Solutions & Comments for Mentors, #1
week of October 6, 2014

1. The US GDP is over 16 trillion dollars. Shouldn't citizens know what this means?

one hundred = 100 = 10^2
five hundred = 500 = 5×10^2
two thousand = 2,000 = 2×10^3
ten thousand = 10,000 = 10^4
one million = 1,000,000 = 10^6
eight million = 8,000,000 = 8×10^6
ten million = 10,000,000 = 10^7
one billion = 1,000,000,000 = 10^9
one hundred billion = 100,000,000,000 = 10^{11}
one trillion = 1,000,000,000,000 = 10^{12}

2. Let's remind students that they have a fine calculator in their very own brain.
Students can see that $(2 \times 7) \times (2 \times 7)$ has to be the same as $(2 \times 2) \times (7 \times 7)$. I'm not too concerned if they're not fluent in the technical reasons; no one used to expect those from kids. (Nowadays they may get exposed to the *commutative* and *associative* rules -- also called properties or axioms. Essentially these mean, respectively, that rearranging the numbers in a product doesn't change the result and neither does changing the order in which the multiplications are done. The above scrambling of the 2's and 7's actually requires several such steps. I suggest you not get bogged down in all that.)

In the second line, $4 \times (50 - 1) = 4 \times 50 - 4 \times 1$ is again something that most students can see. (Again, there is precise terminology. This is an example of *distributivity*; that is, you can distribute multiplication over addition or over subtraction.) On the right-hand side it's important to realize, as many students do not, that the multiplications must be carried out before the subtraction. (That's called precedence; that is, multiplication has greater precedence than subtraction.)

The last step can be done by *counting backwards* from 200 down to (first) 199, then 198, 197, and (4th and last) to 196.

3. This problem hints at the huge number of everyday applications of probability.
There are $3! = 3 \times 2 \times 1 = 6$ possible orderings of people named **A**, **B** and **C** (short for **A**tieno, **B**ao, and **C**himayi). Here they are, in alphabetical order:

ABC **ACB** **BAC** **BCA** **CAB** **CBA**

Let's think of tasks **a**, **b**, and **c** as humming the musical notes **a**, **b**, and **c**, respectively. Here's a picture of the possible assignments:

ABC	ACB	BAC	BCA	CAB	CBA
abc	abc	abc	abc	abc	abc

In the first case, all three people hum their own initial; in the second one, only **A** does so; in the third, only **C** does; in the last one, only **B** does. Match-ups (of a person's initial to task assigned) are underlined. That leaves **BCA** and **CAB** as the only two cases with no match-up. So the stated condition is satisfied in just these 2 cases out of 6. The problem states that all 6 orderings are equally probable, so the probability is 2/6 which equals 1/3.

Problem too hard? Let your student skip it or replace "three" by "two" and leave out "**C**himayi." Too easy? She/he can replace "three" by "four" and add **D**aniele.