TAKE SOME—A FORMAT AND FAMILY OF GAMES

by Henry Hamburger

University of California at Irvine

"Take Some" is a very simple kind of game with probabilistic outcomes. The expected payoffs can be adjusted to conform to some but not all matrix games. Both Prisoner's dilemma and chicken can be played in Take Some format. The format closely parallels certain aspects of real world situations.

EXPERIMENTAL games are typically presented to subjects in matrix format. In this paper we shall describe a different format, called Take Some, which closely parallels significant aspects of real world situations. Prisoner's dilemma can be played in both matrix and Take Some formats and Guyer, Fox, and Hamburger (1974) have found that cooperation levels are substantially higher with Take Some.

The well known game of chicken can also be put into Take Some format, but some other games which have attracted experimental work cannot be reformulated in this way and thus are not members of the Take Some family. In this paper we shall examine the constraints imposed by the Take Some format.

INTRODUCTORY EXAMPLE

In a Take Some game, each of several players asks for an amount of money, which he may or may not get. Players submit their individual requests simultaneously, in ignorance of each other, and all are granted if their sum does not exceed the number which comes up on a roulette wheel. Otherwise all players get nothing.

Specifically suppose that in a two-person game, the roulette wheel has a rectangular distribution over the interval from 0 to 100. If player A chooses to ask for x while player B asks for y, then, if $x + y \leqq 100$, the probability that they win is $(100\text{-}x\text{-}y)/100$, and their expected payoffs are, respectively,

$$P_A = x(100\text{-}x\text{-}y)/100$$

$$P_B = y(100\text{-}x\text{-}y)/100.$$

(if $x + y \geqq 100$, they must get 0). Equilibrium occurs when

$$\partial P_A/\partial x = -x/100 + (100\text{-}x\text{-}y)/100 = 0$$

and

$$\partial P_B/\partial y = -y/100 + (100\text{-}x\text{-}y)/100 = 0.$$

Thus $2x + y = 100$ and $x + 2y = 100$.

These lines are shown in Fig. 1; they intersect at the equilibrium point, E, where $x = y = 33\frac{1}{3}$. Also shown is 0, the symmetric optimum where $x = y = 25$ so that both players receive an expected payoff of $12\frac{1}{2}$, which is better for both than the $11\frac{1}{9}$ which both get at E. Restricting attention to the choices 25 and $33\frac{1}{3}$ yields matrix 1a. In this game each player has a dominant strategy, to ask for $33\frac{1}{3}$. These dominant

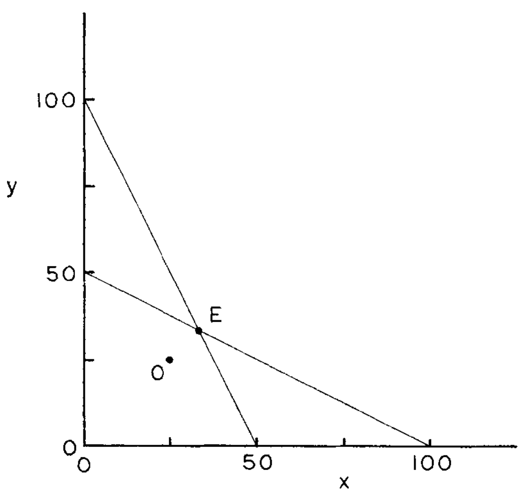

FIGURE 1.

		B ask for 25	ask for 33 1/3
A	ask for 25	12.5, 12.5	10.4, 13.9
	ask for 33 1/3	13.9, 10.4	11.1, 11.1

Matrix 1a

strategies intersect in an equilibrium outcome corresponding to E in Fig. 1.

In a 2×2 game we shall represent a player's best possible outcome by 4, his next best by 3, next 2, and his worst by 1. In this way we represent only the ordinal aspects of payoffs. Matrix 1a then becomes matrix 1b, which is prisoner's dilemma, by definition.

	B_1	B_2
A_1	3,3	1,4
A_2	4,1	2,2

Matrix 1b

Other regions of Fig. 1 have other orderings which correspond to other games. For example, if players are restricted to two alternatives in the range from zero to 25, the resulting game is shown as matrix 2.

	lower	higher
lower	2,2	1,4
higher	4,1	3,3

Matrix 2

Here the equilibrium outcome is relatively beneficial. Matrices 1b and 2 represent only two of the 19 ordinally distinct games that can be presented in Take Some format as we shall show later.

Before leaving this example, it is worth pointing out that the difference between equilibrium payoff and joint optimal payoff, though small here (11.1 versus 12.5), can be substantial in multiperson games. In fact if N is the number of players, then the ratio of equilibrium payoff to optimal payoff approaches $1/N$ as N increases indefinitely.

Use of environmental resources bears some resemblance to a Take Some game. If everybody uses too much, then everybody loses. On the other hand, if total usage is not so great as to damage the environment irreparably, then, in the absence of government regulation or taxation, everybody gets what he took. How big a total is too much is not exactly known to the players and is thus represented by a probability distribution. For example, no one knows just how much total driving by everyone will damage the lungs of a city full of people.

GAME FORMATS AND FAMILIES

The foregoing example shows that the game of prisoner's dilemma, defined by the ordinal properties of matrix 1b, can be presented either in the usual matrix format or in the Take Some format. Yet another format is the oscilloscope screen (Rosenberg, 1968; Sawyer & Friedell, 1965) where each of two players controls a coordinate of a point in two-dimensional space. Payoffs to players are expressed in terms of goal-lines on the screen. A fourth format—in addition to matrix, Take Some, and oscilloscope—is the auction. As a fifth example, consider the situation in which players must divide a fixed amount and for each unit of time that goes by during their deliberations, they lose a certain amount. This might be called a delay-divide format. Labor management contract negotiations resemble this format (Bishop, 1964) as do certain legislative situations.

The sixth and last format to be mentioned here is the separated or decomposed form of a game. Here player i (with $i = 1$ or 2) can take a partial payoff of a_i or b_i, thereby causing the other player to get c_i or d_i, respectively. The two partial payoffs are added. Thus if player 1 takes a_1 and player 2 takes b_2, their respective total payoffs are $a_1 + d_2$ and $b_2 + c_1$. Separated games also can be defined for more than two players and with more than two alternatives to a player. Corresponding real world situations include doing favors and, with certain assumptions, giving contributions (Hamburger 1969; 1973; Pruitt, 1967).

Use of a particular format can implicitly restrict the class of games. For example, in a separated game, as just described, a player always has a dominant strategy—in fact, the restriction is even tighter than this (Hamburger, 1969)—namely to pick the larger of a_i and b_i. We shall show that the Take Some format also imposes restrictions.

As a basis for comparison, consider all the possible two-person, two-alternative games defined on an ordinal scale. Two such games are shown in matrices 1b and 2, above. If each player has clear preferences among his payoffs so that it is appropriate to label them 4, 3, 2 and 1, then there are $4! = 24$ ways each player's payoffs can be distributed into the four cells of the game matrix. There are thus $24 \times 24 = 576$ ways to fill up the matrix, but many of these are the same as others except for an interchange of rows and/or columns and/or players. Rapoport and Guyer (1966) have used these considerations to show that there are in fact 78 ordinally distinct 2×2 games. We shall see that exactly 19 of these can be played in Take Some format.

DEFINITION AND MATRIX

In two-person, two-alternative Take Some, player I takes his choice between two positive numbers, call the smaller a and the larger b. Simultaneously, player II takes his choice between two positive numbers, in general different than those available to player I. Call the smaller of these c and the larger d. The two numbers picked, one by each player, are added together to form a total, T. Then a number L, called the limit, is chosen in random fashion according to a prespecified probability distribution. If T does not exceed L, each player gets a payoff, in points, nickels, or other units, equal to the number he picked. If T does exceed L, each player gets 0.

For the purposes of the following discussion, we assume that the relevant payoff to a player, once players' choices have been made, is his expected number of points. This means that he is indifferent to variance and higher moments, or equivalently, that his utility is linear with money or whatever the unit of reward is.

The total, T, of the two numbers picked by the players, must be one of the four numbers $a + c$, $a + d$, $b + c$, or $b + d$. The limit, L, is a random variable. Define p, q, r, and s as follows:

$$\begin{aligned} p &= \text{Prob}\ (L < a + c) \\ q &= \text{Prob}\ (L < a + d) \\ r &= \text{Prob}\ (L < b + c) \\ s &= \text{Prob}\ (L < b + d). \end{aligned}$$

We are requiring, without loss of generality, that

(1) $$a < b$$

(2) $$c < d.$$

Therefore no matter what the probability distribution for L,

$$p \geqq q \geqq s$$

and

$$p \geqq r \geqq s.$$

For the sake of comparability with the Rapoport-Guyer taxonomy mentioned above, we require that these inequalities be strict. This requirement means that the probability distribution for L is not quite arbitrary. It means, for example, that $\text{Prob}(a + c \leqq L < a + d) \neq 0$. Thus we assume

(3) $$p > q > s$$

(4) $$p > r > s.$$

It is now possible to put Take Some into matrix form as matrix 3.

ap, cp	aq, dq
br, cr	bs, ds

Matrix 3

MOST-THREATENING STRATEGIES

From (3) and (4) above, it follows that $ap > aq$ and $br > bs$. Thus whatever player I (row player) does, he prefers column player to choose the first column. In such a case we say, by definition (Hamburger, 1973), that column player has a most-threatening strategy, namely the second column. Similarly, row player's second row is most threatening to column.

The notion of a most-threatening strategy bears a resemblance to that of dominant strategy. In a two-person game in normal form, let a player's payoffs be $\{a_{ij}\}$ where the player himself picks i and the other player picks j. Then i_k dominates i_m iff for all j

$$a_{i_k j} \geqq a_{i_m j}$$

with strict inequality in at least one case. Similarly j_n is more threatening than j_p iff for all i

$$a_{ij_n} \leqq a_{ij_p}$$

with strict inequality in at least one case. Just as a dominant strategy for a player is one which dominates all his others, so a most-threatening strategy for a player is one which is more threatening than all his others.

The generalization from two to many players is not equally straightforward for the two concepts. The notion of dominance generalizes with no complications. But a strategy for player A which is most threatening with respect to player B may or may not be most threatening with respect to player C. In a Take Some game, however, each player has one particular strategy which is most threatening with respect to all other players.

We next show that in only 21 of the 78 ordinally distinct 2×2 games do both players have available a most-threatening strategy. One possible method of proof is similar to that used by Rapoport and Guyer in the result cited above. However, we shall use a different approach which will be more useful later.

Players' payoffs are labeled 4, 3, 2, and 1, in order of decreasing preference. By convention, put the most-threatening strategies in the second row and the second column, respectively. From this it follows that row player's best payoff, 4, must appear in the first column, so that 4 may appear in either of two positions. The other payoff to row player in the same row with this 4 may be any of the numbers, 1, 2, or 3. These payoffs having been assigned, the remaining payoffs are uniquely determined by the above convention. There are thus $2 \times 3 \times 1 \times 1$ possible configurations of payoffs for row player. These are shown as matrices 4(a)–(f). Corresponding considerations apply to column player.

4,--	3,--
2,--	1,--

(a)

4,--	2,--
3,--	1,--

(b)

4,--	1,--
3,--	2,--

(c)

2,--	1,--
4,--	3,--

(d)

3,--	1,--
4,--	2,--

(e)

3,--	2,--
4,--	1,--

(f)

Matrices 4

Thus there are $6 \times 6 = 36$ ordered pairs of payoff configurations. Of these, six are symmetric game matrices while the remaining 30 comprise 15 nonsymmetric games each appearing twice due to interchange of players. Altogether there are $6 + 15 = 21$ ordinally distinct 2×2 games in which each player has a most-threatening strategy. Without the most-threatening restriction, this enumeration technique leads to $12 \times (12 + 1)/2 = 78$ games, the number given by Rapoport and Guyer. Similarly, there are $6 \times (6 + 1)/2 = 21$

games where each player has a dominant strategy and $4 \times (4+1)/2 = 10$ separable games (Harris, 1972).

2 × 2 TAKE SOME GAMES

In this section we shall show how to construct a Take Some game with given ordinal payoffs. In so doing we shall show that exactly 19 of the 21 2 × 2 games with a most-threatening strategy for each player can be played in Take Some format.

It may help the reader to glance back at matrix 3, inequalities (1)–(4), and the surrounding discussion. Define

$$e = b/a > 1$$
$$f = d/c > 1.$$

Reordering players if necessary, we require by convention that $q \geqq r$, which in turn implies $a + d \leqq b + c$ by definitions of q and r. In view of (3) and (4)

$$p > q \geqq r > s.$$

Also define

$$g = p/q$$
$$h = q/r$$
$$j = r/s.$$

The payoff orderings in the various matrices 4(a)–(f) place certain constraints on the quantities just defined. Thus in matrix 4(a), the 3 and the 2 imply

$$aq > br,$$

in view of matrix 3, which can be rewritten

(5a) $h > e$.

Similarly, corresponding to matrices 4(b)–(f) we have

(5b) $gh > e, \quad hj > e \quad \text{and} \quad e > h$

(5c) $gh > e > hj$

(5d) $e > ghj$

(5e) $ghj > e, \quad e > gh \quad \text{and} \quad e > hj$

(5f) $hj > e > gh.$

For column player, similar considerations lead to matrices 5(a)–(f) and inequalities 6(a)–(f).

--, 4	--, 2
--, 3	--, 1

(a)

--, 4	--, 3
--, 2	--, 1

(b)

--, 4	--, 3
--, 1	--, 2

(c)

--, 2	--, 4
--, 1	--, 3

(d)

--, 3	--, 4
--, 1	--, 2

(e)

--, 3	--, 4
--, 2	--, 1

(f)

Matrices 5

(6a) $f < 1/h$

(6b) $g > f, \quad j > f \quad \text{and} \quad f > 1/h$

(6c) $g > f > j$

(6d) $f > ghj$

(6e) $f > g, \quad f > j \quad \text{and} \quad ghj > f$

(6f) $j > f > g.$

Note that when $h = 1$, the constraints on e and f are identical.

To show that a particular ordinally defined game can be played in Take Some format, we take the following steps:

(i) Check that each player has a most-threatening strategy and interchange rows and/or columns as needed so that most-threatening strategies appear in the second row and second column.

(ii) Match row player's matrix entries against one of the matrices 4(a)–(f). The corresponding inequality 5(a)–(f) must hold. Similarly for column player with respect to matrices 5(a)–(f) and inequalities 6(a)–(f).

(iii) Pick any values of g, h and j such that $g > 1$, $h \geqq 1$ and $j > 1$. Pick e and f in accordance with the inequalities in (ii).

(iv) Pick any value of p with $0 < p \leqq 1$, and pick any positive value of c.

(v) Pick any value of a such that $a \geqq c(f - 1)/(e - 1)$, with equality just in case $h = 1$.

This will ensure that

$$c(f - 1) \leqq a(e - 1)$$
$$d - c \leqq b - a$$
$$a + d \leqq b + c,$$

which is consistent with $q \geqq r$, and hence with $h \geqq 1$.

We now look for ways in which this procedure can break down. If a 2×2 game has a most-threatening strategy for each player, then steps (i) and (ii) are straightforward. Further, step (iv) clearly is always possible, as is step (v) provided that values of e and f are obtained at step (iii).

Turning to step (iii), inequality 6(a) can never be satisfied since $f > 1 \geqq 1/h$. As a result, matrix 7 cannot be played in

4, 4	3, 2
2, 3	1, 1

Matrix 7

4, 3	1, 4
3, 2	2, 1

Matrix 8

Take Some format. Further, inequalities 5(c) and 6(c) both require that $g > j$ while 5(f) and 6(f) both require the opposite. Therefore matrix 8 is not a Take Some game. Matrix 7 may be called "invisible hand" since each player, by seeking his own well-being, incidentally benefits the other, though in lesser degree.

There are five symmetric Take Some games. Leaving out matrix 7, they are the six symmetric games in which each player has a most-threatening strategy. These games are displayed as matrices 9-13. Matrix 9 (same as 1b) is prisoner's dilemma; 10 is chicken (Rapoport & Chammah, 1969); 11 (same as 2) is the matrix version of Rosenberg's (1968) convergent oscilloscope game (Hamburger, 1969); and 12 has been called "stupid competition" (Kubicka, 1968) and "maximizing difference game" (McClintock & McNeel, 1966). Two symmetric games which have been used in experiments but which cannot be put into Take Some format are "hero" and "leader" (Rapoport, 1967; Guyer & Rapoport, 1969).

3, 3	1, 4
4, 1	2, 2

Matrix 9

3, 3	2, 4
4, 2	1, 1

Matrix 10

2, 2	1, 4
4, 1	3, 3

Matrix 11

4, 4	2, 3
3, 2	1, 1

Matrix 12

4, 4	1, 3
3, 1	2, 2

Matrix 13

As a multiple example of how to form a Take Some game satisfying an ordinal matrix, matrices 9–12 are formed, respectively, from matrices 4(e) and 5(e), 4(f) and 5(f), 4(d) and 5(d), and 4(b) and 5(b). With $g = 2$ and $j = 3$, and setting $h = 1$ for player symmetry, the inequalities at step (iii), are:

Matrix 9: $3 < e < 6$ and $3 < f < 6$
Matrix 10: $2 < e < 3$ and $2 < f < 3$
Matrix 11: $6 < e$ and $6 < f$
Matrix 12: $1 < e < 2$ and $1 < f < 2$.

It is thus possible to let $e = f$ throughout. Also, $a = c = 1$ is consistent with step (v). Finally, let $p = 1$.

We now have the following framework:

each of two players may pick 1 or a known larger amount, e. If both take 1, they are assured of getting it. If both pick e, the probability is $\frac{1}{6}$ that they get it. If they choose oppositely, then with probability $\frac{1}{2}$ each gets the amount he picked. For e between 1 and 2, such a game has expected payoffs ordinally equivalent to matrix 12; for e between 2 and 3, matrix 10; for e between 3 and 6, matrix 9; and for e above 6, matrix 11.

Finally note that for matrix 14, the Take Some format imposes the following condition which cannot be expressed in ordinal terms:

A_1, A_2	B_1, C_2
C_1, B_2	D_1, D_2

Matrix 14

$$A_1D_1/B_1C_1 = A_2D_2/B_2C_2 .$$

This relationship can be verified by setting matrix 14 equal, term by term, to matrix 3.

REFERENCES

Bishop, R. L. A Zeuthen-Hicks theory of bargaining. *Econometrica,* 1964, 32, 410–417.

Guyer, M., Fox, J., & Hamburger, H. Format effects in the prisoner's dilemma game. *J. conflict Resolut.*, 1974, in press.

Guyer, M., & Rapoport, A. Information effects in two mixed-motive games. *Behav. Sci.*, 1969, 14, 467–482.

Hamburger, H. Separable games. *Behav. Sci.*, 1969, 14, 121–132.

Hamburger, H. N-person prisoner's dilemma. *J. math. Sociol.*, 1973, in press.

Harris, R. J. An interval-scale classification system for all 2 × 2 games. *Behav. Sci.*, 1972, 17, 371–383.

Kubicka, L. The psychological background of adolescents' behavior in a two-person non-zero sum gam. *Behav. Sci.*, 1968, 13, 455–466.

McClintock, C., & McNeel, S. Reward level and game playing behavior. *J. conflict Resolut.*, 1966, 10, 98–102.

Pruitt, D. Reward structure and cooperation: The decomposed prisoner's dilemma game. *J. pers. soc. Psychol.*, 1967, 7, 21–27.

Rapoport, A. Exploiter, leader, hero, martyr: The four archetypes of the 2 × 2 game. *Behav. Sci.*, 1967, 12, 81–84.

Rapoport, A., & Chammah, A. The game of chicken. In I. Buchler & H. Nutini (Eds.) *Game theory and the behavioral sciences.* Pittsburgh: Univ. Pittsburgh Press, 1969, 151–175.

Rapoport, A., & Guyer, M. A taxonomy of 2 × 2 games. *General Systems*, 1966, 11, 203–214.

Rosenberg, S. An experimental test of a stochastic model of interpersonal conflict in a two-person situation. *J. math. Psychol.*, 1968, 5, 281–299.

Sawyer, J., & Friedell, M. F. The interaction screen: An operational model for experimentation of interpersonal behavior. *Behav. Sci.*, 1965, 10, 446–460.

(Manuscript received March 21, 1973)