

College Bound Math Solutions #15
week of February 9, 2015

Note: One of you said the calendar questions were fun (good! thanks.) and suggested more of them, so here they are. As noted on the Problem page, the answers are in **large** type. That is, the numbers **9**, **11** and **12** on the calendar are not only days of the month, they are the answers. Feel free to point this out and provide other help. This is supposed to be fun and spark some interest.

9 is a perfect square because it is equal to 3^2 . It's the sum of two different cubes because $2^3 + 1^3 = 8 + 1 = 9$. A systematic approach is a table of sums of cubes:

sums of cubes	$1^3=1$	$2^3=8$	$3^3=27$	$4^3=64$
$1^3=1$	[2]	9	28	65
$2^3=8$	9	[16]	35	72
$3^3=27$	28	35	[54]	91
$4^3=64$	65	72	91	[128]

The bracketed sums on the diagonal are disallowed since two different cubes are asked for. Also, sums below the diagonal (written small) are matched by those above it (addition does not depend on order), so there are only six candidates under 100 and only 9 is a winner. Making a larger table and simplifying in the above ways, it doesn't take long to show that the next solution after 9 is 576.

11 is symmetric. One way to convince your student of this is to write it in dark ink on thin paper, flip it, hold it up to the light and you get a pair of ones: **11**. Of course the serifs are now backward so maybe you should omit them and make it look like this: **||**. Next $11^2 = 121$, which is also symmetric. Then, multiply 121 by 11 to get 1331, which, times eleven, gives 14641, all symmetric.

Any one-digit number is symmetric, but the problem disallows them. The two-digit palindromes other than 11 are 22, 33, 44, 55, 66, 77, 88 and 99. Of these, only 22 has a symmetric square (484), and the cube of 22 is *not* symmetric.

12 is called a dozen (as well as twelve), so we're halfway there. If you think of searching in Google for "dozen dozen" (using those quotes), your very first hit will be the Wikipedia entry for a "gross," which, we are told, "refers to a group of 144 items (a dozen dozen)," in other words the square of 12.

My sense of "dozen" is that it doesn't always promise an exact count. "Twelve" seems to me to indicate greater precision. French, which is the source of the word "dozen" uses "douzaine" this way, at times, and also has similar words that can mean *approximately* 10, 15, 20, 30, 40, 50, 60 and 100. The first two would be dizaine and quinzaine. They all have the same ending as "douzaine."