

**College Bound Math Solutions #17**  
week of March 2, 2015

$$1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2}}}$$

**It's an Application.** Problem #2 introduces a foundation of computer science, the algorithm. An algorithm consists of steps and the rules that control which step comes next. Algorithms don't have to involve numbers, but this one does. The paper-and-pencil methods of adding, subtracting, multiplying and dividing are also algorithms.

**Inside the machine.** A student may protest that one can do these operations with a calculator or computer. To this I suggest as an answer, "but what's going on inside that machine?" Answer: a process under the control of (ta-da!!) an algorithm. Please also read the notes at the top and bottom of the Problem sheet. (FYI, Problem #2 is adapted from the "continued fraction" above and at <http://www.cut-the-knot.org/proofs/SqContinuedFraction.shtml>)

**Problems 1,2,3 are related.** I hope Problem #1 will be accessible to 8<sup>th</sup>-graders. It's included as a check and a warm-up, to prepare for Problem #2. Then #3 does some analysis of the results of #2.

**Calculator use:** A lot of today's students like to use a calculator and are very good at it. They should use it for #3, where it will help them to maintain focus on the point of that problem. (However, it should not be used on #1 or #2.)

1. (a) Reciprocal:  $\frac{4}{7} \rightarrow \frac{7}{4}$                        $\frac{12}{5} \rightarrow \frac{5}{12}$   
 (b) Improper fraction:  $1\frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$                        $2\frac{2}{5} = \frac{10}{5} + \frac{2}{5} = \frac{12}{5}$
  
2. (a) As noted in the introduction to Problem #2, the value of  $\sqrt{2}$  must lie between 1 and 2. Being right in the middle of that range,  $1\frac{1}{2}$  is a reasonable starting point. (But other arguments lead to other choices, for example,  $1\frac{1}{3}$ .)  
 (b) The third time through the steps, rows 2 through 5 become, respectively:       $2\frac{5}{12}$                        $\frac{29}{12}$                        $\frac{12}{29}$                        $1\frac{12}{29}$
  
3. (a) 1.4142  
 (b) 1.5000                      1.4000                      1.4167                      1.4138  
 (c) Subtracting 1.4142 from each answer in (b) yields these numbers.  
       0.0858                      -0.0142                      0.0025                      -0.0004  
 (d) Your student may notice the signs going from positive to negative and back, though if we're interested in how close we're getting to the answer, it's the magnitudes (ignoring the sign) that we care about. Each magnitude is about one-sixth ( $\frac{1}{6}$ ) of the preceding one. For example,  $\frac{0.0025}{0.0004} = 6.25$ .