CS483 Analysis of Algorithms
Lecture 04 – Graph *

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*this lecture note is based on *Algorithms* by S. Dasgupta, C.H. Papadimitriou, and U.V. Vazirani and *Introduction to the Design and Analysis of Algorithms* by Anany Levitin.

Introduction

What can be represented as graph?

- Internet
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What are the problems that can be solved using graphs?

- Network routing

Graph Representation

- Adjacency matrix
  - Space

- Adjacency list
  - Space

Explore graphs

Graph Search

- What parts of the graph are reachable from a given vertex? (i.e., connected components)
- Many problems require processing all graph vertices (and edges) in systematic fashion
- Basic tools to safely explore an unknown environment
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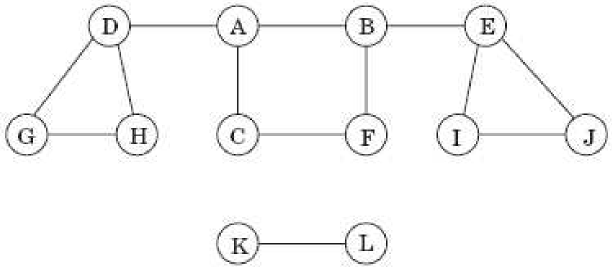
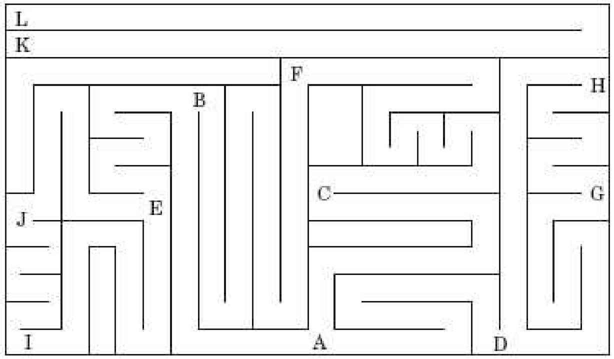




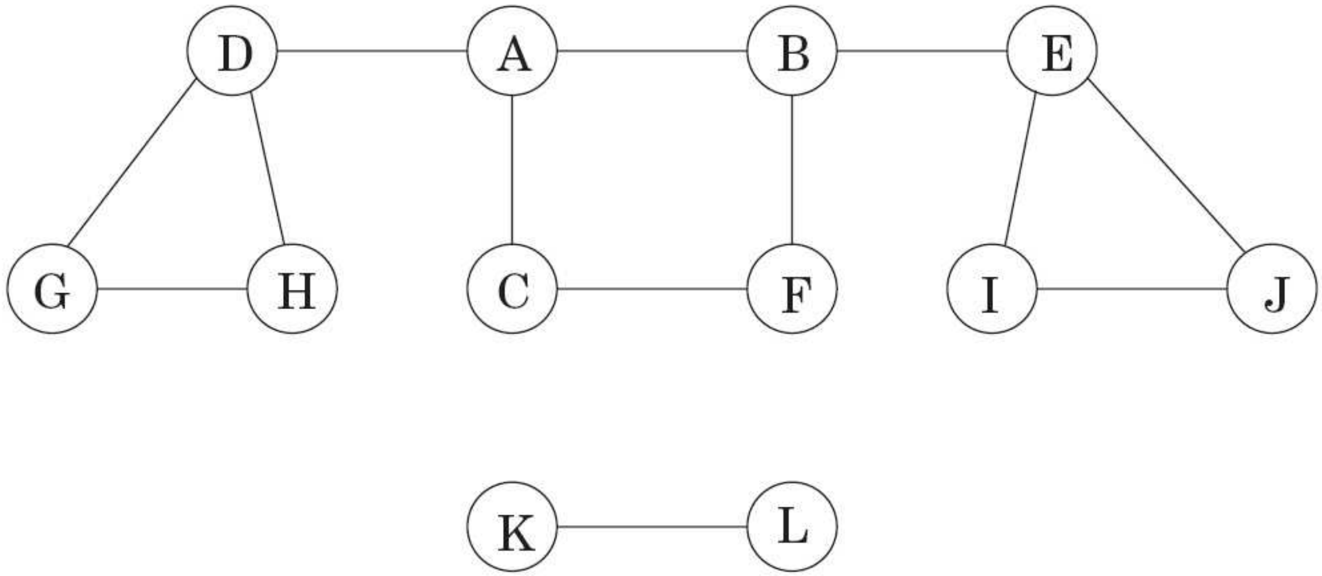
- Basic exploration algorithm

Algorithm 0.1: EXPLORE($G = \{V, E\}, v \in V$)

- Can the algorithm always work?
  - *proof*

Graph Search

- Example: EXPLORE(B)



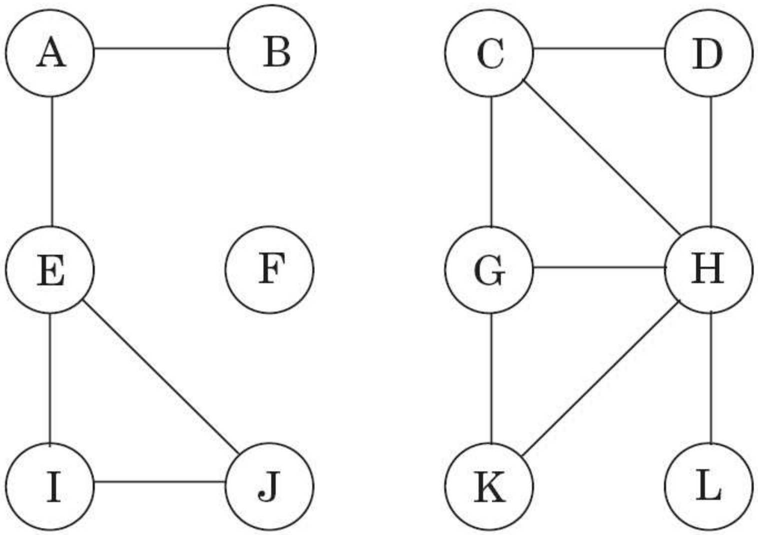
Depth-first search

- DFS

Algorithm 0.2: DFS$(G = \{V, E\}, v \in V)$

Depth-first search

- Example:



- Time complexity:

- Connect component
  - Given a graph G, report the number of connect components in G.
  - Given a graph G, can you preprocess G so that you can check if two nodes u and v from G are from the same connect component?

DFS Application

- Ancestor/Descendant relationship of tree
  - Given a tree T, can you preprocess T so that you can answer if u is the ancestor of v in *constant time*, where u and v are two nodes from T.

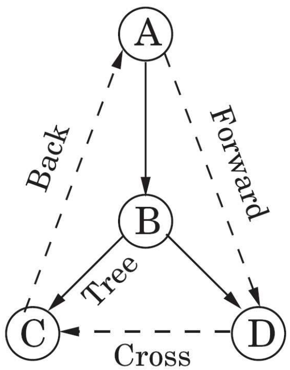
DFS Application

- ☐ Given a *directed* graph G, convert G to a tree whose nodes and edge are the vertices and edges of G

DFS Application

- Types of edges

  DFS tree

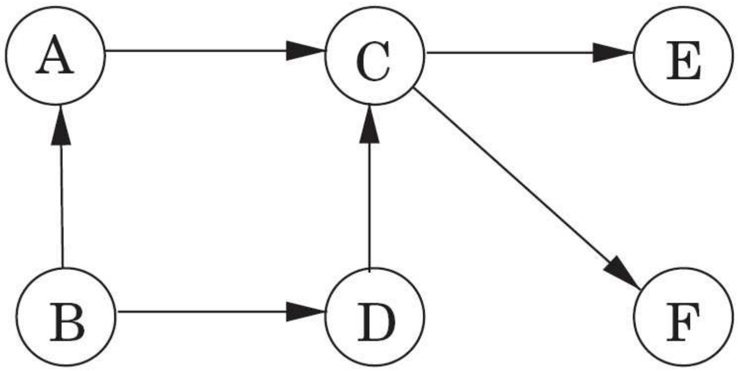


- To identify the type of an edge: pre/post ordering

Topological sort

Directed acyclic graphs

- A graph G without (directed) cycle is a *directed acyclic graphs* (DAG)
- DAG can be found in modeling many problems that involve prerequisite constraints (construction projects, document version control)

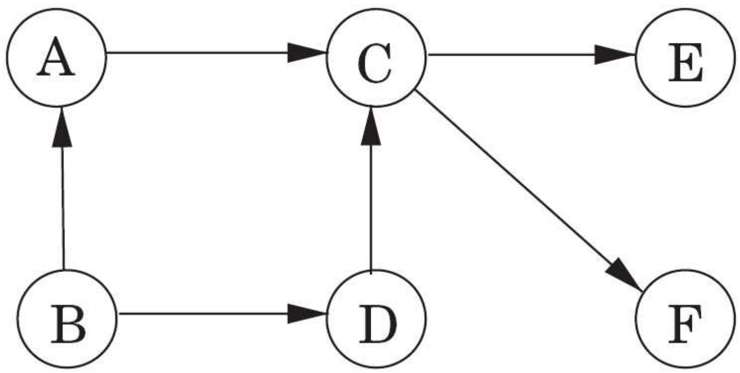


- Given a *directed* graph G, identify *cycles* in G

  – *proof*

DAG and Topological Sort

- **Topological sorting** or **Linearization**: Vertices of a DAG can be linearly ordered so that:
  - Every edge its starting vertex is listed before its ending vertex
  - Being a DAG is also a necessary condition for topological sorting be possible
- Example:



Topological Sort: Using DSF

- Compute DSF and reverse the visit order

Algorithm 0.3: $\mathrm{TS}(G = \{V, E\})$

- Why does it work?

- Time complexity?

Topological Sort: Using Source Removal

- Identify and remove sources iteratively.
  - A source is a vertex without incoming edges.

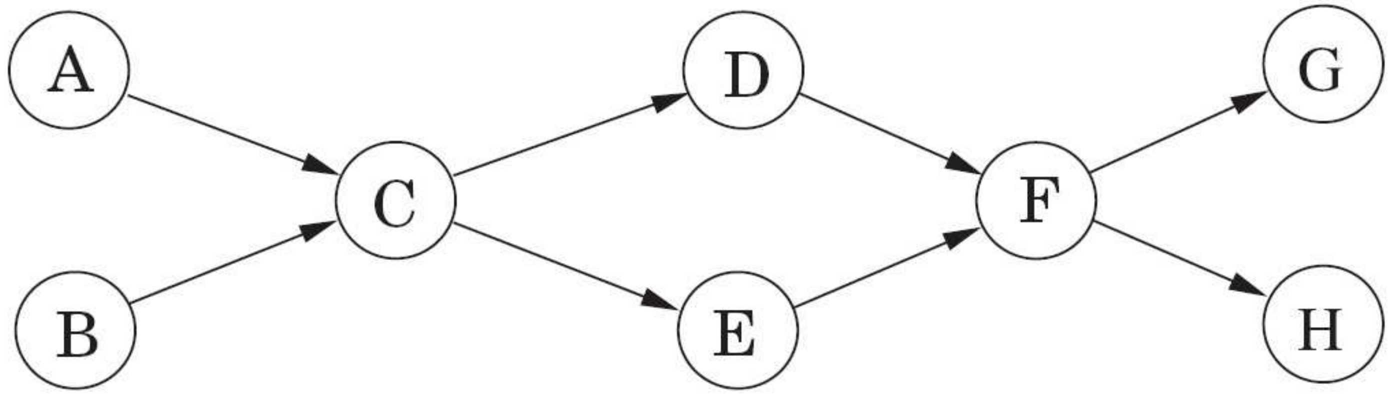
Algorithm 0.4: $\text{TS}(G = \{V, E\})$

- Why does it work?

- Time complexity?

Example

- Example:

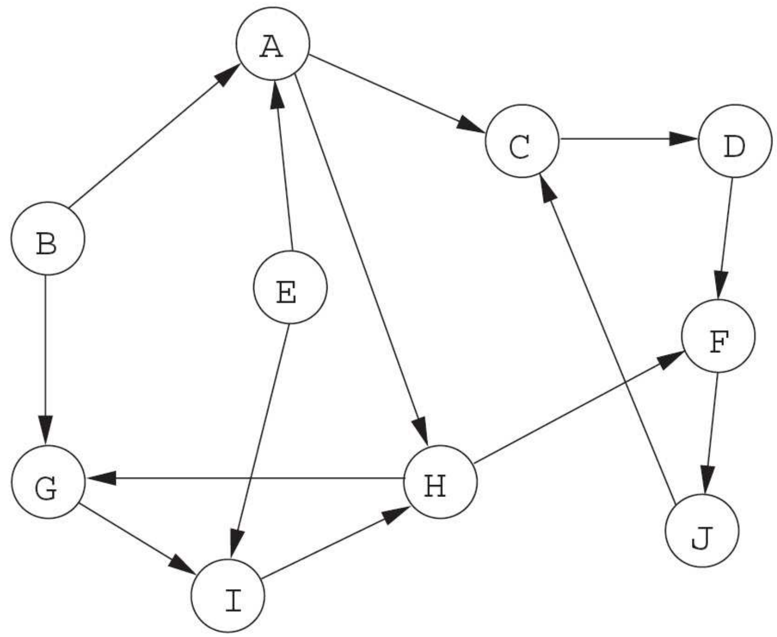


Strong connected components

Strongly connected components

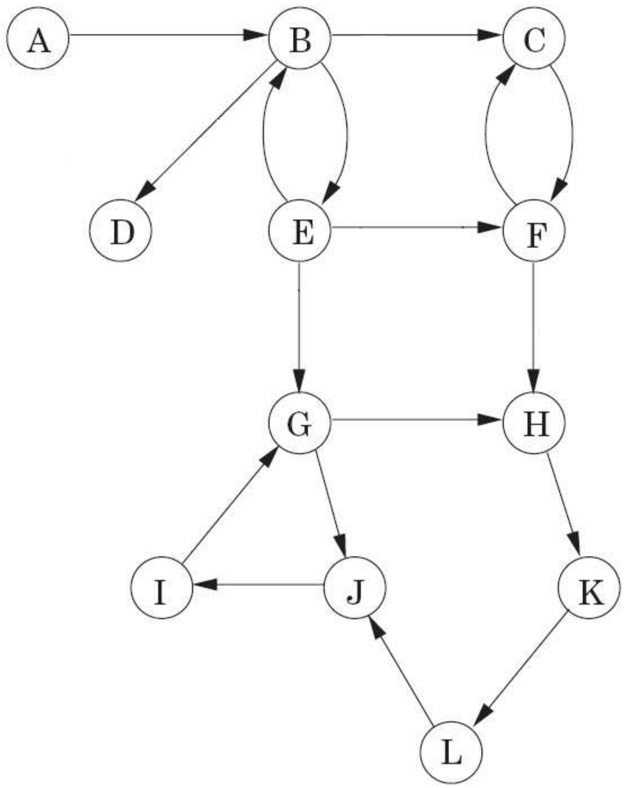
- **Definition**: Two nodes u and v are from the connected if and only if there is a path from u to v and a path from v to u.

- **Definition**: A set of vertices form a strongly connected component (SCC) iff any pairs of vertices are connected.



Strongly connected components

- Connected components in directed graph is less intuitive than that of undirected graph.
  - How many connected components are there in the graph below?



- How to compute SCCs from a directed graph?

Strongly connected components and DAG

- **Observation 1**:
- **Observation 2**:

- Our strategy to find all SCC:
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Strongly connected components and DAG

- **Task**: Find a vertex u in DAG sink node
- **Fact**: It is easier to find a source node.
  -
  -
    - *proof*:
- How to find a vertex u in DAG sink node?
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Strongly connected components and DAG

- **Task**: Remove all nodes from the previous SCC and identify a new sink node

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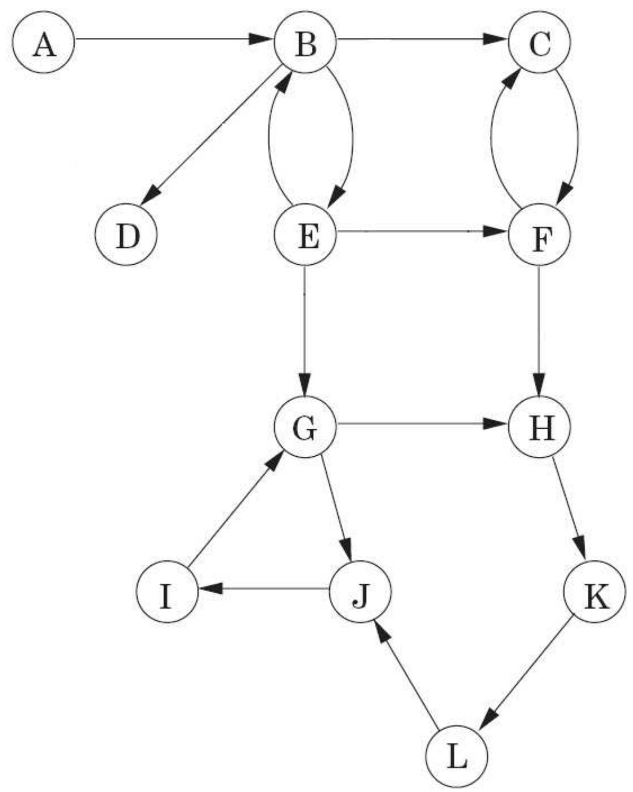
- Example:



Conclusion

Summary

- Graphs can be very useful for many problems.
- DFS can be used for
  - Explore the graph
  - Reveal relationship between the graph nodes and types of edges
  - Linearization for DAG
  - Identify cycles, connected components, strongly connected components
- Assignment
  -

More graph algorithms

- Next week we will discuss problems related to paths in graph
  - shortest path in undirected and directed graphs
  - shortest path in weighted graphs
  - shortest path in graphs with negative edges
  - shortest path in DAG