Red-Black Tree Review

Properties
A Binary Search Tree with 4 additional properties
1. Every node is red or black
2. The root is black
3. If a node is red, its children are black
4. Every path from root to null has the same number of black nodes

Insertion and Invariants

1. Color swap between grandparent and (parent and uncle) does not change black height
2. Rotation at grandparent followed by recoloring does not change black height
Red-Black Tree Deletion

number of kids
Let $x$ be the node in RB tree to be deleted

1. $x$ has two children nodes
   - swap value with the node $y$ with the largest value in the left subtree
   - then delete node $y$
   - node $y$ must have one or zero child node

2. $x$ has one child
3. $x$ has zero child

color of node
Let $x$ be the node in RB tree to be deleted

1. $x$ is red, delete as usual
   - black depth remains the same
   - no two adjacent nodes are red

2. $x$ is black, many hairy cases
   - black depth along the path containing $x$ is one less
   - two adjacent nodes may be red
Case Analysis

Four possible cases

1. $x$ is red
   1.1 $x$ has one child
   1.2 $x$ has no child

2. $x$ is black
   2.1 $x$ has one child
   2.2 $x$ has no child

$x$ is black

1. deleting $x$ is going to reducing the black height of the path containing $x$ by one

2. key idea: recoloring $x$’s sibling $s$ to red, and reducing the black height of the path containing $s$ by one

- Question: is it possible that $s$ is null?
- Question: what to do if $s$ is already red?
Some simple practice

Questions

1. How to get your sibling?
2. How to get your uncle?
More Case Analysis

Now we reduced our analysis to: $x$ is black and $x$’s kid $k$ is also black (or null). We further let $s$ be the sibling, $L$ and $R$ are the left and right children of $s$, respectively, and let $P$ be the parent.

Five possible cases

1. $s$ is red
   1.1 What are the colors of $P$, $L$, $R$?

2. $s$ is black
   2.1 Both kids $L$, and $R$ are all black
   2.2 Both $R$ and $L$ are red
   2.3 $R$ is red, $L$ is black
   2.4 $L$ is red, $R$ is black