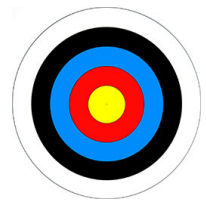


College Bound Math Solutions #20
week of March 30, 2015

1. Probabilities (of all the possible and mutually exclusive outcomes of an event) are defined (in general) to add up to 1.
- (a) $\frac{1}{4}$ In this problem, it's given that the 4 letters all have the same probability, which must be $\frac{1}{4}$ so that they'll add up to 1.
- (b) $\frac{3}{4}$ You either *do* or *do not* get an A and each excludes the other. Therefore the answers to part (a) and to part (b) must add up to 1.
- (c) $\frac{3}{16}$ On two tosses, there are $4 \times 4 = 16$ possibilities: AA, AB, AC, AD, BA, ..., DD. Each has probability $\frac{1}{16}$ since they are all equal and add up to 1. Only AB, AC and AD are cases of "getting an A and then something else." $\frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{16}$.

2. Again, probabilities add up to 1. Again, you either *do* or *do not* get something, in this case, an earthquake in the coming decade.
- (a) 0.6 This is the only number that can be added to 0.4 to get a total of 1.
- (b) 0.24 The "rule" is that you multiply: $0.6 \times 0.4 = 0.24$. But it may help to make it look like #1(c). Suppose Mother Nature lets you determine whether or not a major earthquake will occur in the coming decade -- by picking a card from among five cards labeled 1, 2, A, B and C, placed face down on a table. A number means a quake and a letter means no quake. You have no information. You pick a card, they are taken away, shuffled, put down again, and you choose again. There are $5 \times 5 = 25$ outcomes with equal probabilities that total 1, so each is $\frac{1}{25}$. Six of them meet the conditions: A1, A2, B1, B2, C1, C2 and $\frac{6}{25} = \frac{3 \times 2}{5 \times 5} = \frac{3}{5} \times \frac{2}{5} = 0.6 \times 0.4 = 0.24$.
- (c) 0.144 There are now $5 \times 5 \times 5 = 125$ outcomes, of which $3 \times 3 \times 2 = 18$ satisfy the conditions: AA1, AA2, AB1, ..., CC2 so that's $\frac{18}{125} = 0.6 \times 0.6 \times 0.4 = 0.144$.

3. Let r be the radius of the bull's-eye, so its area is πr^2 . Then $2r$ is the radius of the yellow+red circle and $\pi(2r)^2 = 4\pi r^2$ is its area. Also, the blue+red+yellow circle has radius $3r$ and area $\pi(3r)^2 = 9\pi r^2$. Now the area of the red ring can be found by subtracting areas of circles: $4\pi r^2 - 1\pi r^2 = 3\pi r^2$. The blue ring area is $9\pi r^2 - 4\pi r^2 = 5\pi r^2$.



- (a) $\frac{5}{9}$ With random scattering, the probabilities are proportions of areas: $\frac{5\pi r^2}{9\pi r^2} = \frac{5}{9}$.
- (b) $\frac{64}{729}$ The probability of hitting the bull's-eye is $\frac{(1)\pi r^2}{9\pi r^2} = \frac{1}{9}$, so the probability of a miss is $1 - \frac{1}{9} = \frac{8}{9}$ and the probability of [miss, miss, hit] is $\frac{8}{9} \times \frac{8}{9} \times \frac{1}{9} = \frac{64}{729}$. Note the similarity to #2(c): a hit corresponds to an earthquake in a given decade.