

# *Expanding design spaces through new design variables*

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*This paper describes the concept that the introduction of new design variables into a design process leads to solutions which may not have been possible earlier. This is useful where design constraints are in conflict and hence there are no feasible solutions. It is also useful when a better design is desired. The concept is demonstrated with examples solved using optimization techniques illustrating how new solution spaces emerge with the introduction of new design variables. The paper concludes with a discussion relating this concept to models of creative design.*

*Keywords:* design variables, design spaces, creative design

Design concerns the development of the description of an artifact to satisfy a set of requirements. These requirements can be cast as either objectives which point towards directions of improving the behaviour of the artifact or as a set of constraints which limit the ranges of behaviour variables or structure or decision variables. These constraints may not be possible to satisfy in some cases as described in the next section. This paper introduces with examples a concept about how constraints may be satisfied under such circumstances. It may be extended further to satisfy constraints in different ways resulting in new designs.

## *1 Design as constraint satisfaction*

Design is defined in many different ways. A common definition of design includes that it is a purposeful, goal-directed, constrained, decision-making activity which occurs within a context<sup>1</sup>. Design normally commences with an initial set of requirements which comprises a set of objectives and constraints to be satisfied by the design. It may not be possible to satisfy all the constraints at the same time. One of the reasons could be that the constraints are in conflict with each other. There have been different approaches to solving this problem. Some of the more

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**1** Gero J S Design prototypes; a knowledge representation schema for design *AI Mag* Vol 11 No 4 (1990) 27-36

common ones concentrate on relaxing the constraints<sup>2,3</sup> and finding some sort of a compromise between them. However, these approaches may not be acceptable to the designer or end user. The designer or end user may want all the constraints to be satisfied under all circumstances. Constraint relaxation in many ways determines the very nature of design in that it manipulates the requirements until it can find a solution. Design, fundamentally, is not about changing the requirements but designing for them. One of the main factors driving the relaxation approach is that it assumes that the world is not fixed. The word 'world' is used here in a metaphorical sense implying the design solution space. What needs to be stressed is that the world is capable of being changed if only we have the appropriate tools and techniques to do so. Thus, the problem lies with developing such tools and techniques and not the world. The moment the world is changed, the activity of design can be carried out more purposefully. A changing world may lead to the satisfaction of any set of constraints and it may lead to new designs hitherto unknown.

Another situation in design which occurs frequently is that although the constraints are satisfied, the designer or the end user may not be satisfied with the solution. In other words, a better or different solution may be desired. This again may not be possible in a fixed world as the solution already found may be the 'best' solution. However, even this problem may be solved by changing and expanding the world. In both cases, the design has to be carried out in an expanded design solution space. This leads to the idea of creative design discussed in Section 4. This paper describes how changing the world leads to new designs. We do not deal with specific processes required to change the world but merely show by examples how this leads to new designs which may not have been possible earlier and may be better than the solutions found in the original world.

## 2 When to introduce new design variables

In this section, we propose some general guidelines concerning circumstances when it might be useful to introduce new design variables. Generally, these circumstances fall into one of the following three groups:

- When routine design procedures do not yield any design solution
- When optimization procedures indicate there are no feasible solutions
- When the designer or user desires better or different designs

The first group implies that there are no solutions that can be produced in the solution space bounded by the current set of design variables. In this case, the solution space needs to be expanded by introducing new variables. Possibilities of new solutions arise as this is done. In the second

**2** Steffik, M 'Planning with constraints (MOLGEN: Part 1)', *Artificial Intelligence*, Vol 16 (1981), 111-140

**3** Steffik M 'Planning and meta-planning (MOLGEN: Part 2)', *Artificial Intelligence*, Vol 16 (1981) 141-169

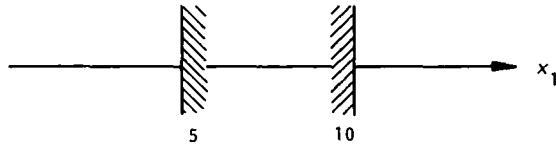


Figure 1 Solution space without a feasible solution

group some constraints are in conflict with each other so that there is no feasible solution. Our contention is that a design procedure should be such that it meets all the specifications and satisfies all the constraints without relaxing them even if some of them are in conflict. Under such circumstances, the introduction of new design variables may generate new solutions because it expands the solution space as required to yield a solution.

### 3 Illustrative examples

In this section, we elaborate on how the introduction of a new design variable allows the generation of a solution which was not feasible earlier. The idea is presented with illustrative examples solved using optimization techniques. In all these examples, there are no feasible solutions initially. Subsequently, we introduce new design variables and feasible solutions emerge.

Consider the situation of a design variable,  $x_1$ , which has to satisfy the following constraints

$$x_1 \geq 10 \quad x_1 \leq 5$$

The solution space is presented in Figure 1 as a one-dimensional space. The two constraints on the variable  $x_1$  are in conflict with each other and there is no feasible solution.

Consider now the situation of introducing another variable,  $x_2$ , and reformulating the constraints in terms of both  $x_1$  and  $x_2$ . These reformulated constraints might look like the following (the specific coefficients,  $a_1$  and  $a_2$ , of course, will depend on the details of the specific situation).

$$x_1 + a_1 x_2 \geq 10 \quad x_1 - a_2 x_2 \leq 5$$

The solution space for this reformulated problem is presented in Figure 2 as a two-dimensional space. This solution space collapses to that shown in Figure 1 when  $x_2 = 0$ . The new design variable  $x_2$  changes the set of constraints and in effect the initial problem is transformed into another problem which embeds the initial problem in it. Due to the change in

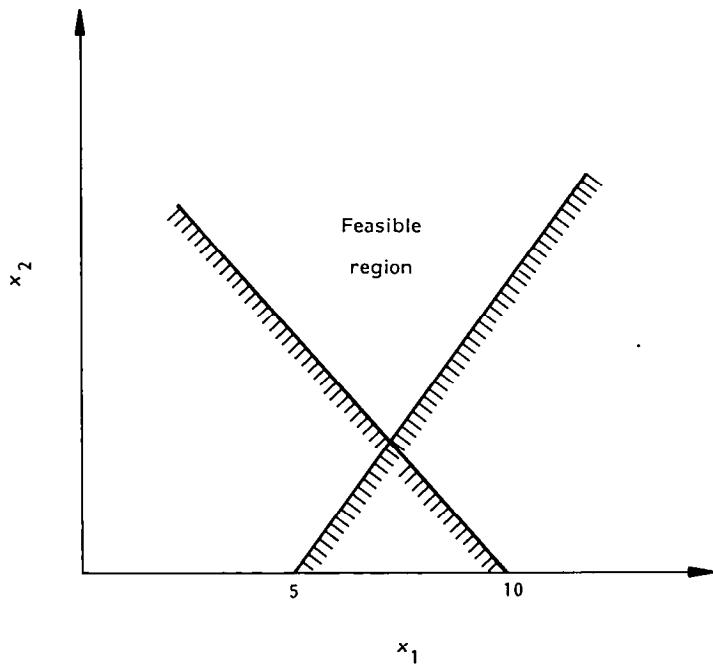


Figure 2 Development of feasible region in expanded solution space with introduction of an additional variable,  $x_2$

constraints, the boundaries of the solution space have changed and thus there are possibilities of new and feasible solutions. In the following sections, we illustrate this concept with specific design examples.

### 3.1 Example 1: housing design

The design problem considered here<sup>4</sup> concerns optimizing the number of 1- and 2-bedroom units for a housing complex to maximize profit. There are constraints on cost, number of people in the development and planning penalty points. The available capital is \$1 800 000. The cost of each type of unit is the following:

Apartment type	Capital cost (in \$1000s)
1-bedroom	100
2-bedroom	180

The profit for each type of unit is as follows:

Apartment type	Profit (in \$1000s)
1-bedroom	20
2-bedroom	24

<sup>4</sup> Radford, A D and Gero J S  
Design by Optimization in  
Architecture, Building and Con-  
struction, Van Nostrand  
Reinhold, New York (1988)

Clearly, it would be most profitable to build all apartments as 1-bedroom

units. However, the planning regulations discourage the construction of only smaller units and impose planning penalty points for each type of unit as the following:

Apartment type	Planning penalty points
1-bedroom	120
2-bedroom	60

The maximum planning penalty points that any development on this site can accrue is limited to 960 by the planning regulations.

The average number of people for each type of unit is the following:

Apartment type	Average number of people
1-bedroom	1.75
2-bedroom	3.0

The planning regulations recommend that the lower limit on the number of people for this site to be 32. Casting these constraints and the objective into mathematical expressions, we obtain the following formulation of the optimization problem:

Objective function

$$\text{maximize: } 20x_1 + 24x_2 \text{ (profit)}$$

subject to

$$100x_1 + 180x_2 \leq 1800 \text{ (cost constraint)}$$

$$120x_1 + 60x_2 \leq 960 \text{ (planning penalty constraint)}$$

$$1.75x_1 + 3.0x_2 \geq 32 \text{ (number of people constraint)}$$

where  $x_1$  is the number of 1-bedroom units and  $x_2$  is the number of 2-bedroom units.

Figure 3 is a graphical illustration of the solution space bounded by the three constraints.

It is clear from Figure 3 that the first and third constraints are in conflict with each other. Thus, this problem does not have a feasible solution. In order to find a solution for this problem, an attempt is made to introduce another design variable and reformulate the problem. After some reasoning, it is decided to reformulate the problem by introducing another design variable in the form of the number of 3-bedroom units,  $x_3$ . The computational processes for such an introduction of a new variable are not

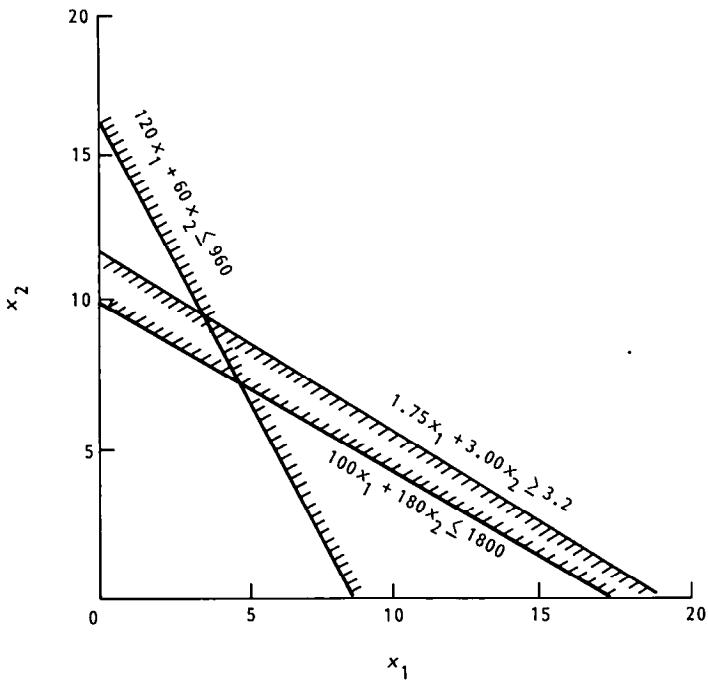


Figure 3 Constraints for housing problem

the concern of this paper. The constraints for this design situation remain the same as the earlier one except that each of them have to also take into account the 3-bedroom units. The cost of a 3-bedroom unit is \$240 000, the profit for each 3-bedroom unit is \$26 000, there are 20 penalty points for a 3-bedroom unit and the average number of people in a 3-bedroom unit is 5. This information is provided by the domain. The reformulated problem is:

Objective function is:

$$\text{maximize: } 20x_1 + 24x_2 + 26x_3 \text{ (profit)}$$

subject to:

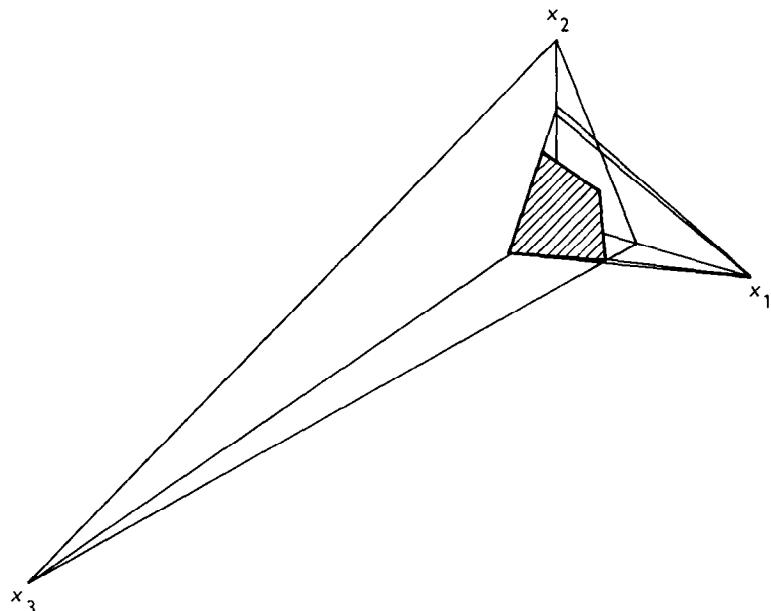
$$100x_1 + 180x_2 + 240x_3 \leq 1800 \text{ (cost constraint)}$$

$$120x_1 + 60x_2 + 20x_3 \leq 960 \text{ (planning penalty constraint)}$$

$$1.75x_1 + 3.00x_2 + 5x_3 \geq 32 \text{ (number of people constraint)}$$

where  $x_1$  is the number of 1-bedroom units;  $x_2$  is the number of 2-bedroom units; and  $x_3$  is the number of 3-bedroom units.

Figure 4 illustrates the feasible region (shaded portion) for the reformulated problem where the new variable,  $x_3$ , has been introduced from outside the problem.



*Figure 4 Constraints for reformulated housing problem.  
(2) feasible solution space*

The mechanisms of such an introduction are not addressed in this paper but it can be achieved by analogical reasoning<sup>5-7</sup>. This may involve finding a precedent of similar past design cases and choosing a variable from one of those designs.

The solution to this reformulated problem is:

Objective function = 265.53

$$x_1 = 5.24$$

$$x_2 = 5.01$$

$$x_3 = 1.56$$

Clearly, the introduction of  $x_3$  to this problem has generated a solution which was not possible earlier.

**5** Gero J S and Maher M L  
Mutation and analogy to support creativity in computer-aided design, *Proc. of CAAD Futures '91*, ETH, Zurich (1991), pp 241-249

**6** Navinchandra, D *Exploration and Innovation in Design*, Springer-Verlag, Berlin (1990)

**7** Zhao F and Maher M L 'Using analogical reasoning to design buildings', *Engng with Comput* Vol 4 (1988) 107-119

**8** Gero J S and Cowan H J *Design of Building Frames* Applied Science, London (1976)

### 3.2 Example 2: beam design

This design problem<sup>8</sup> concerns the minimum weight design of a cantilever beam subjected to a concentrated load at the free end (see Figure 5).

The cross-section of the beam is triangular. The allowable stress in bending tension is 165 MPa, the allowable stress in bending compression is 130 MPa, the average allowable stress in shear is 100 MPa and the elastic modulus is 200 GPa. The maximum deflection to span ratio is 1:360. The maximum depth has been set by architectural considerations as

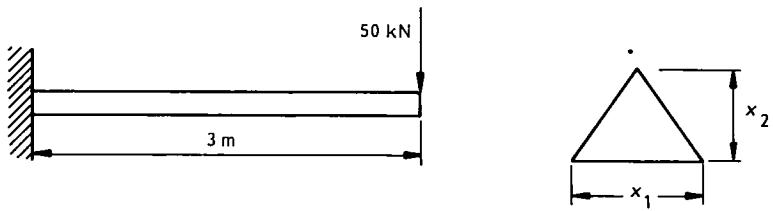


Figure 5 Original problem with triangular beam cross-section

600 mm. The objective is to find the size of the triangular cross-section that minimizes the weight. The constraints and the objective function formulated as mathematical expressions are as follows:

Objective function is:

$$\text{minimize: } \rho x_1 x_2 / 2 \text{ (weight/unit length)}$$

subject to:

$$\begin{aligned} x_1 x_2^2 &\geq 15 \times 10^6 && \text{(bending stress compression constraint)} \\ x_1 x_2^2 &\leq 22.8 \times 10^6 && \text{(bending stress tension constraint)} \\ x_1 x_2 &\geq 1 \times 10^3 && \text{(shear stress constraint)} \\ x_1 x_2^3 &\geq 9740 \times 10^6 && \text{(deflection constraint)} \\ x_2 &\leq 600 && \text{(architectural constraint)} \end{aligned}$$

where  $x_1$  is the base width of the cross-section;  $x_2$  is the depth of the cross-section, and  $\rho$  is the density

This problem does not have a feasible solution. However, when we change the shape of the section by introducing another design variable to produce a potentially trapezoidal section (Figure 6) we obtain a solution. The problem has to be reformulated in terms of the new variable. The reformulated problem is as follows:

Objective function is:

$$\text{minimize: } \rho x_2 (x_3 + x_1) / 2 \text{ (weight/unit length)}$$

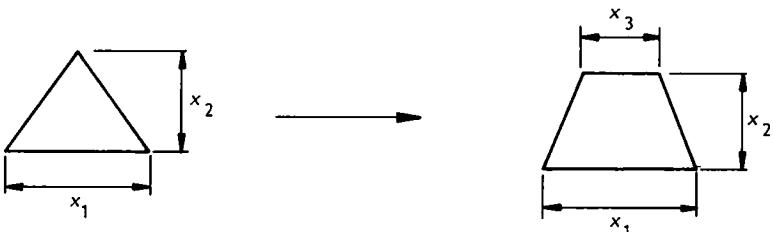


Figure 6 Transformation of a cross-section from a triangle to a trapezium by introducing new variable  $x_3$

subject to:

$$x_2^2(x_3^2+4x_3x_1+x_1^2)/(2x_3+x_1) \geq 15 \times 10^6 \quad (\text{bending stress compression constraint})$$

$$x_2^2(x_3^2+4x_3x_1+x_1^2)/(x_3+2x_1) \geq 11.4 \times 10^6 \quad (\text{bending stress tension constraint})$$

$$x_2(x_3+x_1) \geq 1 \times 10^3 \quad (\text{shear stress constraint})$$

$$x_2^3(x_3^2+4x_3x_1+x_1^2)/(x_3+2x_1) \geq 9740 \times 10^6 \quad (\text{deflection constraint})$$

$$x_2 \leq 600 \quad (\text{architectural constraint})$$

where  $x_1$  is the base width of the cross-section;  $x_2$  is the depth of cross-section;  $x_3$  is the top width of the cross-section; and  $\rho$  is the density

The solution for this reformulated problem is:

objective function =  $\rho 11433.33$

depth = 600.0

top width = 11.2

bottom width = 26.8

The new variable introduced in this case has come from within the problem. Once again, the mechanism for introducing such variables is not addressed in this paper. However, such an introduction of variables can be achieved by mutating the current variables in some way. Some guidelines on such mutation in design can be found in the literature<sup>5,9,10</sup>.

### 3.3 Example 3: beam example revisited

It can be quite easily shown that the introduction of a new variable may not only help in finding solutions where initially there are no feasible solutions but it can also result in better design solutions. For example, let us consider the beam design example in the previous section but replace the architectural constraint so that the limit on depth is now 700 mm instead of 600 mm. The problem formulation is given below.

Objective function is:

$$\text{minimize } \rho x_1 x_2 / 2 \quad (\text{weight/unit length})$$

subject to:

$$x_1 x_2^2 \geq 15 \times 10^6 \quad (\text{bending stress compression constraint})$$

$$x_1 x_2^2 \geq 22.8 \times 10^6 \quad (\text{bending stress tension constraint})$$

$$x_1 x_2 \geq 1 \times 10^3 \quad (\text{shear stress constraint})$$

$$x_1 x_2^3 \geq 9740 \times 10^6 \quad (\text{deflection constraint})$$

$$x_2 \leq 700 \quad (\text{architectural constraint})$$

**9** Cagan J *Innovative Design of Mechanical Structures*, PhD Thesis (unpublished), Department of Mechanical Engineering, University of California, Berkeley (1990)

**10** Jo J and Gero J S 'Design mutation as a computational process', *The Technology of Design*, Australian and New Zealand Architectural Science Association, University of Adelaide, Adelaide (1991), pp 135-143

The solution for this problem is as follows:

objective function  $\geq \rho 16285.7$   
depth = 700.0  
base width = 46.5

We reformulate the problem by introducing an additional variable and replacing the triangular section by a trapezoidal section as before. The reformulated problem is as follows:

Objective function is:

minimize  $\rho x_2(x_3+x_1)/2$  (weight/unit length)

subject to:

$x_2^2(x_3^2+4x_3x_1+x_1^2)/(2x_3+x_1) \geq 15 \times 10^6$  (bending stress compression constraint)  
 $x_2^2(x_3^2+4x_3x_1+x_1^2)/(x_3+2x_1) \geq 11.4 \times 10^6$  (bending stress tension constraint)  
 $x_2(x_3+x_1) \geq 1 \times 10^3$  (shear stress constraint)  
 $x_2^3(x_3^2+4x_3x_1+x_1^2)/(x_3+x_1) \geq 9740 \times 10^6$  (deflection constraint)  
 $x_2 \leq 700$  (architectural constraint)

The solution for this reformulated problem is as follows, resulting in a 40% improvement over the original design.

objective function =  $\rho 9799.9$   
depth = 700.0  
top width = 8.27  
bottom width = 19.72

It can be shown that by introducing more variables one either obtains better solutions or solutions which are no worse than the previous solution.

#### 4 Discussion

The concept of introducing new design variables opens up new directions for research. As has been shown in this paper, the introduction of new variables promises to be of use in not only producing solutions where feasible solutions do not exist in the current solution space, it can also improve on solutions already found. This gives us ideas about designing innovative and creative structures. The concepts of innovative and creative design are still in their infancy and this can be seen as adding to

that research. One of the reasons why the introduction of new variables is not common in routine design is that the reformulations such as those illustrated in the examples of Section 3 may not be simple to carry out. Gero<sup>1</sup> indicates the importance of reformulation in design and points out that in the main there are two situations when reformulation can occur in design:

- The process of synthesis can change the range of expected behaviours of the structure and through them the function being designed for leading to reformulation
- When the evaluation of the comparison between the behaviour of the structure and the expected behaviour is unsatisfactory and cannot be made satisfactory by manipulating the existing structure.

The reformulations in the foregoing examples belong to the second category. There are important unresolved questions regarding the selection of design variables to introduce in order to reformulate the problem. Kumar and Gero<sup>11</sup> suggest a methodology for the selection of design variables to introduce.

There are two classes of new design variables which can be characterized as either homogeneous or heterogeneous. New homogeneous variables are those which are the same type as an existing variable. For example in Section 3.2 a new variable top width was introduced. However, this was of the type as base width. Thus, the reformulation did not require further domain knowledge. New heterogeneous variables, on the other hand, are those which are not of the same type as an existing variable. For example in Section 3.1 if we had introduced floor location as a new variable this would have been a heterogeneous variable.

The distinction between routine and creative design is subjective but an operational definition can be given<sup>5</sup>. Routine design is the result of making design decisions in the context of a design situation where all the decision variables are known *a priori*. Thus, in routine design the designer operates within a defined, closed state of possible designs where the differences between designs can be characterized largely by the values selected for the design variables. To a lesser extent it can be further characterized by the selection of which variables to include in the design from a predefined set of variables. Creative design may occur when new design variables are introduced in the process of designing. Thus, in creative design the designer operates within a changing state space of possible designs; a state space which increased in size with the introduction of each new variable (Figure 7).

<sup>11</sup> Kumar B and Gero J S 'Designing for conflicting constraints' *Working Paper*, Design Computing Unit, University of Sydney, Sydney (1991)

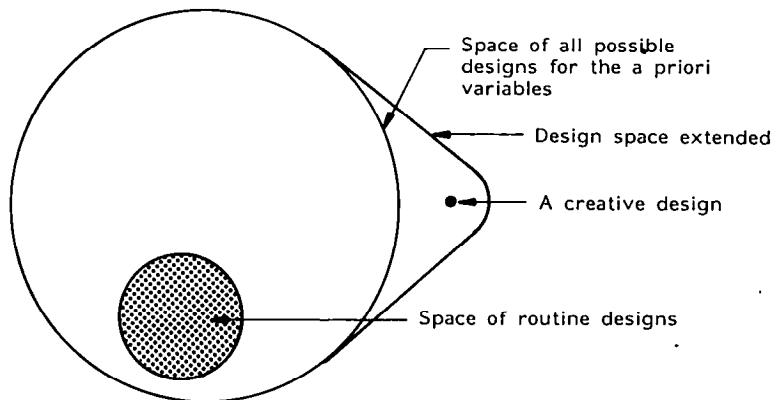


Figure 7 State space of routine and creative designs<sup>5</sup>

Models of creative design have been proposed<sup>5</sup>. Analogical reasoning<sup>12-14</sup> and mutation<sup>10</sup> have been suggested in these models as processes for the introduction of new variables. This paper demonstrates the utility of these approaches.

## 5 Conclusions

This paper has illustrated how the introduction of new variables can lead to new designs. Introduction of new variables has been suggested to be the operational definition of creative design. Some general guidelines on which it might be useful to introduce new variables have been suggested. The computational processes of introducing a new variable have not been addressed here. It was briefly pointed out that the new variables can be introduced either by analogical processes or by mutation of the current variables. The development and implementation of precise computational process models of either of these approaches for creative design still remain important research issues.

## 6 Acknowledgments

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**14** Carbonell J G 'Derivational analogy: a theory of reconstructive problem solving and expertise acquisition', in R S Michalski, J G Carbonell and T M Mitchell (Eds) *Machine Learning II: An Artificial Intelligence Approach* Morgan Kaufmann, Los Altos, CA (1986), pp 371-392