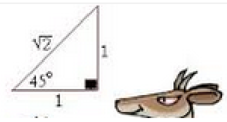
College Bound Math Problems #17
week of March 2, 2015



Note: These problems are all connected. Start with #1, a warm-up on the ideas used in #2, which is a series of steps that can be programmed to run on a computer**.** Then #3 analyzes how well the process has worked**.**

1. (a) Write the reciprocal of each: $\frac{4}{7}$ ____ $\frac{12}{5}$ ____

(b) Write as an improper fraction: $1\frac{1}{2}$ ____ $2\frac{2}{5}$ _____

2. Finding $\sqrt{2}$ with paper and pencil. The square root of 2 is *the number whose square is 2*. This number *can't* be an integer because 0^2 and 1^2 are too small (less than 2), while 2^2, 3^2 and all the larger squares are too big (greater than 2). However, we can approximate $\sqrt{2}$ by repeatedly using the sequence of steps in the box below. The values in Row #5 are the approximations. They get closer and closer to $\sqrt{2}$.

Row	**Steps**	**1st time**	**2nd time**	**3rd time**
1	Guess (first time only). Then use last result in preceding column.	$1\frac{1}{2}$	$1\frac{2}{5}$	$1\frac{5}{12}$
2	Add 1.	$2\frac{1}{2}$	$2\frac{2}{5}$	**?**
3	Convert to an improper fraction.	$\frac{5}{2}$	$\frac{12}{5}$	**?**
4	Take the reciprocal.	$\frac{2}{5}$	$\frac{5}{12}$	**?**
5	Add 1 and copy the result to the top of the next column.	$1\frac{2}{5}$	$1\frac{5}{12}$	**?**
6	Do it all again or stop.			

(a) Why is $1\frac{1}{2}$ a reasonable guess in step #1 at the beginning of the process?

(b) Replace each "**?**" in the column called "**3rd time**" by the value that will go there.

3. Use a calculator to see how close these results are to the actual square root of 2.

(a) To 4 decimal places, what is the square root of 2?

(b) To 4 decimal places, write the decimal form of $1\frac{1}{2}$ and each number in Row #5.

(c) Subtract the value of $\sqrt{2}$ found in part (a) from each answer in part (b).

(d) Compare the results found in part (c).

Note: As we'll see next week, there are better ways to arrive at approximations. One of them was devised by Isaac Newton 300+ years ago, another by the Babylonians 3,000+ years ago. The beauty of Newton's method lies in its generality as well as how quickly it gains accuracy, compared to your results in Problem #3. For more, see: http://en.wikipedia.org/wiki/Square_root_of_2