CS583 Lecture 09
All-Pairs Shortest Paths

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some materials here are based on Prof. Shehu, and Prof. Wang's past lecture notes

A DP approach

- sub-problem (use path length):
- recursive definition:

A DP approach

- Algorithm (1st attempt)

- time complexity

A DP approach

- Can we do better?
  - Key observation:
  - New algorithm:
  - time complexity?

Floyd-Warshall

- another DP algorithm
  - sub-problem
  - recursive definition

Floyd-Warshall

- algorithm

  - time complexity?

Transitive closure

- definition:
- solve using Floyd-Warshall's algorithm:

Transitive closure

- another (simpler) way:

- time complexity?

Johnson's Algorithm

- If graph is sparse (not many edges), then we can run Dijkstra's algorithm for each vertex
- time complexity:

- However, Dijkstra's algorithm handles only non-negative weights
  - Donald Johnson found a way to **re-weight** the graph

Johnson's Algorithm

- Re-weighting the graph
  - idea:
- Algorithm:

- time complexity: