

On measuring the visual complexity of 3D solid objects

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Abstract: A computational model of the visual complexity of solids is presented. It is based on using a qualitative representation for 3-dimensional objects as semantic graphs. Then information-theoretic measures are constructed for these graphs. This measure of visual complexity allows for the comparison of different architectural forms. An example of the application of this model is presented.

Keywords: visual reasoning; complexity; 3D modelling

1 Introduction

Drawings are the most common form of design representation. They are used in a variety of ways including memory aids and external representations that can be re-interpreted [PG 97]. Drawings are the most evident of design outcomes both during and at the completion of the process of designing. In research into computer support for designing, the focus on drawings has been primarily as the end result although there has recently been interest in drawings as a medium for the representation of ideas. If computer support for designing is to extend beyond the mere increase in the efficiency of the production of drawings then we need to be able to derive and “measure” attributes and qualities of the content of drawings. Earlier work focussed on deriving design semantics from drawings [GRO 96; JG 98]. Computer support during the process of designing will have to be involved with the design drawings before they are finalised. If we are going to use measures of designs based on their drawings we need to be able to use a measure that can differentiate them based on their drawings

How do we measure the difference between two designs visually? Commonly we measure some aspect of their behaviours such as cost, environmental efficiency or structural sufficiency. However, all of these assume that a detailed design is available. What is needed is a means of measuring the difference between designs at a visual level that does not depend on a knowledge of the detailed design itself so that such measures could be utilised at any stage during the design, including at the very early stages when only the form is known. This implies that what is being measured cannot depend directly on quantitative descriptions but rather on qualitative representations.

Qualitative symbolic modeling represents objects as classes rather than as instances. As a consequence they can be used at the early stages of designing when objects do not yet have a fixed geometry, only fixed topology and restrictions on geometry, based on their topology. The framework of qualitative representations allows

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application of AI tools for symbolic reasoning, including developing specialized representations based on features. The possibility of identifying an appropriate feature space and then re-representing objects in this space provides the opportunity to add another layer of reasoning to a design system. This process of feature identification and re-representation can be organized cyclically, when more abstract features are identified on the basis of the currently available features, then a new re-representation on the basis of these new features is carried out, and so on. Gero and Damski [GD 99] have developed a feature-based qualitative representation for 3D solid objects. This representation is a semantic network or graph.

One way to measure a drawing of 3D objects is to measure its visual complexity. The notion of visual complexity in visual-based design is intuitively appealing, but the notion itself remains vague and ill defined. One way to conceive of visual complexity is to think of it as a measure of the different ways a drawing can be interpreted. Many attempts [BIR 32; SG 78; ARN 71; SAL 97] have been made to formulate a general approach to visual complexity based on some principle that would be universally applicable across all domains. These attempts have generally failed because the results were not consistent.

The research described in this paper is based on a semantic graph representation. However, instead of simply working in a feature space, we re-represent our problem from the feature space onto the space of distributions in the feature space. The reasoning process, which we construct in this paper, operates in terms of those distributions and the objective here is to construct some overall measure of 3D solids, based on these distributions, which correlates with the notion of visual complexity of those 3D objects. We will restrict ourselves to 3D objects that are described by their external surfaces only and that consist of plane facets. We will neglect any internal structure of the object. We do this since our interest, at this stage, is in the visual complexity of the object in terms of an external observer.

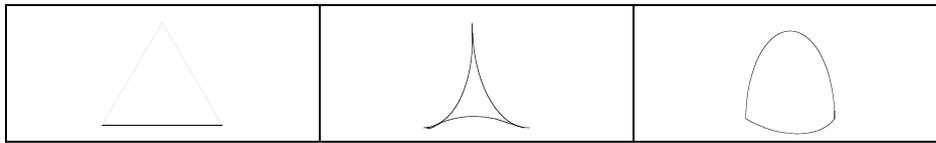
2 Symbolic representation of objects

2.1 Basic formalism

In order to represent shape in terms of such a representation one needs first to place landmarks on the shape and then map the shape angles in each of these landmarks onto the above-described symbolic sequence. Such a symbolic encoding, based on qualitative representation and placement of landmarks on shapes, is called Q-codes [GP 97]. This representation encodes the outlines of 2D shapes as symbolic linear strings similar to any natural language such as English. This linguistic analogy works very well for such shapes, where a number of Q-code words, sentences, etc. which carry design meaning were identified [GP 97]. For example, for the angle (for the measured between two surfaces or tangents to surfaces in three-dimensional objects measured from the line that lies in both surfaces), these intervals can be chosen as $\{(0,0), (0,\pi), (\pi,2\pi), (2\pi,0)\}$ which are mapped on to $\{A_{nil}, A^+, A_0, A^-\}$. Thus, the Q-code $3^*(A^+)$ represents all the shapes in Table 1. This allows us to treat all these shapes as exemplars of a single class of shapes. Thus, all three shapes there are, in some sense, triangles. Despite different appearances they are all equivalent in

the world of Q-codes.

Table 1. All these shapes have the same Q-code of A+A+A+, adapted from Gero and Park [GP 97]



2.2 Semantic vertex graph model

The geometry of three-dimensional objects can be represented as a set of vertices (points) and edges (lines) connecting the vertices. This representation is implemented here through a semantic vertex graph where each node represents a vertex and the arcs represent the edges between two faces [MAN 88]. Then the labels for vertices are calculated in two steps. First, an auxiliary label "+" or "-" is assigned for every edge which is connected to the vertex. These labels correspond to the internal angle measured between the corresponding adjacent planes are more than (or equal) or less than, respectively. Second, a label is assigned to the vertex by mapping the sequence of "+" and "-" from the edges adjacent to the vertex onto letters according to Table 2. Thus, the can be viewed as a hierarchical two-level encoding of the shape. Here essentially a normal basic 2-D Q-code encoding for edges is carried out during the first stage, and the higher-level Q-coding of these basic Q-codes along elementary minimal circle around each vertex is executed during the second step.

Table 2. Coding of vertex labels

| Edge coding sequence | Vertex label |
|----------------------|--------------|
| -+-+--+ | <i>a</i> |
| --- | <i>b</i> |
| --+ | <i>c</i> |
| +++ | <i>d</i> |
| ++ | <i>e</i> |
| -- | <i>f</i> |
| +--+ | <i>g</i> |
| --+- | <i>h</i> |

2.3 Information-theoretic measures for vertex graphs

Given the semantics vertex representation of 3D object, a number of possibilities for further study of this object arise. First, it becomes possible to identify some useful features in a graph, which carry design meaning. Gero and Damski [GD 99] identified a number of such features, beginning with the labels from Table 1 themselves, then particular vertex cycles, etc. Second, the use of this representation

also makes it possible to study statistical regularities of semantic vertex graphs and then use these statistics to estimate the structural information content of this graph, and consequentially, the information content of the 3D solid itself.

The notion of structural information content (structural diversity) of a graph was first developed in chemical graph theory [MOW 68; BM 94]. It became an important working tool in chemistry (see for example, Devillers and Balaban, [DB 99]). Similar notions were developed for the analysis of software complexity, and now are widely used to estimate the cost of development and maintenance of software [KR 87]. Numerous realizations of this notion of structural diversity measure for graphs have been developed but conceptually they all operate in a similar fashion, by constructing a finite probability schema from a graph and then by computing the Shannon entropy for this schema. The probability schema is created by partitioning the N elements of graph structure into k equivalent classes of N_k equivalent elements according to a specified equivalence relation E :

| | |
|--------------------------|------------------------|
| Equivalence classes | $1, 2, \dots, k$ |
| Equivalence partition | N_1, N_2, \dots, N_k |
| Probability distribution | p_1, p_2, \dots, p_k |

In this schema $p_i = N_i/N$ is the probability of a randomly chosen element belonging to the class i having elements N_i , and $N = \sum N_i$. The structural information content of the graph is then defined as

$$I(E) = -\sum_1^k p_i \log_2(p_i) \quad (1)$$

The majority of equivalence relations that are employed for calculating the structural information content are based on subgraph equivalences. In the simplest case this equivalence is determined by the vertex labels only, and then more and more complex subgraphs could be employed. As a rule, the combined structural information measure based on a number of different equivalence relations is used [BM 94].

$$I_{combined} = \sum_j w_j I(E_j), \sum_j w_j = 1. \quad (2)$$

where w_j are non-negative weights. These weights should be adjusted to provide the proper contributions by accounting for the various aspects of diversity that can be deduced from the graph, including vertex diversity, branching diversity, etc.

In order to illustrate how the structural information content of a graph is calculated, we will compute it for the graph of the object, which is shown in Figure 1(a), for a number of different equivalence relations.

For the equivalence that is based on the labels only, we estimate the probabilities of appearance of vertices with different labels by their relative frequencies in the graph. Here only two labels b and c are used and their frequencies/probabilities are $p_1 = p(\text{label } b) = 10/12$, $p_2 = p(\text{label } c) = 2/12$.

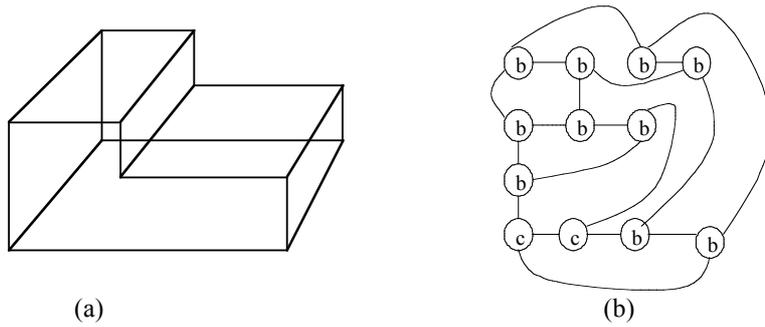


Figure 1. (a) L-shaped object, and (b) its vertex graph

The label-based structural information content then is found as $I(E_{\text{labels}}) = -p_1 \log_2(p_1) - p_2 \log_2(p_2) = 0.65$. For the equivalence, which is based on minimal circuits, the probability schema determines probabilities of appearances of all minimal circuits in a graph. The graph, which is shown in Figure 1(b), contains only three circuits and their probabilities/frequencies are:

$$P\left(\begin{array}{cc} (b) & (b) \\ (b) & (b) \end{array}\right) = 4/8, \quad P\left(\begin{array}{cc} (c) & (c) \\ (b) & (b) \end{array}\right) = 2/8, \quad P\left(\begin{array}{ccc} (b) & (b) & (b) \\ (b) & (b) & (c) \end{array}\right) = 2/8.$$

The structural information content, which is based on distributions of circuits, is then found again from Shannon's formula by summing $-\log_2(p)$ products for each of these probabilities. This yields $I(E_{\text{circuits}}) = 1.50$. For the equivalence relationship, based on subgraphs, which include one middle edge and all the edges that branch out of its ends, this probability schema defines the probabilities of such subgraphs in a graph. The graph shown in Figure 1(b) contains only 5 such subgraphs, whose probabilities are:

$$P\left(\begin{array}{ccc} (b) & & (b) \\ | & & | \\ (b) & - & (b) \\ | & & | \\ (b) & & (b) \end{array}\right) = 6/17, \quad P\left(\begin{array}{ccc} (b) & & (c) \\ | & & | \\ (b) & - & (c) \\ | & & | \\ (b) & & (b) \end{array}\right) = 4/17, \quad P\left(\begin{array}{ccc} (b) & & (c) \\ | & & | \\ (b) & - & (b) \\ | & & | \\ (b) & & (b) \end{array}\right) = 4/17,$$

$$P\left(\begin{array}{ccc} (b) & & (b) \\ | & & | \\ (c) & - & (c) \\ | & & | \\ (c) & & (c) \end{array}\right) = 2/17, \quad P\left(\begin{array}{ccc} (b) & & (c) \\ | & & | \\ (b) & - & (c) \\ | & & | \\ (b) & & (b) \end{array}\right) = 1/17.$$

Again the Shannon formula yields the edge based structural information content as $I(E_{\text{edges}}) = 0.25$.

For the equivalence, which is based on the vertex with all the vertices connected to it by single edges (we shall call it a bunch), the probability schema determines probabilities of appearances of all bunches in a graph. The graph shown in Figure 1(b) contains only three bunches and their probabilities/frequencies are:

$$P\left(\begin{array}{cc} (b) & (b) \\ & | \\ (b) & (b) \end{array}\right) = 5/11, \quad P\left(\begin{array}{cc} (b) & (c) \\ & | \\ (b) & (b) \end{array}\right) = 4/11, \quad P\left(\begin{array}{cc} (b) & (b) \\ & | \\ (c) & (c) \end{array}\right) = 2/11.$$

The Shannon formula yields $I(E_{\text{bunches}}) = 1.03$.

The application of this complexity measures to a range of objects of different apparent complexity produces results that prima facie match human perception. The results of these experiments will be reported in another paper.

3 Architectural examples

Figure 2 shows four buildings selected to demonstrate the capacity of the approach to distinguish between the complexities of different buildings. The buildings are the Chichen-Itza Pyramid in Mexico, Bank of China in Hong Kong, the Sears Tower in Chicago and the Transamerica Pyramid in San Francisco.



(a)



(b)



(c)



(d)

Figure 2. (a) Chichen-Itza Pyramid, Mexico, (b) Bank of China Tower, Hong Kong, (c) Sears Tower, Chicago, and (d) Transamerica Pyramid, San Francisco.

Figures 3, 4, 5 and 6 show an outline model comprised of the planar external surfaces and the resulting semantic vertex graph for each of the buildings in Figure 2. The results of the calculation of the measures of complexity for labels, circuits and edges for each of the semantic vertex graphs are shown in Table 3.

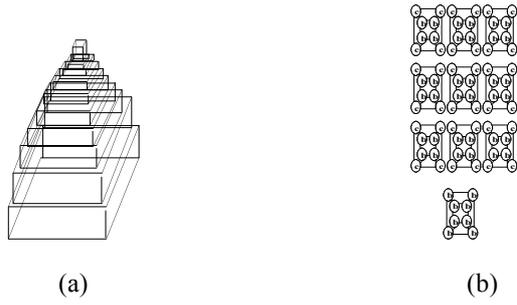


Figure 3. (a) Outline of simplified Chichen-Itza Pyramid shape (without stairs), and (b) resulting semantic vertex graph.



Figure 4. (a) Outline of Bank of China Tower shape, and (b) resulting semantic vertex graph.

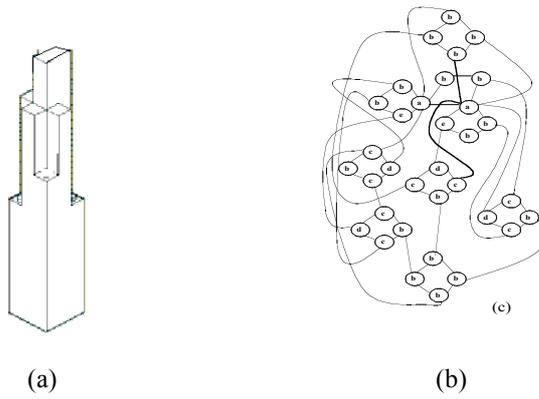


Figure 5. (a) Outline of Sears Tower shape, and (b) resulting semantic vertex graph.

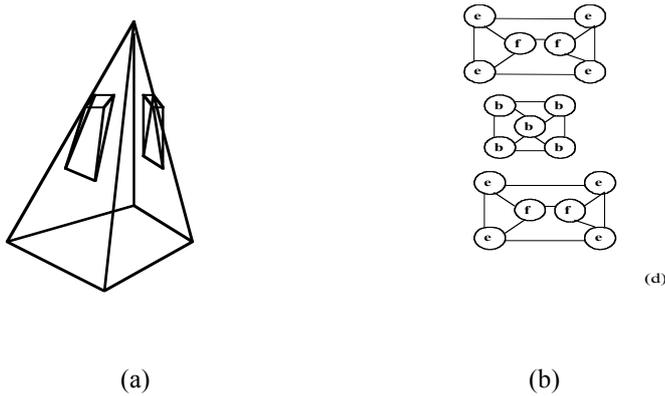


Figure 6. (a) Outline of Transamerica Tower shape, and (b) resulting semantic vertex graph.

Table 3. The structural information content for models of simplified Chichen-Itza Pyramid, Bank of China Tower, Sears Tower and the Transamerica Pyramid.

| | $I(E_{\text{labels}})$ | $I(E_{\text{circuits}})$ | $I(E_{\text{edges}})$ |
|---------------------------------|------------------------|--------------------------|-----------------------|
| Chichen Itza Pyramid simplified | 0.99 | 1.34 | 1.57 |
| Bank of China Tower | 1.79 | 3.32 | 4.00 |
| Sears Tower | 1.82 | 2.61 | 3.70 |
| Transamerica Pyramid | 1.26 | 1.58 | 2.16 |

The view of the visual complexity of 3D objects we are taking is that it maps onto these three measures of information content of the buildings' graph theoretic representations. Figure 7 shows the results of graphing the visual complexities, from Table 3, of these four buildings. We can see that in general the Bank of China building is measured as the most visually complex, whilst the simplified Chichen-Itza pyramid is consistently measured as the least complex. The Sears Tower has a higher complexity than the Transamerica Pyramid.

4 Discussion

Measuring the content of drawings and other graphical representations of objects is notoriously difficult. One way is to measure the information content of the representation used to describe the object being depicted. This provides the basis for the measurement of the visual complexity of the object itself. With such a measurement of visual complexity we are in a position to compare that complexity with the complexity of other objects and other representations of the same object.

In general we utilise representations on the basis of their computational efficiency without much concern for the implications of using one representation over another. What this experiment has shown is that the representation plays a significant role in

the development of the measurement. By choosing a particular qualitative representation we are able to characterise a particular design as an instance of a class of designs and make measurements accordingly. The effect of representation is an under-researched area in design computing research. Complexity measurements of the kind developed here provide the foundation for the comparison of different representational schemas

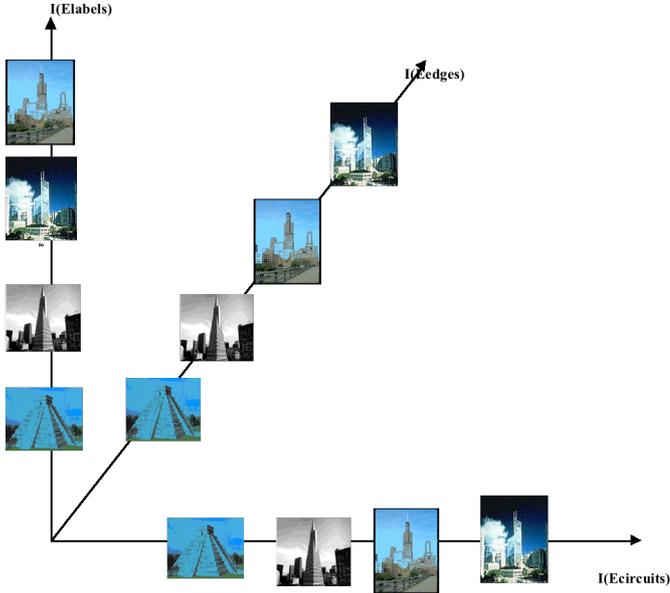


Figure 7. Comparative plot of the visual complexities of the models of the simplified Chichen-Itza Pyramid, the Bank of China Tower, the Sears Tower and the Transamerica Pyramid.

In addition to allowing different designs to be compared and potentially different representations of the same design to be compared, the ability to measure the complexity of an object provides the opportunity to compare the visual complexity of the visual depiction of the object with that of the CAD representation to see whether a correlation exists. Measuring the complexity of the CAD representation is an obvious next task. However, there is no direct mapping between a CAD representation in a CAD database and its qualitative visual representation. The ability to measure the visual complexity of designs as depicted by their representation in drawings opens up avenues for computational support at different stages of designing.

5 Acknowledgements

This research is supported by a grant from the Australian Research Council.

Computational resources are provided by the Key Centre of Design Computing and Cognition.

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This is a copy of the paper: Gero, JS and Kazakov, V (2004) On measuring the visual complexity of 3D objects, *Journal of Design Sciences and Technology* (to appear).