CS311 Data Structures
Lecture 06 — Trees

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Logistics

Reading

- Weiss Ch. 7 Recursion
- Weiss Ch 18 General Trees
- Weiss Ch 19 BSTs

Today

- Tree Traversals
- Recursive traversals
- Recursion practice for tree properties

Ordering

List property

There is a well defined ordering of first, next, last objects in the data structure,

- Wide ranging uses
- Supported in List data structure (`LinkedList`, `ArrayList`)
- Supported structurally in Lists
- *A property of the Data Structure*

Sorting property

There is a well defined ordering relation over all possible data of a type

- ”bigger than” ”less than” ”equal to” are well defined
- A property of the *Data*
- A data structure can try to mirror the data ordering structurally
- Useful for searching, walking through stored data in order

Sorted Lists

Definition is straight-forward

- "Smallest" things are structurally "first", "Biggest" last
- Ordering on elements (`Comparable/Comparator`)
- `add/insert` put elements in proper place

Question: For a sorted `List L`, what is the complexity of `L.insert(x)` which preserves sorting?

`L` is an `ArrayList`

How long to

- find insertion location?
- complete insertion?
- traverse elements in order (e.g. for printing)?

`L` is a `LinkedList`

How long to

- find insertion location?
- complete insertion?
- traverse elements in order (e.g. for printing)?

Alternatives to the Linear Data Structures

Hash Tables

- Abandon list property
- Abandon sorting property
- $O(1)$ insertion/retrieval
- $O(N)$ traversal, not ordered

Trees

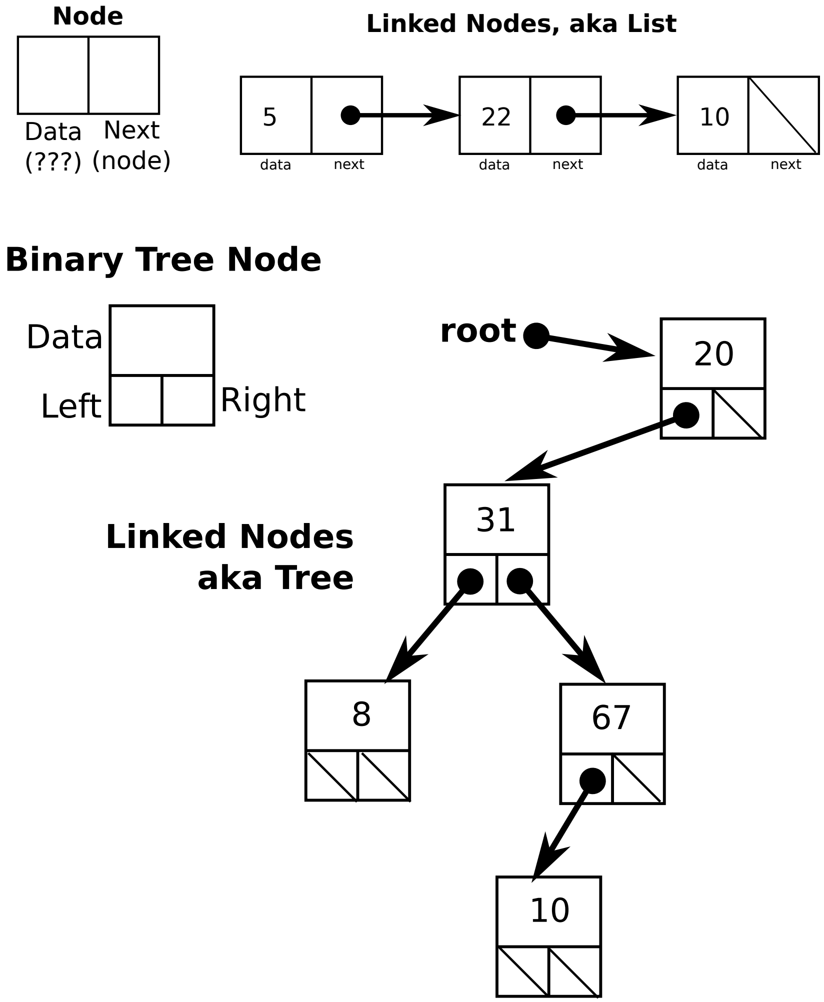
- Abandon list property
- Preserve sorting property
- $O(\log N)$ insertion/retrieval
- $O(N)$ traversal, ordered
- Commonly Binary Trees
- Other variants

Roots

Source

- Next few sessions we'll talk about roots
- For simplicity, we'll call them trees

Mutated Nodes



Node structures should be familiar for linked lists

- Singly linked: `next/data`
- Doubly linked: `next/previous/data`

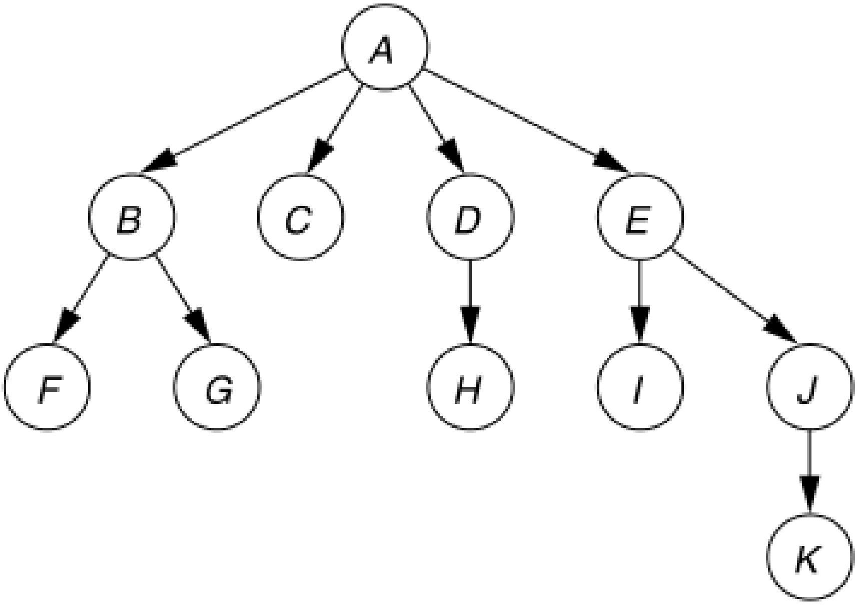
Trees use Nodes as well

- `children`, `data`, possibly `parent`
- Arbitrary Trees: `List<Node>` of `children`
- Binary Trees: `left` and `right` children

Tree Properties of Interest

- Root of tree
- Leaves
- Data at nodes

- Size (number of nodes)
- Height of tree
- Depth of a node



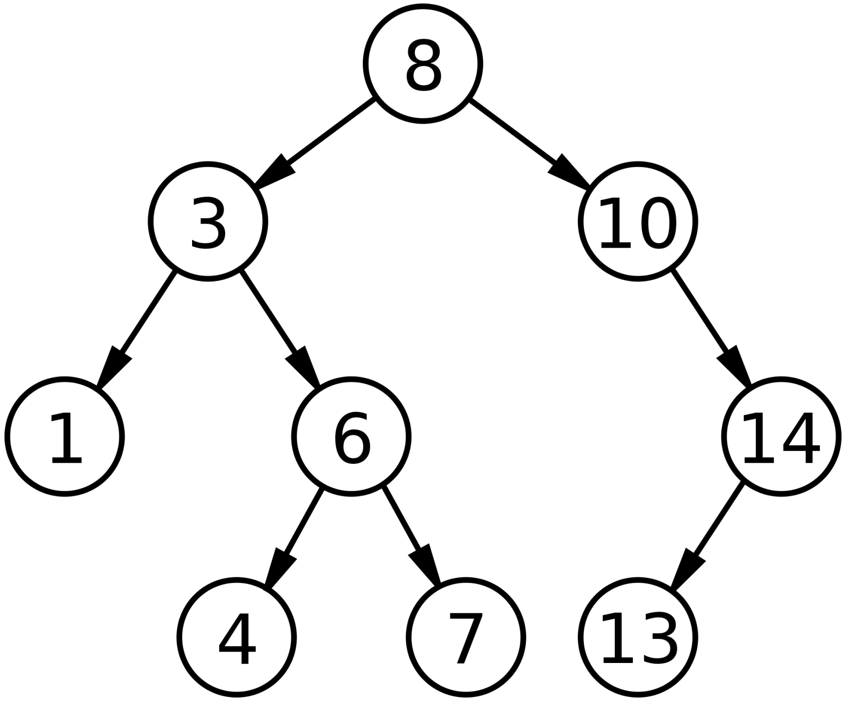
Node	Height	Depth
A	3	0
B	1	1
C	0	1
D	1	1
E	2	1
F	0	2
G	0	2
H	0	2
I	0	2
J	1	2
K	0	3

Binary Tree

Binary Tree Nodes

```
class Node<T>{
  T data;
  Node<T> left, right;
}
void main(){
  Node root = new Node();
  root.data = 8;
  root.left = new Node();
  root.right= new Node();
  root.left.data = 3;
  root.right.data= 10;
  root.left.left = new Node();
  ...
```

Structure



Recursive Example: Binary Tree Size Method

int size(Node<T> t)

Number of nodes in tree t

```
public Tree<T>{
  Node<T> root;

  // Entry point
  public int size(){
    return size(this.root);
  }
  // Recursive helper
  public static <T>
    int size( Node<T> t ){
    if(t == null){
      return 0;
    }
    int sL = size(t.left);
    int sR = size(t.right);
    return 1 + sL + sR;
  }
}
```

Usage

```
Tree<Integer> myTree = new Tree();
// add some stuff to myTree
int s = myTree.size();
```

Recursive Example: Binary Tree Height Method

Exercise

- Define a recursive `t.height()`
- `t.height()` is the longest path from root to leaf
- Empty tree has height=0

`int height(Node<T> t)`

Depth of deepest node in `t`

```
public Tree<T>{
  Node<T> root;
  public int height(){
    return height(this.root);
  }
  // Depth of deepest node
  public static <T>
  int height( Node<T> t ){
    // Recursive version?
  }
}
```

Recursive Implementation of height()

Slight difference of definitions from textbook

- Empty tree has size=0 and height=0
- 1-node tree has size=1 and height=1

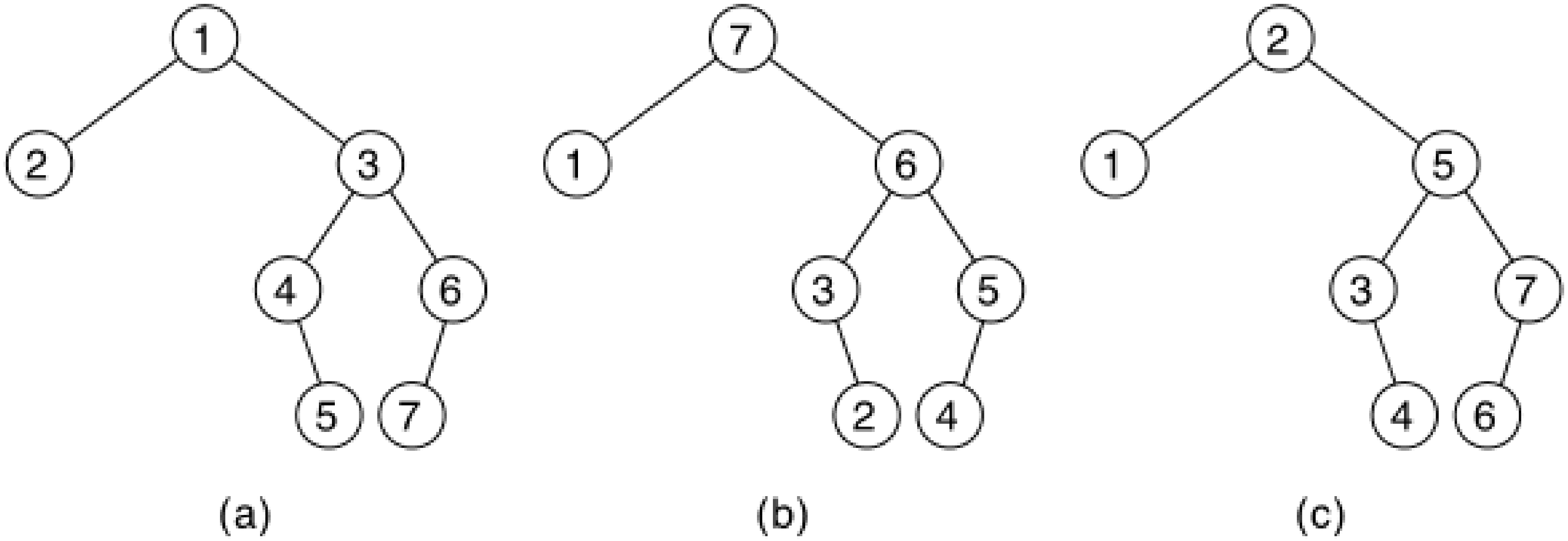
```
// Depth of deepest node
public Tree<T>{
  Node<T> root;
  public int height(){
    return height(this.root);
  }

  public static <T>
    int height( Node<T> t ){
    if(t == null){
      return 0;
    }
    int hL = height(t.left);
    int hR = height(t.right);
    int bigger = Math.max(hL,hR);
    return 1+bigger;
  }
}
```

The Many Ways to Walk

No linear property: several orders to traverse tree, mostly starting from the root

- (a) Pre-order traversal (this, left, right)
- (b) Post-order traversal (left, right, this)
- (c) In-order traversal (left, this, right)



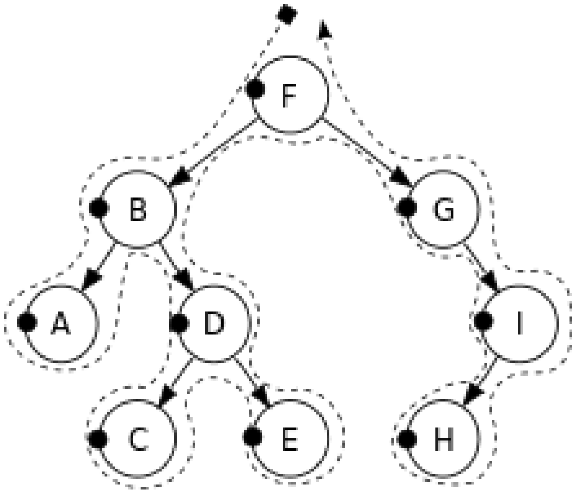
Picture shows the order nodes will be visited in each type of traversal

The Many Ways to Walk

No linear property: several orders to traverse tree

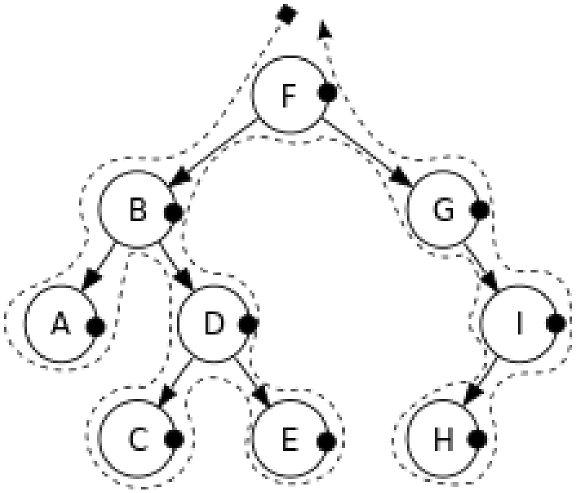
Pre-order traversal

this, left, right



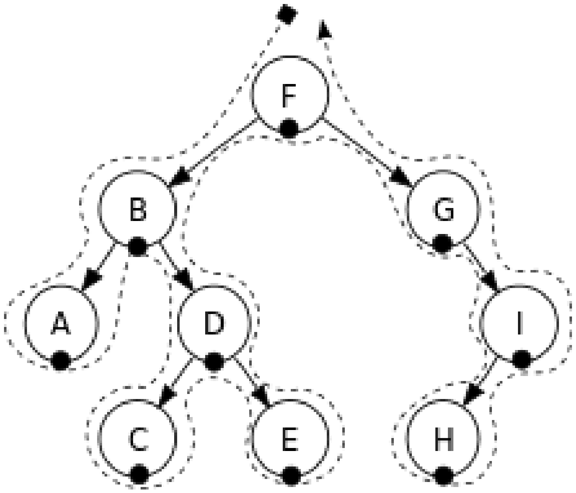
Post-order traversal

left, right, this

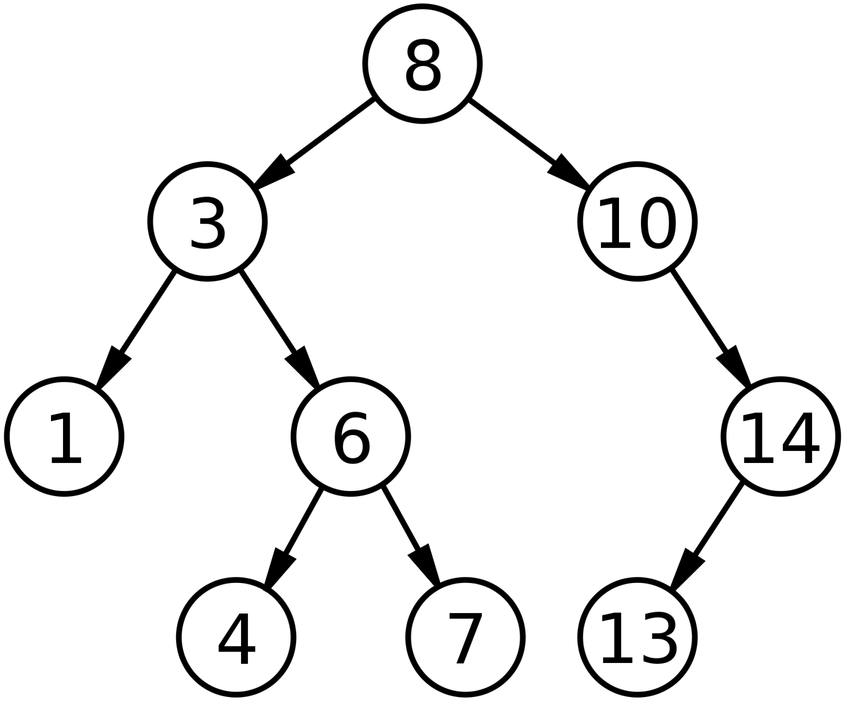


In-order traversal

left, this, right



Walk This Tree



Show

- (a) Pre-order traversal (this, left, right)
- (b) Post-order traversal (left, right, this)
- (c) In-order traversal (left, this, right)

Which one "sorts" the numbers?

Implementing Traversals for Binary Trees

```
class Tree<T>{
  private Node<T> root;

  public void printPreOrder(){
    preOrder(this.root);
  }
  private static void
  preOrder(Node<T> t){
    ... print(t.data) ...
  }

  public void printInOrder(){ }
  private static void
  inOrder(Node<T> t){ }

  public void printPostOrder(){ }
  private static void
  postOrder(Node<T> t){ }
}
```

```
class Node<T> {
  T data;
  Node<T> left, right;
}
```

Implement Print Traversals

- `preOrder(this.root)`
- `postOrder(this.root)`
- `inOrder(this.root)`

2 Ways

- Recursively (first)
- Iteratively (good luck. . .)

Recursive Implementation of Traversals

```
inOrder(Node t){
  if(t != null){
    inOrder(t.left);
    print(t.data);
    inOrder(t.right);
  }
}
```

```
preOrder(Node t){
  if(t != null){
    print(t.data);
    preOrder(t.left);
    preOrder(t.right);
  }
}
```

```
postOrder(Node t){
  if(t != null){
    postOrder(t.left);
    postOrder(t.right);
    print(t.data);
  }
}
```

Evaluate

- Correct?
- Time complexity?
- Space complexity?
- What makes this so easy?

Iterative Implementation?

Compare to Iterative Implementation of Traversals

```
// Pseudo-code for post order print
void postOrder(root){
  Stack s = new Stack();
  s.push( {root, DOLEFT });
  while(!s.empty()){
    {tree, action} = s.popTop();
    if(tree == null){
      // do nothing;
    }
    else if(action == DOLEFT){
      s.push({tree, DORIGHT});
      s.push({tree.left, DOLEFT});
    }
    else if(action == DORIGHT){
      s.push({tree, DOTHIS});
      s.push({tree.right, DOLEFT});
    }
    else if(action == DOTHIS){
      print(tree.data);
    }
    else{
      throw new YouScrewedUpException();
    }
  }
}
```

- No call stack
- Use an explicit stack
- Auxilliary data action

  `DOLEFT` work on left subtree

  `DORIGHT` work on right subtree

  `DOTHIS` process data for current

Evaluate

- Correct?
- Time complexity?
- Space complexity?

Weiss's Traversals

Implemented as `iterators`

- See `TestTreeIterators.java`
- Uses `BinaryTree.java` and `BinaryNode.java`
- Must preserve state accross `advance()` calls

```
BinaryTree<Integer> t = new BinaryTree<Integer>( );
... // fill tree

TreeIterator<AnyType> itr = new PreOrder<Integer>( t );
for( itr.first( ); itr.isValid( ); itr.advance( ) ){
  System.out.print( " " + itr.retrieve( ) );
}
```

- Much more complex to understand but good for you
- Play with some of these in a debugger if you want more practice

General Notes

Iterative Traversal Implementation Notes

- Can augment tree nodes to have a parent pointer

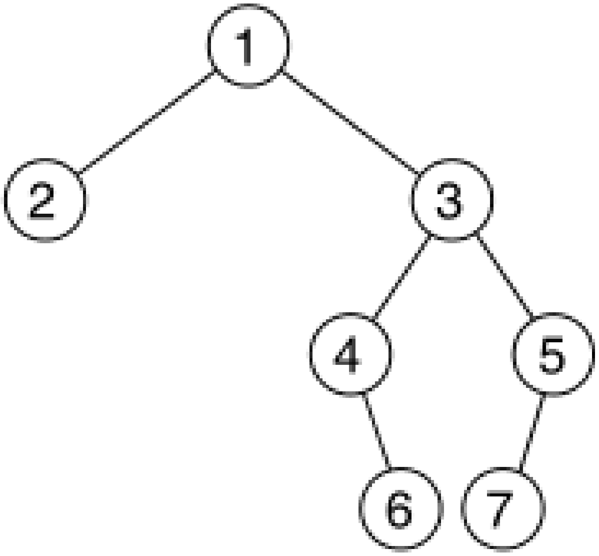
  ```
  class Node<T>{
    T data; Node left, right, parent;
  }
  ```

- Enables stackless, iterative traversals with great cleverness

Iterative vs Recursive Tree Methods

- Multiple types of traversals of `T`
- Other Tree methods: `T.find(x)`, `T.add(x)`, `T.remove(x)`
- Recursive implementations are simpler to code but will cost more memory
- Iterative methods are possible and save memory at the expense of tricky code

Level-order Traversal



Level Order Traversal: 1 2 3 4 5 6 7

- Top level first (depth 1: 1)
- Then next level (depth 2: 2 3)
- etc.

This is a bit trickier

- Need an auxilliary data structure: Queue
- Does recursion help?