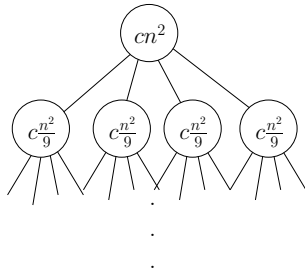


Show all work clearly and in order. Justify your answers whenever possible; You have 20 minutes to take this 10 point quiz.

1. (4 points) Determine if following statements are correct or not and provide a one-sentence explanation to your answer, e.g., correct; because both functions have logarithmic order of growth.

1. $n! \in O(n^n)$?
2. $\sum_{i=1}^n i \in \Theta(n \log n)$?
3. $(50n)! \in \Theta(n!)$?
4. $5\sqrt{n+9} + \log n^{3000} + \pi \in \Omega(\log n)$?

2. (5 points) Given the recursion tree below, answer the following questions.



1. Write the recursion function of the tree: $T(n) =$
2. Solve the recursion by analyzing the tree (show details)
3. Solve the recursion using the Master theorem (show details)
4. Prove your answer using induction

3. (1 point) What is the running time of insertion sort?

Master Theorem

- If we have a problem of size n and our algorithm divides the problems into b instances, with a of them needing to be solved. Then we can set up our running time $T(n)$ as: $T(n) = aT(n/b) + f(n)$, where $f(n)$ is the time spent on dividing and merging.
- If $f(n) \in \Theta(n^d)$, with $d \geq 0$, then

$$T(n) = \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \log n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$