College Bound Math Solutions #20
week of March 30, 2015

1. Probabilities (of all the possible and mutually exclusive outcomes of an event) are defined (in general) to add up to 1.

   (a) ¼ In this problem, it's given that the 4 letters all have the same probability, which must be must be ¼ so that they'll add up to 1.

   (b) ¾ You either *do* or *do not* get an A and each excludes the other. Therefore the answers to part (a) and to part (b) must add up to 1.

   (c) 3/16 On two tosses, there are $4 \times 4 = 16$ possibilities: AA,AB,AC,AD,BA,..., DD. Each has probability 1/16 since they are all equal and add up to 1. Only AB, AC and AD are cases of "getting an A and then something else." 1/16 + 1/16 + 1/16 = 3/16.

2. Again, probabilities add up to 1. Again, you either *do* or *do not* get something, in this case, an earthquake in the coming decade.

   (a) 0.6 This is the only number that can be added to 0.4 to get a total of 1.

   (b) 0.24 The "rule" is that you multiply: $0.6 \times 0.4 = 0.24$. But it may help to make it look like #1(c). Suppose Mother Nature lets you determine whether or not a major earthquake will occur in the coming decade -- by picking a card from among five cards labeled 1, 2, A, B and C, placed face down on a table. A number means a quake and a letter means <u>*no*</u> quake. You have no information. You pick a card, they are taken away, shuffled, put down again, and you choose again. There are $5 \times 5 = 25$ outcomes with equal probabilities that total 1, so each is 1/25. Six of them meet the conditions: A1,A2,B1,B2,C1,C2 and $\frac{6}{25} = \frac{3\times2}{5\times5} = \frac{3}{5} \times \frac{2}{5} = 0.6 \times 0.4 = 0.24$.

   (c) 0.144 There are now $5 \times 5 \times 5 = 125$ outcomes, of which $3 \times 3 \times 2 = 18$ satisfy the conditions: AA1, AA2, AB1, ..., CC2 so that's $\frac{18}{125} = 0.6 \times 0.6 \times 0.4 = 0.144$.

3. Let r be the radius of the bull's-eye, so its area is πr^2. Then $2r$ is the radius of the yellow+red circle and $\pi(2r)^2 = 4\pi r^2$ is its area. Also, the blue+red+yellow circle has radius $3r$ and area $\pi(3r)^2 = 9\pi r^2$. Now the area of the red <u>*ring*</u> can be found by subtracting areas of circles: $4\pi r^2 - 1\pi r^2 = 3\pi r^2$. The blue ring area is $9\pi r^2 - 4\pi r^2 = 5\pi r^2$.

   (a) 5/9 With random scattering, the probabilities are proportions of areas: $\frac{5\pi r^2}{9\pi r^2} = \frac{5}{9}$.

   (b) 64/729 The probability of hitting the bull's-eye is $\frac{(1)\pi r^2}{9\pi r^2} = \frac{1}{9}$, so the probability of a miss is $1 - 1/9 = 9/9 - 1/9 = 8/9$ and the probability of [miss, miss, hit] is $8/9 \times 8/9 \times 1/9 = 64/729$. Note the similarity to #2(c): a hit corresponds to an earthquake in a given decade.