

# Shepherding Behaviors with Multiple Shepherds\*

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**Abstract**—Shepherding behaviors are a type of group behaviors in which one group (the shepherds) tries to control the motion of another group (the flock). Shepherding behaviors can be found in many forms in nature and have various important robotic applications. In this paper we extend our previous work of shepherding behaviors with a single shepherd to multiple shepherds. More specifically, we study how a group of shepherds can work cooperatively without communication to efficiently control the flock.

**Index Terms**—Computer Animation, Flocking, Shepherding, Multiple Shepherds.

## I. INTRODUCTION

Simulating the coordinated behavior of a flock has been studied in many fields of research including robotics, computer animation and games. Shepherding behaviors are a class of flocking behaviors in which one or more external agents (called *shepherds*) try to control the motion of another group of agents (called a *flock*). This behavior is readily found in nature, e.g., sheep herding.

Shepherding behaviors, in general, have applications beyond building robots to herd a group of animals [10], [15], [18]. For example, shepherding behaviors can be used to build robots that work cooperatively to collect oil that has been spilt from oil tankers. These behaviors can also be applied to a group of robots whose function is to keep animals off airport runways. Similarly these behaviors can be applied to keeping people from dangerous areas such as unsafe waters, construction zones or other restricted areas. They could also have applications serving as guides, e.g., through a museum, or in simulations of neuron migration [10].

In this paper we extend our previous work [10] in which various shepherd *locomotions* and shepherd *behaviors* were described for a *single* shepherd. The locomotion of a shepherd is the motion that the shepherd uses to control the flock. Several shepherding behaviors, including herding, covering, patrolling and collecting, can be generated using these common locomotions. Our study in [10] shows that a single shepherd can control the flock's motion better by intelligently positioning itself. However, our experiments also show that a single shepherd cannot adequately control the flock when the size of the flock gets large or if the flock's behavior makes it difficult to influence.

In this paper, we explore how multiple shepherds can be used to gain better control of more difficult or larger flocks.

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Fig. 1. Three shepherd dogs attempt to control a herd of cattle.

In real working situations, it is typical to see several dogs herding a flock (see Fig. 1). In such circumstances, the shepherds need to work cooperatively to guide the flock.

In this paper we study how *multiple* shepherd agents can work cooperatively to effectively control another group of agents that react based on (repulsive) forces exerted by the shepherds and obstacles in the environment. Our work assumes that each shepherd is independent of the other shepherds and that there is no communication between shepherds. To our knowledge, this is the first work that studies shepherding behaviors with multiple shepherds and flocks of large size. Key contributions of this work include:

- Developing herding strategies for multiple shepherds.
- Demonstrating situations in which multiple shepherds can control a flock's motion better than a single shepherd.

We begin our discussion by reviewing related work in Section II and define terms used in this paper in Section III. We study different shepherd formations to steer, turn, stop and merge the flock cooperatively in Sections IV, V and VI. We present experimental results studying how effectively one or more shepherds are able to herd flocks of various sizes with varying separation tendencies.

## II. RELATED WORK

Reynolds' influential flocking simulation [14] established the feasibility of modeling such a system. His work showed that flocking is a dramatic example of *emergent behavior* – global behavior arising from the interaction of simple local rules. Each individual member of the flock has a simple rule set stating how it should move with its neighbors. This concept has been used successfully by researchers both in computer graphics [2], [5], [7] and robotics [9], [15], [18].

Shepherding is an interesting flocking behavior that has received comparatively little attention. Schultz et al. [15] applied a genetic algorithm to learn rules for a shepherd robot to control the movement of another robot (sheep). The sheep reacts to the shepherd by moving away from it. Vaughan et al. [18] simulate and construct a robot that shepherds a flock of geese in a simple (circular) environment. Funge et al. [7] have simulated an interesting shepherding behavior in which a T-Rex chases raptors

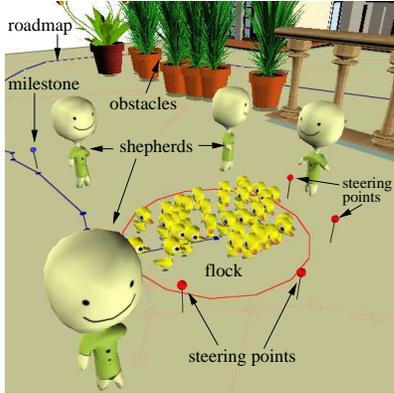


Fig. 2. An environment and associated terms.

out of its territory. Potter et al. [12] studied herding behavior using three shepherds and a single sheep in a simple environment. None of the above mentioned methods are able to combine shepherding with navigation in the presence of obstacles. Recently, Bayazit et al. [4] use a roadmap to integrate global navigation and shepherding in environments with obstacles. However, without considering the influence of the shepherd’s motion on the flock, the flock is often disturbed and separated and becomes hard to control. In [10], we propose several strategies by which a shepherd can position itself more intelligently so that the flock will stay together better. Based on these improved locomotions, we can simulate several interesting behaviors, including herding, covering, patrolling and collecting.

Research in multiple robot systems considers how robots can cooperate to accomplish a task. The survey from Parker [11] provides an overview of these systems. Shepherding with multiple robots is one such system. From the perspective of multiple robot cooperation, the task of shepherding requires *inherent* cooperation, in which the success of a robot in the team depends on the actions of other robots. Unlike non-inherent tasks, such as covering, inherent tasks, such as shepherding, cannot be decomposed into sub-tasks that can be solved independently and thus are generally more difficult.

Shepherding behaviors with multiple robots can also be viewed as a type of manipulation task. Several researchers have attempted to use multiple robots to manipulate or move *passive* objects cooperatively such as pushing a box [19] and kicking a ball [16]. A passive object will move only if external forces are applied to it. On the other hand, shepherding behaviors attempt to manipulate the motion of *active* objects, which have the ability to change their own movement even without external forces and, thus, are usually more difficult to control. To our knowledge, no methods have been proposed to manipulate multiple active objects using multiple robots.

Multiple robots often form a formation, such as a line, a column, or a V shape [3], to accomplish a given task. In this paper, shepherds also organize themselves into a formation during flock steering.

### III. PRELIMINARIES

In this section, we define terms and concepts used in this paper. It is important to note that we use *shepherding* as a broad term to describe any flocking behavior in which outside agents influence the movement of a flock, i.e., our definition of shepherding behaviors is not limited to herding behaviors such as those described in [4], [15], [18].

A *shepherd* is an external agent that influences the movement of the flock. A *flock* is a collection of agents that move in a coordinated fashion [14] while simultaneously reacting to external factors such as the shepherd(s) or other obstacles – the distance the flock maintains from the shepherd(s) depends on the magnitude of the *repulsive forces* they exert. The shepherd’s task is to steer the flock to desired locations or along desired routes. In addition to steering, the shepherd unites separated flock *groups*. In a group, each member can *see* at least one member in that group. Flock separation can be caused by repulsive forces exerted from obstacles or shepherds.

A *milestone* is a position toward which the shepherd attempts to *steer* the flock, and a *steering point* is a position to which the shepherd moves himself in order to influence the movement of the flock (see Fig. 2). As in [4], a milestone is usually a node of a *roadmap* close to the flock. A roadmap is an abstract representation of the connectivity of the feasible space in a given environment. A shepherd can use a roadmap to plan a path to the goal position.

**Shepherd Locomotion.** We define a shepherd’s locomotion as the manner in which the shepherd moves to control the movement of a flock [10]. We divide the shepherd’s locomotion into two sub-problems: *approaching* and *steering*. In the approaching problem, we study how the shepherd moves to the steering point from its current position. In the steering problem, we study how the shepherd moves the flock toward the (next) milestone.

Various shepherd locomotions were defined in [10] for a single shepherd to control a flock (see Figs. 3 and 4). In this work, we will use similar locomotions when trying to influence the flock with multiple shepherds.

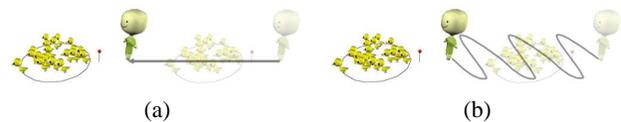


Fig. 3. Steering using (a) straight line, or (b) side-to-side motion.

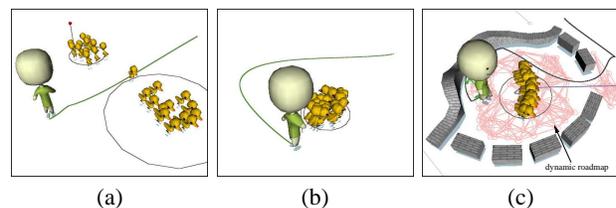


Fig. 4. Approaching using (a) a straight line, (b) a safe zone, or (c) a dynamic roadmap.

#### IV. APPROACHING AND FORMATION

Approaching is a locomotion that shepherds take to move themselves through the environment to reach their steering points near a flock group so that they can control that group. To accomplish approaching, the shepherds must answer the following three questions.

- 1) Where should the steering points be?
- 2) Which shepherd should go to which steering point?
- 3) How should a shepherd go to its assigned steering point?

We place the steering points in a way such that the shepherds will be arranged in a particular formation with respect to the flock and its current and desired direction of movement. We study two formations, line and arc, in Section IV-A. Once the positions of the formation have been determined, each shepherd needs to decide which position in the formation it will take. In Section IV-B, we discuss three shepherd/steering point matching methods, a simple intuitive method, a greedy method, and an optimal distance based method. Note that all three methods use only distance information and communication is *not* required between shepherds.

After a shepherd decides on its steering point, it must move itself to that point in order to influence the flock motion. To do so, the shepherd can perform one of the three approaching locomotions studied in [10] (also shown in Fig. 4).

##### A. Shepherd formations

We have identified two simple flock formations, a line formation and an arc formation. The positions along the formation define the steering points for the shepherds.

**Line Formation.** A line formation places steering points on a straight line segment behind the flock. This line is perpendicular to the line between the flock center and the current milestone. A line formation is useful in distributing the amount of force that is applied at any one point along the flock while pushing the flock toward the milestone. An example of such a formation is shown in Fig. 5(a).

**Arc Formation.** An arc formation is a semi-circular arc behind the flock, centered at the flock center and facing the current milestone. The radius of the formation is obtained from the radius of the flock’s minimum enclosing circle plus some (user defined) epsilon. The arc formation helps in pushing the flock toward the milestone while at the same time keeping the flock compact. See Fig. 5(b).

Note that the steering point formation is independent from the problems of steering point assignment and the shepherd approaching locomotion. In fact, for the problem of merging separate flock groups, discussed in Section VI, as we place steering points around different groups (which can be considered as another “formation”), we can still use the same steering point assignment method and the shepherd approaching locomotion discussed in the following sections.

##### B. Matching shepherds with steering points

Once the shepherd has determined the possible (formation of) steering points, it needs to choose which steering point it will use. These steering points must be chosen by

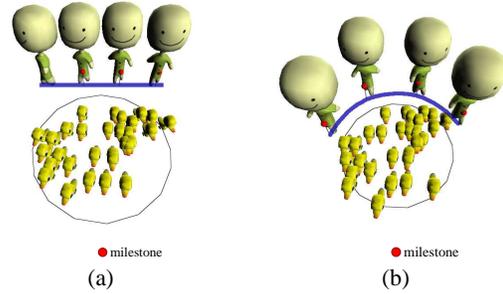


Fig. 5. Steering points (and shepherds) form (a) a straight line (b) an arc behind the flock.

each shepherd independently, i.e., without communication between shepherds. However, all shepherds are assumed to know the current positions of all other shepherds. All methods described in this section assign steering points to shepherds using only the distance between the shepherds and the steering points.

Let  $S = \{s_i\}$  and  $P = \{p_i\}$ ,  $0 \leq i \leq n_s - 1$ , be two sets of points indicating the positions of the  $n_s$  shepherds and the  $n_s$  desired steering points, respectively. We denote  $\text{dist}(s_i, p_j)$  as the distance between shepherd  $s_i$  and steering point  $p_j$ .

**Vector Projection.** In this method, a shepherd picks its steering point by counting the number of shepherds on its left (or right) hand side. If there are  $k - 1$  shepherds on the left (or right), then the shepherd will go to the  $k$ -th steering point from the left (or right). See Fig. 6(a). To determine this ordering,  $S$  and  $P$  are projected onto the line perpendicular to the line through the center of the flock and the milestone. Assume that  $S$  and  $P$  are both sorted along this line from left to right. Then, this method will assign  $p_j$  to  $s_i$  if  $i = j$ .

This strategy is intuitive and fast. It intuitively reflects how a human may select an unoccupied seat. Moreover, it takes only  $O(n_s)$  time for each shepherd. However, due to the dynamics of the system, the line that the shepherds are projected to is constantly changing. This sometimes causes shepherds to change their steering points rapidly and disturb the flock, especially during flock merging.

**Greedy Distance Minimization.** In this method, a shepherd will go to its *closest available* steering point. A steering point  $p \in P$  is called available for a shepherd  $s_i \in S$  if  $p$  is closer to  $s_i$  than it is to all other unassigned shepherds  $s_j \in S$ . No shepherds are assigned to steering points initially.

To determine the closest steering point, a shepherd constructs a bipartite<sup>1</sup> graph  $G = \{V, E\}$ , where  $V = S \cup P$  and initially  $E$  consists of all possible edges  $(v_s, v_p)$ ,  $v_s \in S$  and  $v_p \in P$ . In our example, the vertices in  $G$  are partitioned into the two sets  $S$  (shepherds) and  $P$  (steering points), and edges represent possible assignments of shepherds to steering points. This greedy method iteratively selects the shortest edge  $(v_s, v_p) \in E$  as the next

<sup>1</sup>A bipartite graph has vertices that can be partitioned into two sets such that its edges only connect vertices from different sets, i.e., there are no edges between vertices in the same set.

assignment and then removes all edges incident to  $v_s$  or  $v_p$ . This process continues until  $E = \emptyset$ , i.e., until all shepherds are assigned a steering point. Fig. 6(b) shows an assignment using this strategy.

This greedy strategy mimics the commonly known *selfish* behavior in the flocking system, i.e., a shepherd greedily picks the closest steering point without considering how this action will affect others. This method is more stable than the vector projection method since a shepherd will not change its target position if it is already on a target position. However, some shepherds will travel farther than necessary. This method takes  $O(n_s^2 \log n_s)$  time. Note that because the vertices of  $G$  are points on a plane, this problem can be solved more efficiently in  $O(n_s \log n_s)$  time by using a more specialized geometric query data structure such as a range tree [13].

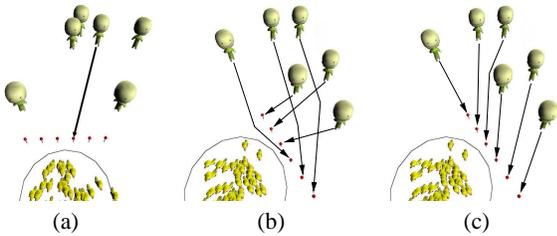


Fig. 6. Matching shepherds with steering points: (a) vector projection, (b) greedy distance minimization, and (c) global distance minimization.

**Global Distance Minimization.** Ideally, one would minimize the traveling distance of all shepherds. Fortunately, the previously mentioned graph  $G$  can be used for this purpose as well and we can solve the global distance minimization problem by converting it to the *weighted minimum-bipartite-matching problem* [6]. Fig. 6(b) shows an assignment using this strategy.

The weighted minimum-bipartite-matching problem can be solved in  $O(nm)$  time [6] for a graph with  $n$  vertices and  $m$  edges. In our case,  $n = 2n_s$  and  $m = n_s^2$ , so it takes  $O(n_s^3)$  time for each shepherd to compute the assignment. As with the previous case, an optimized implementation is possible using a range tree [13] that solves the problem in  $O(n_s^{1.5} \log n_s)$  time [17].

## V. STEERING, TURNING AND STOPPING THE FLOCK

After approaching a flock, the shepherds need to steer the flock toward the goal. Shepherds may need to *steer*, *turn* or *stop* the flock depending on the flock's heading direction.

Let  $\vec{v}_f$  be the mean heading direction of the flock and let  $\vec{v}_m$  be the direction pointing from the flock's center to the milestone. If the angle (denoted as  $\angle_{fm}$ ) between  $\vec{v}_f$  and  $\vec{v}_m$  is smaller than a predefined value (denoted as  $\angle_t$ ), then the shepherd will steer the flock straight ahead. If  $\angle_t \leq \angle_{fm} < 90^\circ$ , the shepherds will attempt to turn the flock to change its heading direction. If  $\angle_{fm} \geq 90^\circ$ , the shepherds will completely stop the flock and then change flock's heading direction. In the following, we discuss how the shepherds handle these cases.

**Steering the Flock Straight Ahead.** The shepherd's locomotion previously described to steer a group was by straight-line or side-to-side steering behind the flock. If a shepherd is working with other shepherds to steer a flock group straight ahead, then straight-line steering (in formation) is used to apply force evenly to the flock group. When a single shepherd is assigned to a flock group, then it will use side-to-side steering to more uniformly influence the flock. This type of steering was shown in [10] to perform better than straight-line steering for a single shepherd.

**Turning the Flock.** When turning and stopping a flock in coordination with other shepherds, the shepherds arrange themselves in a formation. When turning a flock, the shepherd formation for steering that flock is aligned in the opposite direction of the turn.

**Stopping the Flock.** Because the turning movement of the shepherds is sometimes not strong enough (e.g., due to a large flock) or the turn along the path is too sharp, the shepherds can not always turn the flock. To turn the flock completely, the shepherds will need to stop the flock. When stopping a flock, the shepherds generally need to make a formation aligned in the opposite direction of the flock's heading direction.

## VI. MERGING SEPARATED FLOCK MEMBERS

During the process of steering, flock members may separate into multiple groups of one or more flock members. It is the shepherd's task to merge them back to minimize the number of separated flock groups as this makes it easier for the shepherds to control the flock.

In the case of a single shepherd, the shepherd will always go to the separated group(s) and steer them back to the main group. When working together, the shepherds also need to ensure that the flock stays united as much as possible. Let  $n_g$  be the number of flock groups and  $n_s$  be the number of shepherds. We define three possible flock states:

- 1) One Group:  $n_g = 1$ .
- 2) At least as many Shepherds as Groups:  $1 < n_g \leq n_s$ .
- 3) More Groups than Shepherds:  $1 \leq n_s < n_g$ .

During each state the shepherds may have to change their motion to better control the flock group. In this section, we discuss how the shepherds should react to flock groups in States 2 and 3 to change the flock into state 1 by merging groups. A shepherd will consider the following questions during the merging process.

- 1) How to define the flock separation?
- 2) How to distribute the steering points and shepherds among the separate groups?
- 3) Where should the shepherds steer the separate groups to?

In the following, we discuss our answers to these questions for different states. Again, our method does not require the shepherds to communicate to accomplish these tasks.

### A. Flock separation

We consider two flock members as separated if they cannot see each other. Similarly, two groups are separated if no member in one group can see any member of the

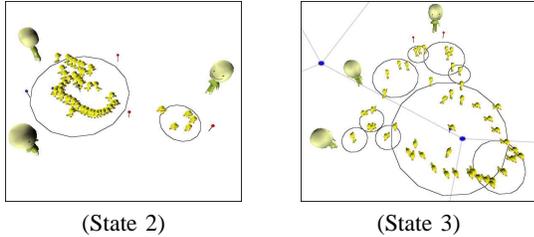


Fig. 7. (State 2) More shepherds than groups. (State 3) More groups than shepherds.

other group. This definition has been used in [4], [10]. Unfortunately, by this definition alone, the shepherds are often not able to keep the flock compact, especially for the large flock. Fig. 8(a) shows an example of scattered flock members in one group.

To handle this problem, we define a *compact area* for each group. The compact area of a group is the smallest circle that could potentially contain all group members. Ideally, shepherds should control the flock so that all members are in its compact area. This is an inverse version of the *packing circles in a circle problem* [8] which tries to find the maximum radius of  $K$  equal-sized smaller circles that can be packed into a unit circle and is known to be extremely difficult.

Let's consider a group with  $n_f$  flock members in which each individual member can be enclosed in a radius  $R_f$  circle. Since we do not have to compute the exact value of the radius  $R_c$  of the compact area, we approximate it by considering a bounding square of  $n_f$  flock members and then the compact area will be the minimum circle enclosing the square. Then, we define  $R_c$  as:

$$R_c = R_f \times \sqrt{2n_f} \quad (1)$$

Finally, we consider all members outside the compact area as separated from members inside the compact area even if they can see each other. Fig. 8(c) shows the separate groups using the compact areas.

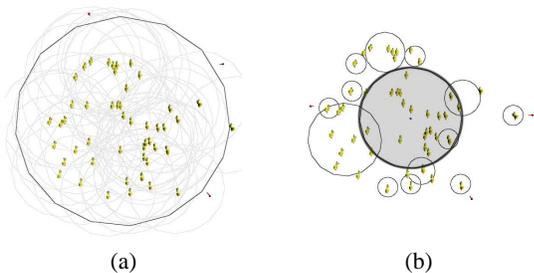


Fig. 8. (a) A flock group with scattered members. The smaller circles indicate range of view for the flock members. (b) Separated flock members as defined by the *compact area* (shaded area).

### B. Distributing shepherds

The shepherds need to be distributed to the separate flock groups. When the flock is in State 2, i.e.,  $1 < n_g \leq n_s$ , there is guaranteed to be at least one shepherd for each flock group. The number of shepherds for each group

depends on the number of flock members in that group, i.e., the  $i$ -th group with  $n_i$  members will have at least  $\lfloor \frac{n_i}{n} \times n_s \rfloor$  shepherds, where  $n$  is the total number of flock members. The steering points of each group will be arranged in a formation as described in Section IV-A. Once all steering points from all groups are decided, the shepherds will select their own steering points using one of the three assignment methods in Section IV-B and then will approach the separated flock members.

When the flock is in State 3, i.e.,  $N_g > N_s$ , there are more groups than shepherds. Thus, not all groups can have shepherds assigned to them. In this case, shepherds need to decide which groups require immediate attention and which can be postponed. We use the distance between each group and its milestone as the criterion, i.e., shepherds will distribute themselves to merge the groups that are farther away.

### C. Steering direction

Once the shepherds are at the steering points, they need to decide whether they should steer all groups to a common point or if they should stop a group to wait for other groups to arrive. Note that we do not want shepherds to push the flock to the goal in individual groups.

Whether a group should be steered to a common point or if it should be stopped depends on the existence of a *dominant* group. A dominant group, e.g., the larger group in Fig. 7 (State 2), is a group with more than half of the total flock members. If there is a dominant group, shepherds will stop that dominant group and steer smaller groups to it. On the other hand, when no dominant group exists, all shepherds will push their groups toward a common point which in our case is the closest milestone, e.g., State 3 in Fig. 7.

## VII. IMPLEMENTATION AND EXPERIMENTS

The purpose of these experiments is to estimate the performance of the proposed strategies for herding tasks with different levels of difficulty. Herding is a behavior in which shepherds need to move all flock members from a start region to a goal region. Herding is the most studied shepherding behavior [15], [18], [4], [10]. However, even for herding, little work has been done to enhance the shepherd's ability to control the flock.

In our experiments, we compare the time to accomplish a herding task for flocks with different sizes and with different levels of separation tendency. We use two environments in the following experiments, one with no obstacles (*empty* environment) and one with obstacles (*farm* environment); see Fig. 9(a). Due to the difficulty of presenting these simulations in a paper, we encourage readers to view animations at [1].

**Shepherd Performance v.s. Flock Size.** In this experiment, we study the performance of the shepherds with different numbers of the flock members. The purpose of this experiment is to show that it is useful to have more shepherds to herd a flock with a large number of members.

Fig. 9(b) and (c) shows results from both environments. We study flocks with 10, 20, and 40 members that are herded by 1 to 5 shepherds. After 2,000 time steps, the

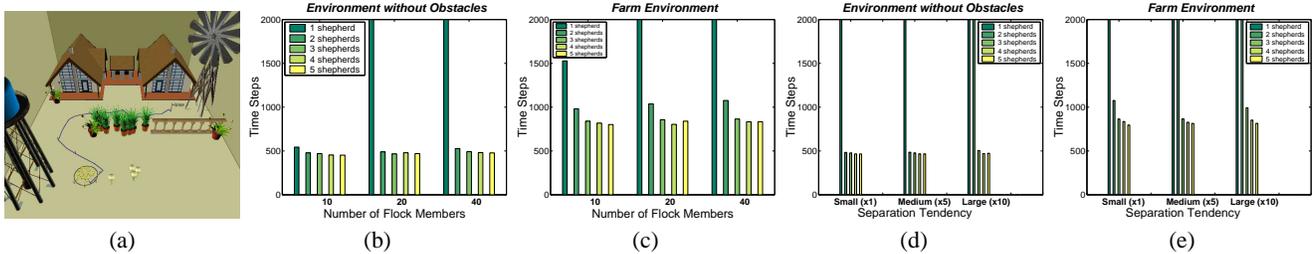


Fig. 9. (a) The farm environment. Results for flocks with different sizes in the (b) empty and in the (c) farm environments. Results for flocks with different separation tendencies in the (d) empty and the (e) farm environments. Each time step is 0.04 seconds.

shepherds fail the task if the flock is not home. A flock with more members is considered to be more difficult to steer than a flock with fewer members. From Fig. 9(b) and (c), one can clearly see that more shepherds consistently perform better when the number of flock members increases. Note that a single shepherd failed to herd 10 flock members in the farm environment but 4 shepherds successfully herded 40 flock members in the same environment. Hence, each individual shepherd can herd better in cooperation than it can herd individually.

**Shepherd Performance v.s. Flock Separation.** In this experiment, we study the performance of shepherds with different levels of separation tendency. Flocks, such as sheep, with lower separation tendencies will stay together when the shepherds approach. In contrast, flocks, such as ducks or cattle, with higher separation tendencies will scatter if the shepherds approach and are therefore harder to control. As shown in [10], a single shepherd, even using the best locomotions, has difficulty in controlling the motion of a flock with high separation tendency.

Fig. 9(d) and (e) shows our results from both environments. We study the flocks with small ( $\times 1$ ), medium ( $\times 5$ ), and large ( $\times 10$ ) levels of separation tendency. All flocks have 40 members and are herded by 1 to 5 shepherds. After 2,000 time steps, the shepherds fail the task if the flock is not home. As expected, Fig. 9(d) and (e) shows that multiple shepherds can herd more efficiently than a single shepherd in both environments.

## VIII. CONCLUSION

In this paper, we have shown that a large flock or a flock composed of members that are more difficult to control can be herded more efficiently using multiple shepherds than with a single shepherd. We proposed several strategies for multiple shepherds to accomplish this task cooperatively without communication between the shepherds. Our experiments show that the herding behavior using these multiple shepherd locomotions is superior to the behavior with a single shepherd.

We are currently investigating how these locomotions, such as the side-to-side steering in Fig. 3 or the formation and shepherd/steering point distribution in Section IV-B, might be learned automatically. Although methods based on genetic algorithms [15] and neural networks [12] have been proposed for learning the control methods for herding behaviors, they only consider herding with a few ( $\leq 3$ )

flock members and shepherds, and hence cannot be applied to the scenarios we are studying.

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