payoffs. If the payoffs were money in an experimental game, polite or altruistic players might move along the cycle in the manner of Alphonse and Gaston, characters in a comic strip of the early 1900s by Frederick Burr Opper, featuring two exceedingly polite gentlemen.

Joint-Best (2-state) Games

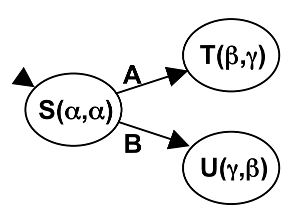
In each of the four remaining 2x2 games with two states, players agree about which state is most preferable, so joint-best is a more precise term than the sometimes used "no-conflict." The games differ in two ways: first, as to whether it is the start state or the other state that both prefer. Second, independent of that, they also differ as to whether the transition structure is a cycle or consists of two transitions from the start state to the other state. In all four combinations of these two aspects, though, each state has an obvious choice for each player: in the preferred state, stay there; in the other state, act (if you have a transition available) to reach the preferred state.

5. Three-State 2x2 Games

Now consider 2x2 games for which the number of states is three. As always, one state is the start state, which we'll call S. In a 3-state game there are two others, call them T and U. Each must, as always, be reachable from the start state. Also, since in a 2x2 game the second "2" is the number of overall strategies each player, no one can be without a transition and no one can have more than one transition (for reasons mentioned in the 2-state case) and so each player must have exactly one transition for a total of exactly two altogether. Those two transitions can link up the three states in two ways: two out-transitions from the start state with one going to each of the others (S to T and S to U) or one from the start state to a middle state (S to T) and another from there to a final state (T to U). Games having the latter structure for their transitions - with players each having one transition and where these can only be exercised in a preset order - can be expressed as a game tree and so can be left for discussion in the context of tree games.

The other possibility for structuring the graph - transitions S to T and S to U - has a state at which both players have out-transitions, giving rise to considerations of timing and not expressible as a game tree. To count up these games, we arbitrarily assign the players to the transitions and then see how many ways we can sprinkle the utility values 3 (best), 2 and 1 (worst) into the states. To get a symmetric game we do the sprinkling only for one of the players, since that and symmetry fix the other player's preferences. Since there are 3! = 6 permutations of the utilities, this yields 6 symmetric games. For the asymmetric case, each of the 6 permutations for one player can be accompanied by any of the 5 for the other that do not yield symmetry. Interchange of players cuts these 6x5 possibilities in half to yield 15 asymmetric games. That's 21 (15+6) 3-state 2x2 games in which timing might play a role. We'll look first at the symmetric ones, beginning with two in which both players like the start state best.

Each of the six symmetric 3-state 2x2 games has a transition from the start state S for each player. Without loss of generality, we stipulate that A's transition is to T and that B's is to U. The diagram covers all six of them. If neither player moves, both get α. If one of them does move, the mover gets β and the other gets γ. We begin with the two games in which the best outcomes are together, that is, where α=3, then take up those in which a player's best result is achieved by acting first, so that β=3 then those where it isn't, so γ=3.



The six symmetric 3-state 2x2 games

Joint-Best (symmetric) Games

Two symmetric games start in (S: 3,3), the state most preferred by each player, so it would seem that neither has cause to abandon that state. Clearly that's true in the game where each player gets lowest possible utility by acting; e.g., (T:1,2) for A and (U:2,1) for B.

Now consider the other symmetric joint-best game starting in (S:3,3). Here A's alternative to inaction is moving to (T:2,1), again a decrease in utility for both players. One could argue that a player may act out of spite, or to avoid the other player doing so, but such an analysis fails because any existing spite should already be part of correctly specified utilities. Still, trouble lurks: a maximin player will take action here, as will a maximizer if the other is viewed as, or is known to be, a maximin player.

Piece of Chicken

A graph game like matrix chicken has start state (S:2,2), contains (T:3,1) and, by symmetry, (U:1,3) as well. To get one's most preferred outcome and avoid the worst one, either player chooses her/his transition action, carrying it out before the other does so. State S is a compromise outcome like the upper left cell of the chicken matrix. From that matrix cell, suppose players are offered the chance to reconsider, so that the row chooser can move down or the column chooser can move to the right. Three cells would then be under consideration, with ordinal payoffs 3 (no move), 4 (self-move) and 2 (other's move), equivalent respectively to 2, 3 and 1 here.

K & K battle it out in the early 60s

Waiting

Two symmetric games remain. Waiting-at-1, containing (S:1,1), (T:2,3) and (U:3,2), is like matrix chicken from the viewpoint of the latter's lower right cell, with its outcome of mutual disaster. The description runs analogously to the one for Piece of Chicken, with matrix players allowed to reconsider. Here the ordinal payoffs are 1 (no move), 2 (self-move) and 3 (other's move), equivalent to 1, 2 and 4 in the matrix. In this game a player has an incentive to act. Since the other does too, waiting - trying to outwait - has a chance to succeed, and if it does it yields the best payoff of all. However, the same reasoning is applicable to the other player, and two would-be outwaiters cannot both succeed.

Waiting-at-2, with (S:2,2), (T:1,3) and (U:3,1), provides a negative incentive for taking action, but each player gains in utility if the other one acts. The incentives are thus for each to wait, presumably in vain, for the other to act. Both waiting games can be seen as cases of shared responsibility. Both can actually model the *same* situation. This is best seen with a physically demanding task whose completion benefits both parties but takes an effort that, for the person doing it, is a factor in the state's overall utility. For folks who are frail or lazy, the payoffs may therefore be those of Waiting-at-2, with its equilibrium of shared inaction. If they're energetic, they may consider themselves to be in Waiting-at-1, perhaps resulting in accomplishment at the expense of equality

With 15 asymmetric games to present and discuss, we need to put them in a sequence that makes structural or strategic sense and facilitates comparisons. We'll first comment on games containing a state best for both. Beyond that, our principal basis for organizing asymmetric games is the number of players - 0, 1 or 2 - that have a direct incentive to move out of the start state. Indirect or defense-oriented incentives to move will also get some attention. A secondary basis for the discussion sequence will be the degree of conflict, operationally defined as the degree to which players' incentives are at odds with each other. We have already spoken of conflict, noting its absence in the case of games where both players have the same preferred state.

Joint-Best Games

Five of the asymmetric games contain a jointly best state, one giving both players a utility of 3. In one of these it is the start state that is best for both and it makes sense to stay there. In the other four games as well, there is evidently no basis for conflict if players fully understand the situation. Although in the latter cases the mutually preferred state is not the start state, there is no apparent reason for players not to reach it since all that is needed is for one player to see that action is called for and to act and for the other to appreciate the advantage of remaining idle.

Notice that joint-best games make up precisely one-third of the 21 three-state games: two of the six symmetric ones and five of the 15 asymmetric ones. This comes as no surprise if we envision filling the three states with one player's payoffs and then the other's, doing each in all possible ways. One out of three is then the proportion of cases in which the second player's best payoff lands in the state occupied by the first player's best payoff.

Asymmetric Race

In each of the ten remaining asymmetric games we now compare each player's payoff in the start state with the payoff to that player upon acting, and ask this question: How many of them would get an improved outcome by taking action: neither of them, just one, or both? The answer "both" occurs just once, and we'll take up that game first: one with states S, T and U as before, but with state payoffs (S:2,1), (T:3,2) and (U:1,3). Each player's transition improves that player's situation, so there is motivation for a race to act, with Player B preferring an unfulfilled attempt to act over yielding without the attempt. In consequence A's transition, unlike B's, improves the lot of both players, a potential advantage if there is to be a settlement or if players anticipate some later possible interaction when cooperation will be available. This combination of features is shared by only one other game, the one with state payoffs (S:2,1), (T:3,3) and (U:1,2). However, note that this is a Joint-Best game and, as noted above, getting to T seems easy in such games.

Rebel *with* Cause

For two games the answer to the above question is "neither." That is, neither gets an improvement by acting so neither has an obvious temptation to do so. Still, at least one of these games is of some interest for its implicit threat potential. Specifically, take (S:3,2), (T:2,3) and (U:1,1). If such a situation goes on long enough Player B might, though not by nature inclined to destructive overthrow of the situation, get frustrated with a kind of second-class citizenship, always ending up less satisfied than what can be made possible by a sacrifice on the part of Player A. In such a case there is the possibility of revolution or at least a rebellious act. This is not to say that these payoff orderings necessarily mean a privileged situation for Player A, since the orderings express only within-player preferences. Such privilege is, however, consistent with these orderings.

Legislation

Similar to the preceding game is one we'll call Legislation. In each of the two the transitions offer no direct temptation to either player, neither game is symmetric and neither belongs to the Joint-Best category. Legislation is the game containing (S,3,2), (T,1,3) and (U,2,1). Here threat is less plausible than in Rebel with Cause, above.

This game seems to us a reasonable model of the U.S. Senate's recent deliberations on health insurance legislation. The exact shape of the bill changed over time, so when we say "the legislation," that should be understood as whatever is under consideration at any moment. It's also well to point out that neither party is monolithic, especially not the Democrats, so when we say that player B is the group supporting the legislation, that is not always all of the Democrats. Similarly the player who opposes the legislation, A, corresponds only approximately to the Republicans. In each party there is a small number of senators who are not part of A or B, but rather are part of the context and are not modeled explicitly. The action available to each player is to yield sufficiently for legislation to pass.

It turned out that the Republicans were content to be labeled the "Party of No," so for player A, yielding leads to the worst outcome, which justifies the worst payoff, 1, to A in state T. The payoffs also suitably reflect the idea that if either of the yielding actions is to occur, each player wishes the other to have been the yielder. If we posit further that A wishes to stymie the legislation but keep it in play as long as possible to use up time that would be spent on other bills that it also opposes, then start state S is A's most preferred state. B, on the other hand, wants to get on with it, though not so badly as to warrant yielding. Thus we arrive at the specification of states and payoffs presented at the start of this discussion.

In each of the 7 remaining asymmetric games, Player B (in the current scheme, but A if players were interchanged) can gain via the transition (B:S→U), indeed getting to his/her most preferred outcome in 4 of the 7. Let's look at those 4 next. In 2 of them (A:S→T) takes A from best to worst, giving A strong incentive not to act. With such a combination of features these games - (S:3,1), (T:1,2), (T:2,3) and - (S:3,2), (T:1,1), (U:2,3), look sure to end at state U. Retaining the property that (B:S→U) gets to B's most preferred outcome, now let (A:S→T) take A from best down only as far as second best outcome. Now A has indirect incentive to try to act first, to avoid a worst outcome which would come about if B pursues self-interest uncontested. The payoffs in these games are (S:3,1), (T:2,2), (T:1,3) and - (S:3,2), (T:2,1), (U:1,3).

Finally, there are 3 asymmetric games in which (B:S→U) improves B's situation but only from worst up to his/her second best outcome. In these games (A:S→T) takes Player A downward in all three possible ways. For example, in the game (S:2,1), (T:1,3), (U:3,2), the transition (A:S→T), takes A from 2 down to 1. In this game we expect an uncontested transition (B:S→U), which yields improvement for both players.

Side Payment

Continuing with asymmetric games with a transition (B:S→U) that takes B up to 2, we now take the case in which (A:S→T) lowers Player A from 3 to 1, which results in the game (S:3,1), (T:1,3), (U:2,2). This sharper decrease gives A an greater incentive not to act than in the preceding game, so it is reasonable to expect U as the outcome. Because the players' preference orderings are in direct opposition, one might be tempted to dismiss the usefulness of discussion. However, if on an interval scale each player would place U closer to the worst than the best of the states and if, also, discussion and side payments are allowed, then such payments could be part of an outcome involving one of the other states.

Suboptimality

Finally there is the game (S:3,1), (T:2,3), (U:1,2), which has an intriguing set of properties, as follows. One player, but not the other, has an incentive to act, but in doing so moves to a state that is the worst for one player, not the best for either, and suboptimal in the sense that there is another state that would be better for both. To arrive at the other state, each would need to recognize, before deciding what to do, the nature of the situation and act (or not) counter what the above-mentioned incentives suggest.

Late in the Day

The eleventh hour comes late in the day and is a metaphor for a situation approaching a deadline, when a previously possible action, moving to a new and possibly desirable state, will no longer be available. The train will "leave the station" or some other vehicle with an immutable departure time will do so, say the airplane at the end of the movie *Casablanca*, with consequences summed up by Rick to Ilsa, "If that plane leaves the ground and you're not with him, you'll regret it. Maybe not today. Maybe not tomorrow, but soon and for the rest of your life." Parameters of deadline situations include: whether negative change will be imposed by nature, as in environmental collapse, or by people, as with an ultimatum or a legal agreement; whether or not the time of the deadline is precise, known, and/or immutable; and which player or players or how many of them, possibly all, can or must act in order to avert the negative outcome.

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