Solutions & Comments for Mentors, #9
week of December 15, 2014

1. Some history is included in case your student is interested. Flags like (c) and (d) can be made with any number of stars that is twice a perfect square ($2n^2$).

(a) 1 star. Ghanaian soccer player and flag, at the 2010 Soccer World Cup.

(b) $7+1=8$. The Big Dipper and North Star pointed the way north for escaping slaves on the Underground Railroad. For more, search "Follow the Drinking Gourd." This is also the flag of Alaska (way up north).

(c) $9 \bullet 8 + 8 \bullet 7 = 8(9+7) = 8 \bullet 16 = 8 \bullet 2 \bullet 8 = 2 \bullet 8^2 = 128$. Too many to count!

(d) Current US flag. Just like (c). $6 \bullet 5 + 5 \bullet 4 = 5 \bullet 10 = 2 \bullet 5^2 = 50$

(e) Briefly US flag, 1959-60, before Hawaii. $7 \bullet 7 = 49$

(f) US flag, 1912-1959, before Alaska and Hawaii. $6 \bullet 8 = 48$

(g) 5 triangles of 6, separated by 5 radiating lines of 4. Also, 1 in center. $30+20+1=51$

2. (a) 10

(b) 5 of them get flipped over at some point. In the language of geometry, they have be *reflected* (as in a mirror). Start with the yellow one that's already there. Call it #1. Now reflect it to get #2. More precisely, do a reflection over the line segment AO. Together #1 and #2 now fill up the quadrilateral ABOE, where E is the left-side counterpart of B. That is, E is at the same vertical position as B and the same distance from the line segment AO.

Now add in 4 more quadrilaterals identical to ABOE, but rotated around point O by various amounts so that their tips (corresponding to A) are at the other 4 points of the star. Each of these new quadrilaterals was formed from 2 triangles: from #3 and #4 up to #9 and #10.

3. (a) Arc AC is one of 5 equal arcs making up the whole circle. A fifth of a circle is $360/5$ = 72°

(b) An angle at the center has the same number of degrees as its intercepted arc, which in this case we know from (a). It's 72°.

(c) By symmetry, $\angle AOB$ and $\angle BOC$ are equal. Their sum is 72°, so each is 36°.

(d) 36°. As stated in the problem, the number of degree in an angle like this - an angle that is inscribed in a circle - is always half the number of degrees in the arc that it intercepts.

Knowing the angles plus some trig and algebra, one can find the star's area as a function of the radius of the circle. It's about $1.12r^2$, which is close to 36% of the circle's area. That's true for a star of *any size* so long as its points "lie at equal distances around a circle," as specified in the problem. See http://www.mathalino.com/reviewer/plane-geometry/area-of-regular-five-pointed-star