

Solutions & Comments for Mentors, #8
week of December 8, 2014

Mentors: These questions are related to each other and have a practical nature. Admittedly, paper has become somewhat less important with the rise of the Internet, but I trust that the USPS still brings you ads on paper. Problems #1 and #2 really did occur in my presence quite recently.

1. Each cut is "in the shorter direction," so the cuts are $8\frac{1}{2}$ inches long, but the side that gets divided is the 11-inch side, as shown. After cutting, the short sides are each one third of 11, which is



$$\left(\frac{1}{3}\right)(11) = \frac{11}{3} = \frac{9+2}{3} = \frac{9}{3} + \frac{2}{3} = 3 + \frac{2}{3} = 3\frac{2}{3}.$$

2. (The statement of this problem contains the answer to #1, so a student may guess that this is so and will have clear sailing on that problem.)

The distance (one inch) between 3 and 4 is divided into 16 intervals each of length $\frac{1}{16}$ ". To reach the desired position, $3\frac{2}{3}$, starting from 3, we need to go two-thirds of the way, i.e., two-thirds of the 16 little intervals, which is

$\left(\frac{2}{3}\right) \times (16) = \frac{2 \times 16}{3} = \frac{32}{3} = \frac{30+2}{3} = \frac{30}{3} + \frac{2}{3} = 10 + \frac{2}{3} = 10\frac{2}{3}$. Since $10\frac{2}{3}$ is closer to 11 than to 10, the required value of n is 11 and the required distance is $\frac{11}{16}$ of the way from 3 to 4. Of course, you might want to eyeball it back slightly (one-third of the way) toward $3\frac{10}{16} = 3\frac{5}{8}$.



3. Paper is often sold in packages of 500 sheets, which is known as a *ream*.
- (a) In any case, one sheet of this paper weighs one five-hundredth of 15 pounds or $\left(\frac{1}{500}\right)(15) = \frac{3 \times 5}{100 \times 5} = \frac{3}{100} = 0.03$ pounds and each third of that is 0.01 pounds. Since each pound is 16 ounces, each cut piece weighs $(16) \times 0.01 = 0.16$ ounces, well within the one-ounce limit.
- (b) (i) A 17×22 sheet is exactly twice as long in each direction as an $8\frac{1}{2} \times 11$ sheet so you can cut the big sheet into $2 \times 2 = 4$ of the smaller sheets.
- (iii) As stated on the printing website, "24-pound bond" refers to the weight of a ream of 17×22 sheets. (A ream is 500 sheets, as noted above.) Since each sheet of that size yields 4 sheets of $8\frac{1}{2} \times 11$, the latter weigh only one quarter as much, that is, $\left(\frac{1}{4}\right)(24) = 6$ pounds per ream. That's only 40% of the 15 pounds per ream given as the weight of a ream of the thick paper in the statement of this problem.