Solutions & Comments for Mentors, #5
for the week of November 3, 2014

Note to mentors: Each problem here follows up on the one numbered the same in Set #4, so you may find some things in the solutions to #4 that are relevant here. These problems go a bit further. A student who hasn't done any of Set #4 yet should do some of them first.

1. (a) $1\frac{4}{5} \times 4\frac{1}{2} = (1+\frac{4}{5})(4+\frac{1}{2})$ by the ***definition*** of mixed numbers.

(b) "No decimals until part (c)," so use F.O.I.L. as in Set #4 to get:

$$(1+\tfrac{4}{5})(4+\tfrac{1}{2}) = (1\bullet 4)+(1\bullet\tfrac{1}{2})+(\tfrac{4}{5}\bullet 4)+(\tfrac{4}{5}\bullet\tfrac{1}{2}) =$$

$$4+\tfrac{1}{2}+\tfrac{16}{5}+\tfrac{4}{10} = 4+\frac{5+32+4}{10} = 4+\tfrac{41}{10} = 4+\tfrac{40+1}{10} = 4+4+\tfrac{1}{10} = 8\tfrac{1}{10}$$

(c) Start as at right, but by ignoring decimal points we have a result that's 100 times too big and we need to move the decimal point 2 places to the left from its implicit position at the right end of "810**.**" to get the correct answer, 8.10 or 8.1, which is consistent with the answer in (b).

$$\begin{array}{r} 1.8 \\ \times\ 4.5 \\ \hline 90 \\ +\ 720 \\ \hline 810 \end{array}$$

2. $3^0 = 1,\ 3^1 = 3,\ 3^2 =$ **9**, $3^3 =$ **27**, $3^4 =$ **81**.

(a) $3^x = 9 = 3^2$, so x must be **2**.

(b) $3^{x-1} = 27 = 3^3$, so $x-1$ must be 3, which requires x to be **4**.

(c) $3^{2x} = 81 = 3^4$, so $2x = 4$ and x is **2**.

3. Since 3 circles span the 3 inches each circle must have a width of $\frac{3}{3} = 1$, measured along its diameter. So that diameter must be 1 and the height of the rectangle *must* be 2, as given. It also follows that each radius is $\frac{1}{2}$.

(a) Since each radius is $\frac{1}{2}$, the area of (any) one circle is $\pi r^2 = \pi(\frac{1}{2})^2 = \frac{1}{4}\pi$.

(b) The total area of 6 circles is $6 \times \frac{\pi}{4} = \frac{6}{4}\pi = \frac{3}{2}\pi$

(c) The gray portion is a 3 by 2 rectangle with the 6 circles removed. That's $6 - \frac{3}{2}\pi$.