

Solutions & Comments for Mentors, #7

week of November 17, 2014

Mentors: This week's questions are closely tied to each other. The topic has fascinated architects and artists, biologists and mathematicians, yet lies within the capacities of an 8th grader. I hope it's fun for your student. If not, I wouldn't push it. It won't be on the final.

1. (a) "Each number is the sum of the two preceding numbers," so the number after 55 is the sum of 34 and 55, which is **89**.
(b) The differences within pairs are given as (1-1), (2-1), (3-2), (5-3), (8-5), ... After that come (13-8), (21-13), (34-21), (55-34). Doing the subtractions yields 0, 1, 1, 2, 3 and then 5, 8, 13, 21. These are precisely the Fibonacci numbers themselves. It's no coincidence. We have just used subtraction to undo addition.
2. (a) The first few ratios were provided: 1.000, 2.000, 1.500, 1.667. After that come 1.600, 1.625, 1.615, 1.619, 1.618
(b) One aspect of the pattern is that the direction of change from one number to the next alternates between up and down. Thus from 1 we go **up** to 2, then **down** to 1.5, **up** to 1.667, **down** to 1.6, etc. Also, the **changes get smaller and smaller** and the value seems to be **zeroing in on 1.618**.
(c) With three decimal places all the remaining ratios are 1.618. With four decimal places, we have $55/34=1.6176$, then $89/55=1.6182$. The number being approached is called the "golden ratio." To eight decimal places it's 1.61803399.
3. This is a matter of taste, of course. My own reaction is that almost all of the images - natural and artificial - are intriguing and some are gorgeous. The one below is artificial. And by the way, the spiral is known as the "golden spiral."

