

College Bound Math Solutions #11

week of January 12, 2015

Mentors: The solutions are below. First, please respond to this questionnaire and send directly to me at henry@collegebound.org *I need and value your feedback.*

1. **Problems:** I hope your student (with your help if needed) is looking through the weekly math problems to find at least one each week that's intriguing and at the right level of difficulty. Is your student doing that? Finding one? Usually? Rarely? About half the time?
2. **Solutions:** The solutions are for you, to make it easier for you to help your student. Do you use them? How can I improve them? Should I discontinue them?

Math materials are emailed weekly or go to cbmath.org
and click on "Weekly Math Problems & Solutions "

3. **MathLab:** Has your student ever come to MathLab? Math-challenged students should start coming in the first week of the semester, not wait until it's too late. Strong math students should be coming to stay strong and get SAT/ACT scores that will get them into a college worthy of their abilities.

For full information on MathLab go to cbmath.org
and click on "MathLab: Free Tutoring"

My e-address is easy to remember: henry@collegebound.org

Solutions

Please have your student try 1 or 2 problems at a suitable level.

12. The divisors of 12 are 1, 2, 3, 4, 6, and 12. It is the smallest number with more than 4 divisors
14. Trial and error may lead to some insight so it's a good way to start and the curious student may enjoy that. However, it's very difficult to come up with an explanation. Saying "I tried it a few ways and none worked" doesn't show that no arrangement of 7 does the job.

So here's the key: A 1-by-2 block covers one square plus its neighbor. On this checkerboard that means one black and one white square. But although there are 14 squares on the board and 14 divided by 2 is indeed 7, it turns out that there are 8(!) black squares and just 6 white ones.

15. The checkerboard for problem "14" has (exterior) sides with these lengths: 3, 3, 1, 1, 3, 3, 1, 1. Those numbers add up to 16 which is therefore the perimeter (by definition of perimeter). The problem states that the rectangle has that same perimeter. So we are to look for a rectangle with perimeter 16 and area 15.

If we assume that the kind problem-maker (me) has made the answer come out nicely with only integers, we can just try all the possible rectangle dimensions with the required perimeter: 1-by-7, 2-by-6, 3-by-5 and 4-by-4. Of these only 3-by-5 gives an area of 15. (That is, $3 + 5 + 3 + 5 = 16$ and $3 \cdot 5 = 15$ and the others don't work; e.g., $2 \cdot 6 \neq 15$).

Some students may think of using algebra: $x + y + x + y = 16$ and $xy = 15$. Using substitution, among other things, yields a quadratic equation, $x(8 - x) = 15$, that Algebra-2 students can solve. The answers turn out to be 3 and 5 even though we did not *assume* that they would be confined to integers.