

Show all work clearly and in order. Justify your answers whenever possible; You have 20 minutes to take this 10 point quiz.

1. (*4 points*) Quicksort $A = [9, 29, 22, 81, 31, 58, 24, 35, 57, 26]$. Use the **last** element as the pivot. Show steps to earn full points.

2. (*4 points*) We learned that if we want to multiply two integers A and B , we can represent $AB = (a10^{\frac{n}{2}} + b)(c10^{\frac{n}{2}} + d) = K_210^n + K_110^{\frac{n}{2}} + K_0$, where $K_2 = ac$, $K_0 = bd$, $K_1 = (a + b)(c + d) - (K_0 + K_2)$. Given $A = 9876$ and $B = 4321$, show what $a, b, c, d, K_1, K_2, K_3$ are in each recursion.

3. (*1 point*) Solve this recursion $T(n) = 3T(n - \frac{2}{9}n) + n^2$ using the Master theorem provided on the back of the sheet.

4. (*1 point*) The strength of RSA encryption is from the fact that solving _____ (a problem) efficiently is intractable.

Master Theorem

- If we have a problem of size n and our algorithm divides the problems into b instances, with a of them needing to be solved. Then we can set up our running time $T(n)$ as: $T(n) = aT(n/b) + f(n)$, where $f(n)$ is the time spent on dividing and merging.
- If $f(n) \in \Theta(n^d)$, with $d \geq 0$, then

$$T(n) = \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \log n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$