

Solutions & Comments for Mentors, #2
week of October 13, 2014

- | | | | | |
|----|-------------------|------------------------------|------------|-------------------------------|
| 1. | 10^4 | <u>ten thousand</u> | 10^{-14} | <u>one hundred-trillionth</u> |
| | 7.2×10^9 | <u>7 billion 200 million</u> | 10^{-10} | <u>one ten-billionth</u> |
| | 10^{14} | <u>one hundred trillion</u> | | |

Powers and prefixes (monopoly, bicycle, triple, quadrangle, quintet, ...) are the keys to these number names. The crucial role is played by powers of one thousand, but remember to subtract 1.

- $1000^1 = 1,000 =$ one thousand
 $1000^2 = 1,000,000 =$ one million
 $1000^3 = 1,000,000,000 =$ one billion
 $1000^4 = 1,000,000,000,000 =$ one trillion
 $1000^5 = 1,000,000,000,000,000 =$ one quadrillion

2. (a) $600 \text{ nanometers} = 6 \times 10^2 \times 10^{-9} \text{ meters} = 6 \times 10^{-7} \text{ meters}$

(b)
$$\text{frequency} = \frac{\text{speed}}{\text{wavelength}} = \frac{3 \times 10^8}{6 \times 10^{-7}} = \frac{30 \times 10^7}{6 \times 10^{-7}} = 5 \times 10^{14} / \text{sec}$$

(d) One can (i) web-search for the "speed of light" (in "miles per second" or (ii) web-search for "conversion" of "speed units" (and use a converter on the speed that's provided in the problem), or (iii) use only what we know ... if "we" know the handy fact that a kilometer is about 62% of a mile. Thus, $3 \times 10^8 \text{ m/sec} =$

$$3 \times 10^5 \text{ km/sec} \approx (0.62) \times 3 \times 10^5 \text{ mi/sec} = 1.86 \times 10^5 \text{ mi/sec} = 186,000 \text{ mi/sec}.$$

3. Draw the line segment \overline{BD} and consider it to be the base of the triangle $\triangle BDA$. If your student likes bases to be horizontal, just rotate the figure... or tilt your heads to the left! Since B and D are opposite corners of a square, \overline{BD} goes through the center, O , so it's a diameter*. Its length is therefore $2r$.

The height of any triangle is measured in a direction perpendicular to its base. The radius \overline{OA} meets that requirement* and, since it's a radius, its length is r .

The area of any triangle is half its base times its height. In the case of $\triangle BDA$, that's $\frac{1}{2}(2r)(r) = r^2$. Since two such triangles make up the square, its area is $2r^2$.

* The two starred claims in #3 can be proved using the definition of a square, but I wouldn't recommend going into proofs. They're both visually plausible and if your student raises the issue, they can be argued on symmetry grounds. For example: If not through the center, on *which side* of the center would the line go?! Looking at the figure in a mirror - or drawing it on a glass plate that we then flip - we get the same figure but with the line passing on the *other side* of the center, an outcome that contradicts the assumption that the line could possibly miss the center!