CS311 Data Structures
Lecture 07 — Binary Search Trees

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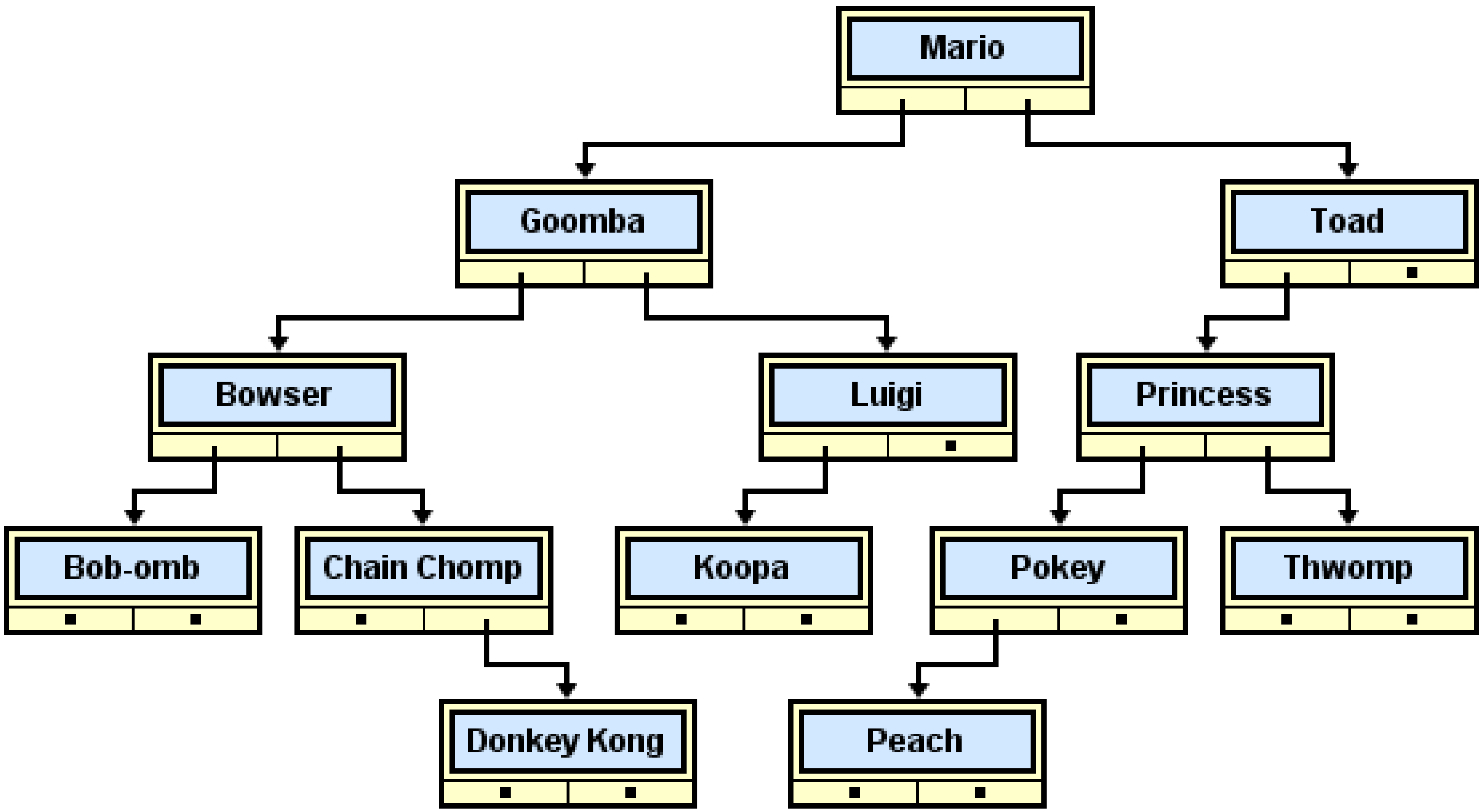
Recall Tree Basics

- What distinguishes a tree from a linked list? What gets stored at each tree `Node`
- What technique becomes useful for implementing operations on trees? Why?
- What is our motivations for looking at trees again?
  e.g. Why not just stick to `ArrayList/LinkedList`?
- New: How might one implement an iterator for a tree?

Binary Search Tree Property

A binary tree where every node `N` in the BST

- Any data in the tree rooted at `N.left` sort before `N.data`
- Any data in the tree rooted at `N.right` sort after `N.data`



Comparisons

How does java guarantee comparability?

`Comparable`

Data can implement Comparable

```
int c = x.compareTo(y);
// neg for x < y, right order
//   0 for x = y, don't care
// pos for x > y, wrong order
```

`Comparator`

Use a Comparator object to do comparisons

```
Comparator<Thing> cmp =
   new ...;
int c = cmp.compare(x,y);
// neg for x < y, right order
//   0 for x = y, don't care
// pos for x > y, wrong order
```

Presence of both hints at a fundamental problem

Define `bst.find()`

- `find(T x)` is publicly accessible

  `tree.find("Mario");`

- Define

  `find(T x, Node<T> t)`

  which works on a given start node

- Compare via `Comparable`:

  `if(x.compareTo(t.data) < 0)`

Give 2 versions

- Recursive
- Iterative

```
public class BinarySearchTree
  <T extends Comparable<T>>
{
  protected Node<T> root;
  // Return x if in tree,
  // null otherwise
  public T find( T x ){
    Node<T> result =
      find(x, this.root);
    if(result == null){ return null;}
    else{ return result.data; }
  }

  // Find node containing x
  // starting at node t
  // Return null if not found
  private static
  Node<T> find(T x, Node<T> t){
    // DEFINE ME
  }
}
```

Recursive `find(x,node)`

Use key of data to search through tree

- Left for less than
- Right for greater than

```
// pseudocode
Node<T> find(x,t){
  if(t == null){
    return null;
  }
  int diff = x.compareTo(t.data);
  if(diff < 0){                    // x < t
    return find(x,t.left);
  } else if(diff > 0){             // x > t
    return find(x,t.right);
  } else {                         // x==t.data
    return t.data;                 // found
  }
}
```

Iterative find(x,node)

See weiss/nonstandard/BinarySearchTree.java

```
private static BinaryNode<T> find(T x, BinaryNode<T> t){
  while( t != null ) {
    if( x.compareTo( t.data ) < 0 )
      t = t.left;
    else if( x.compareTo( t.data ) > 0 )
      t = t.right;
    else
      return t;                  // Match
  }
  return null;                   // Not found
}
```

Complexity of BST Find

- What is the worst-case complexity of `find(x)` in terms of tree properties?
- Construct a tree with this this worst case complexity

Warm-up: Perform BST Insertions

Draw the tree that results from the following sequence of insertions.

```
MyBST<String> t = new MyBST<String>();
t.insert("Mario");
t.insert("Goomba");
t.insert("Luigi");
t.insert("Toad");
t.insert("Wario");
t.insert("Princess");
t.insert("Bowser");
t.insert("Chain Chomp");
```

Insertion: Similar to find(x)

- May need to change a left or right pointers, redefine root
- No duplication, define a TreeSet, exception on duplicate insert

Define Recursive Insert

```
class BinarySearchTree<T> {
  Node<T> root=null; int size=0;
  public void insert( T x ){
    root = insert( x, root );
    this.size++;
  }
  private static Node<T> insert( T x, Node<T> t ){
    // DEFINE ME
  }
}
```

Recursive insert(x,t)

From weiss/nonstandard/BinarySearchTree.java

```
class BinarySearchTree<T> {
  Node<T> root;
  public void insert( T x ){
    root = insert( x, root );
  }
  private static Node<T> insert( T x, Node<T> t )
  {
    if( t == null )
      t = new Node<T>( x );
    else if( x.compareTo( t.data ) < 0 )
      t.left = insert( x, t.left );
    else if( x.compareTo( t.data ) > 0 )
      t.right = insert( x, t.right );
    else
      throw new DuplicateItemException( x.toString( ) );
    return t;
  }
}
```

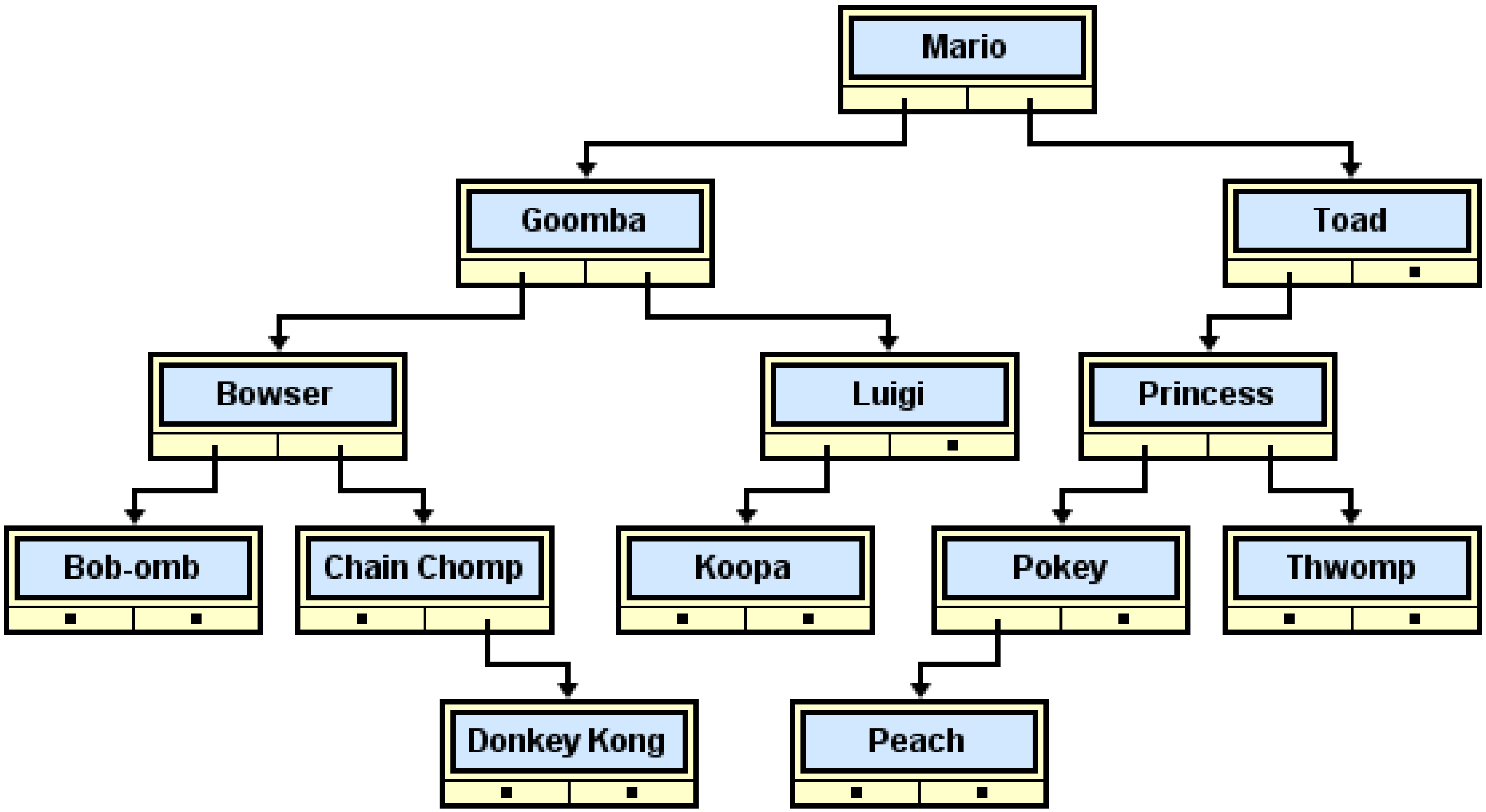
Binary Search Tree `remove(x)`

```
// Public method, eliminate x if present in tree
public void remove( T x );

// Recursive helper method
private Node<T> remove( T x, Node<T> t );
```

- Get rid of a node with data `x` in a binary tree; throw exception if not present (or ignore request)
- More involved than `find`/`insert`
- Preserve Tree Structure
- Recursion greatly eases implementation

Prelims

Consider Mario Tree



- Describe which cases exist `tree.remove(x)`?
- Which of these do you anticipate being easy/hard to code for?

BST General `remove(x)`: Cases

Cases for `t.remove(x)`

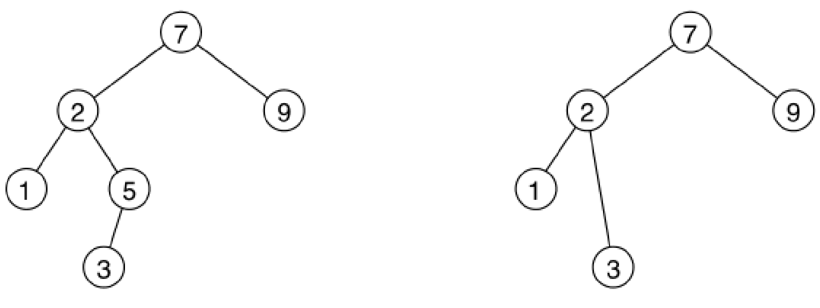
1. `x` not in tree
   - Leave tree as is or raise an exception
2. `x` at a node with no children
   - Get rid of node containing `x`
3. `x` at a node with 1 child
   - "Pass over" node containing `x`
4. `x` at a node with 2 children
   - Find a next node in sorting order
   - Replace `x` with next nodes data
   - Remove next node
   - Next is minimum of right subtree

Exercise: Useful Helper Methods

```
class BST<T> {
  private Node<T> root;

  // Public facing method, find minimum element and return it
  public T findMin(){  return this.findMin(this.root); }

  // Private helper method return the smallest element in the
  // tree rooted at t
  private T findMin(Node<T> t){
    // DEFINE ME
  }

  // Public facing method, eliminate the smallest data in tree
  public void removeMin(){  this.root = removeMin(this.root); }

  // Recursive helper; remove the node with the smallest data
  // in it in the tree rooted at t.  The node returned is used
  // to alter the structure of the tree.
  private Node<T> removeMin(Node<T> t){
    // DEFINE ME
  }
}
```

Children Cases for `remove(t,x)`

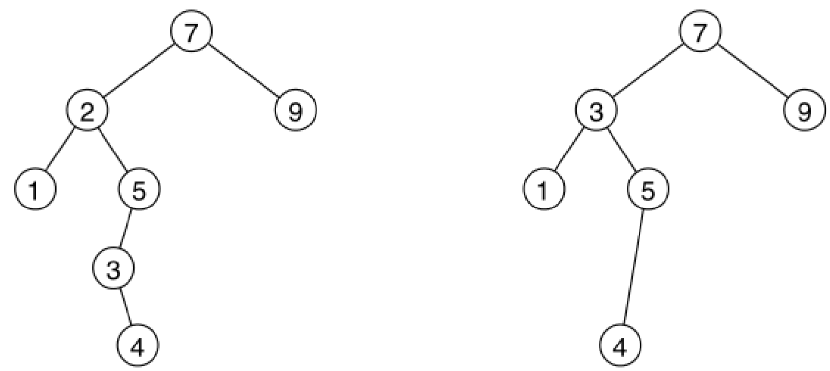
One Child: Remove 5



(a) (b)

1. Find node `t` with data `x`
2. Replace with only child

Two Children: Remove 2



(a) (b)

1. Find node `t` with data `x`
2. Find `min` node of `t.right`:
   `min` must have 0/1 child
3. Replace `t.data` with `min.data`
4. Remove `min`

Recursive Implementation: Think Locally

Lesson from `insert()`

- Recall in `insert(x,t)`, did stuff like

```
t.right = insert(x, t.right);
// a new/existing node is returned by insert()
```

- Take the same approach for `remove(x,t)`
- Assume these helpers are Available

```
T findMin(Node<T> t);   Node<T> removeMin(Node<T> t)
```

Implement Recursive `remove(x,t)`

- How to know if `t` is *the* node?
- What to do if `t` isn't *the* node?
- If `t` is *the* node, are there separte cases for action?

Cases for recursive `remove()`

1. ☒ `t` is `null`
   Throw an exception

   ```
   throw new ItemNotFoundException();
   ```

   Or do nothing to the tree

   ```
   return null;
   ```

2. ☒ `x` less than `t.data` (recurse left)

   ```
   t.left = remove(t.left, x);
   ```

3. ☒ `x` greater than `t.data` (recurse right)

   ```
   t.right = remove(t.right, x);
   ```

4. ☐ `x` equals `t.data` (remove `t`)
   - `t` has 0 children, get rid of `t`
   - `t` has 1 child, pass over `t`
   - `t` has 2 children, replace with next/prev

Case 4: `x` equals `t.data` (remove `t`)

Helper methods defined elsewhere

```
T findMin(Node<T> t);    Node<T> removeMin(Node<T> t)
```

- `t` has 0 children, get rid of `t`

  ```
  return null;
  ```

- `t` has 1 child, pass over `t`

  ```
  (t.left!=null) ? return t.left : return t.right;
  ```

- `t` has 2 children, replace with next or prev

  ```
  t.data = findMin(t.right);
  t.right = removeMin(t.right);
  return t;
  ```

- How are `findMin(t)` and `removeMin(t)` implemented?
  - Where is the minimum node in a tree?
  - How many children does it have?

remove(x)

Adapted from `weiss/nonstandard/BinarySearchTree.java`

```
private Node<T> remove( T x, Node<T> t ){
  if( t == null )
    throw new ItemNotFoundException( x.toString( ) );
  if( x.compareTo( t.data ) < 0 )
    t.left = remove( x, t.left );
  else if( x.compareTo( t.data ) > 0 )
    t.right = remove( x, t.right );
  // Found at this node
  else if( t.left != null && t.right != null ){
    // Two children
    t.data = findMin( t.right );
    t.right = removeMin( t.right );
  }
  else
    // One child or no children
    t = ( t.left != null ) ? t.left : t.right;
  return t;
}
```

So Far

Binary Search Trees

- Defined `find()` / `insert()` / `remove()`
- Helpers: `findMin()` / `findMax()` / `removeMin()` / `removeMax()`
- All ops runtime complexity $O(Height)$
- Discuss balancing trees to ensure that $Height \approx \log(Size)$

Review Questions

1. What is the Binary Search Tree property?
2. Are all trees binary trees? Do all binary trees have the BST property? (give counter-examples)
3. Where is the biggest data element in a BST? The smallest?
4. What are the runtime complexities of BST `tree.find(x)` and `tree.insert(x)`?
5. Which kinds of nodes are easy to remove from BSTs? Which kinds are more difficult?
6. What is a useful strategy for removing difficult nodes?