CS311 Data Structures
Lecture 09— Priority Queue

Jyh-Ming Lien

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Priority Queues

Queue

What operations does a queue support?

Priority: Number representing importance

- Convention lower is better priority

  Bring back life form. Priority One. All other priorities rescinded.
- Symmetric code if higher is better

Priority Queue (PQ): Supports 3 operations

- `void insert(T x,int p)`: Insert `x` with priority `p`
- `T findMin()`: Return the object with the best priority
- `void deleteMin()`: Remove the the object with the best priority

Priority

Explicit Priority

`insert(T x, int p)`

- Priority is explicitly `int p`
- Separate from data

Implicit Priority

`insert(Comparable<T> x)`

- `x` ”knows” its own priority
- Comparisons dictated by `x.compareTo(y)`

Implicit is simpler for discussion: only one thing (`x`) to draw
Explicit usually uses a wrapper node of sorts

```
class PQNode<T> extends Comparable<PQNode>{
  int priority;    T data;
  public int compareTo(PQNode that){
    return this.priority - that.priority;
  }
}
```

Exercise: Design a PQ

Discuss

- How would you design `PriorityQueue` class?
- What underlying data structures would you use?
- Discuss with a neighbor
- Give rough idea of implementation
- Make it as efficient as possible in Big-O sense

Must Implement

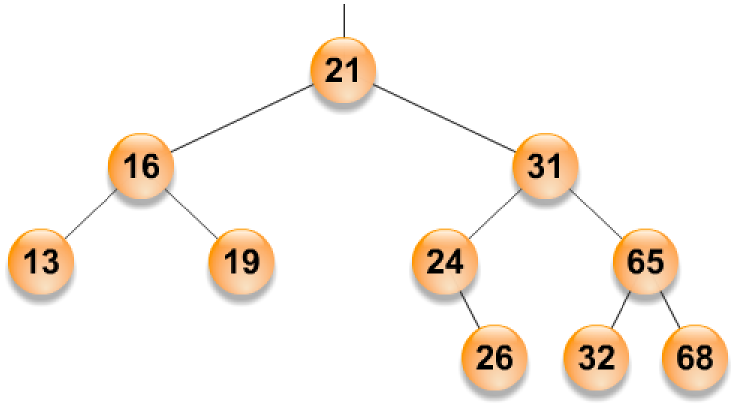
- Constructor
- `void insert(T x)`: Insert `x`, knows its own priority
- `T findMin()`: Return the object with the best priority
- `void deleteMin()`: Remove the the object with the best priority

Binary Heap: Sort of Sorted

- Most common way to build a PQ is using a new-ish data structure, the Binary Heap.
- Looks similar to a Binary Search Tree but maintains a different property

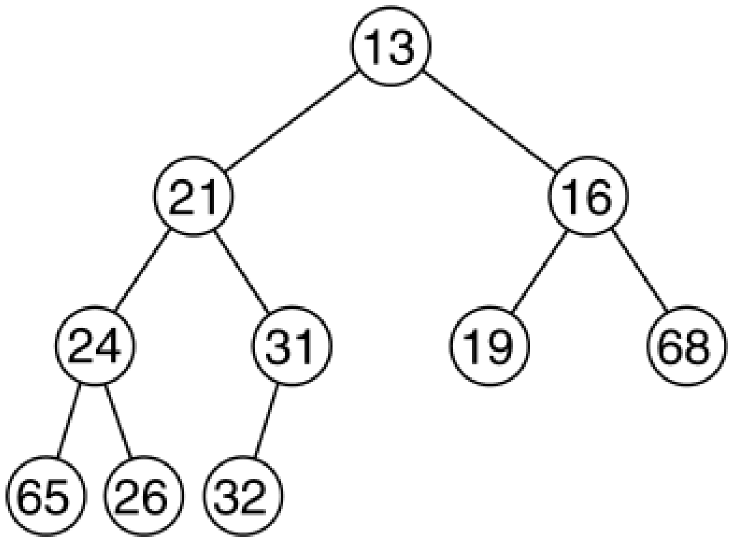
BST Property

A Node must be bigger than its left children and smaller than its right children

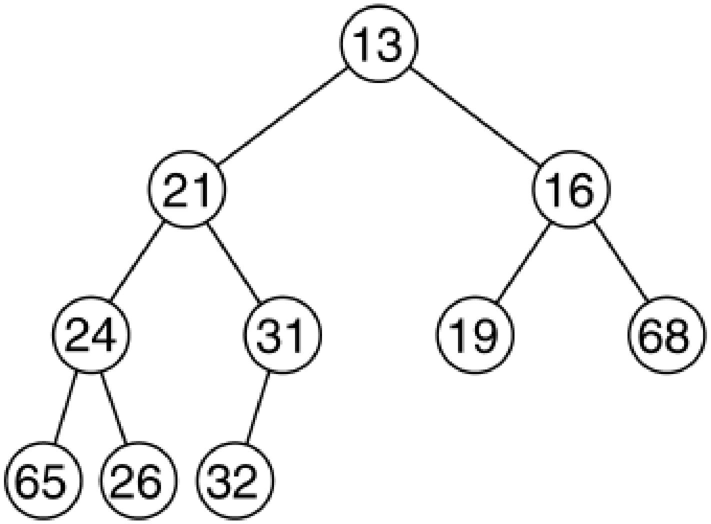


Binary Min-Heap Property

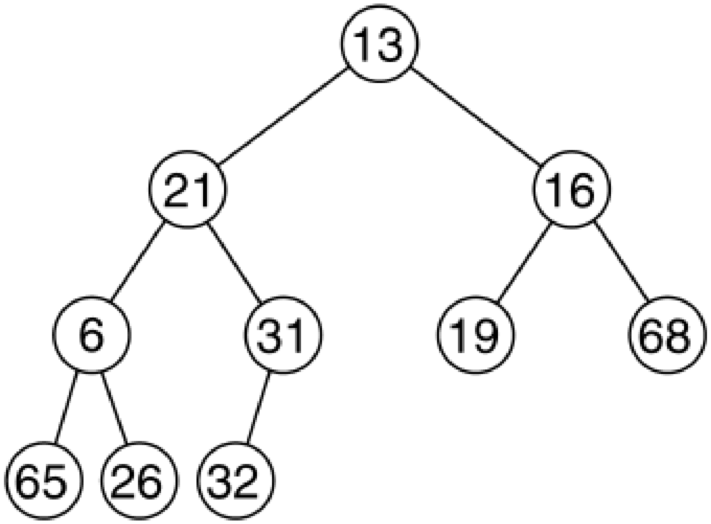
A Node must be smaller than its children



Heap and Not Heap



(a)

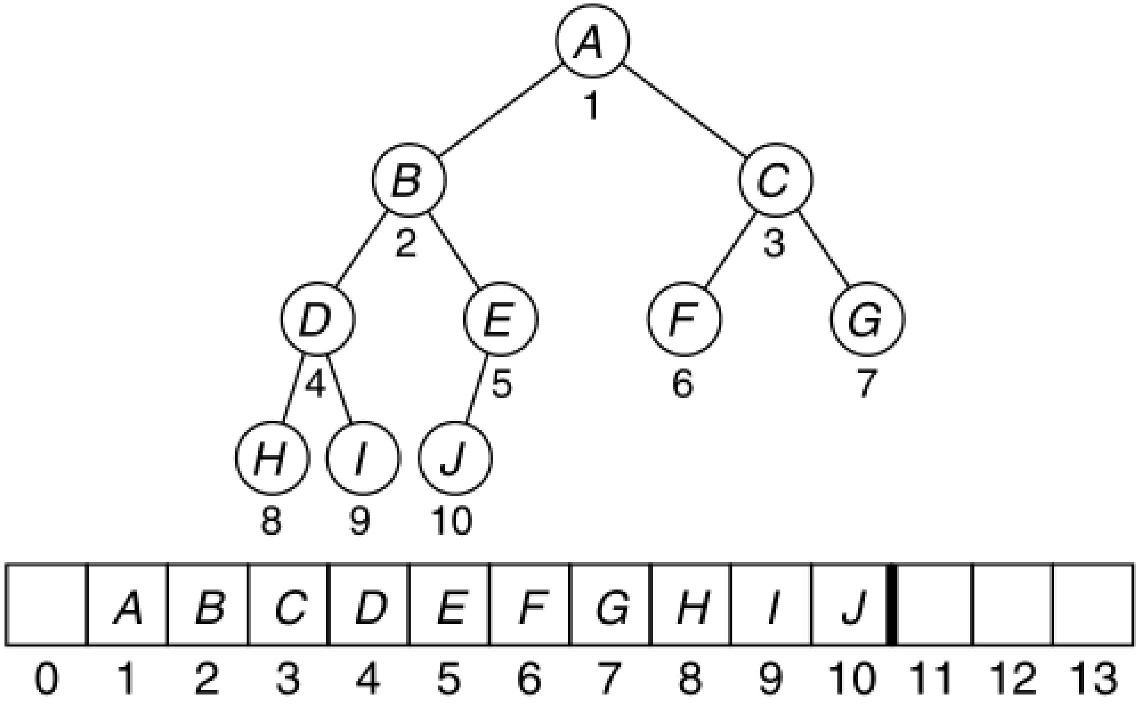


(b)

Which of these is a min-heap and which is not?

Trees and Heaps in Arrays

- Mostly we have used trees of linked Nodes
- Can also put trees/heaps in an array



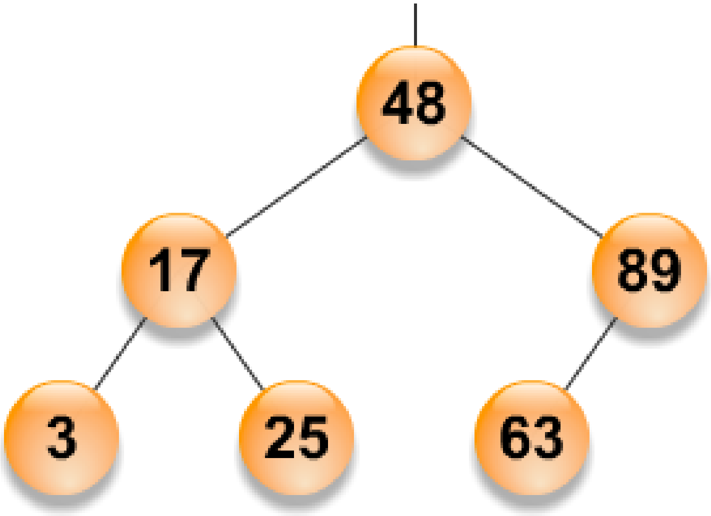
- Root is at 1 (discuss root at 0 later)
- `left(i) = 2*i`
- `right(i) = 2*i + 1`

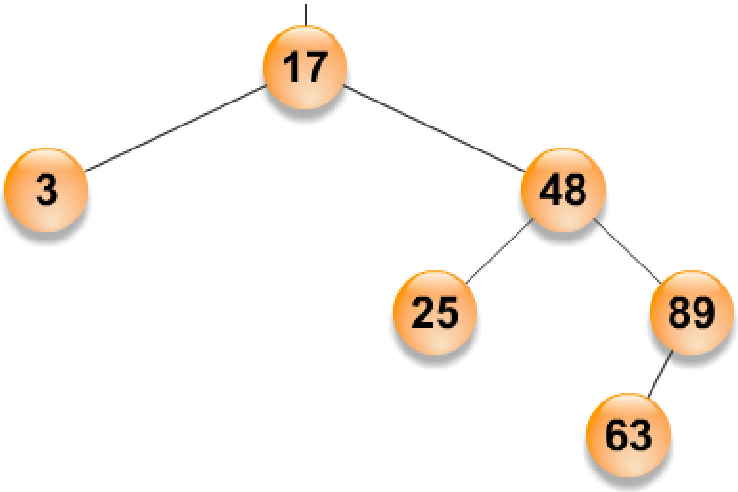
Balanced v. Unbalanced in Arrays

Find the array layout of these two trees

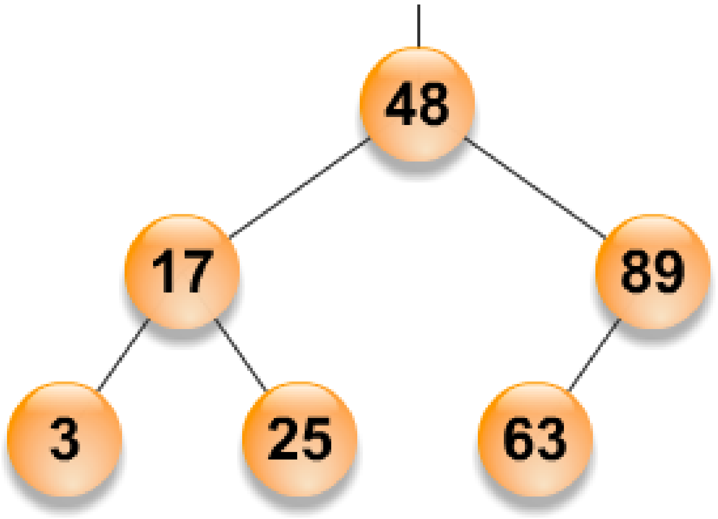
- Root is at 1
- `left(i) = 2*i`
- `right(i) = 2*i + 1`

Q: How big of array is required?

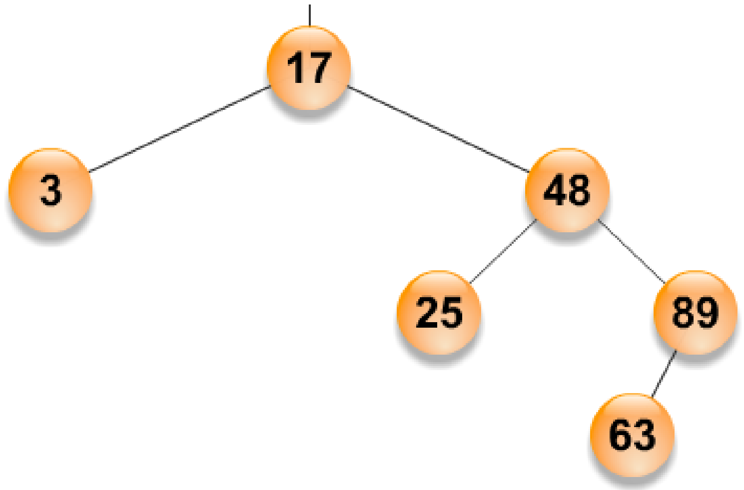




Balanced v. Unbalanced in Arrays



0	1	2	3	4	5	6
	48	17	89	3	25	63

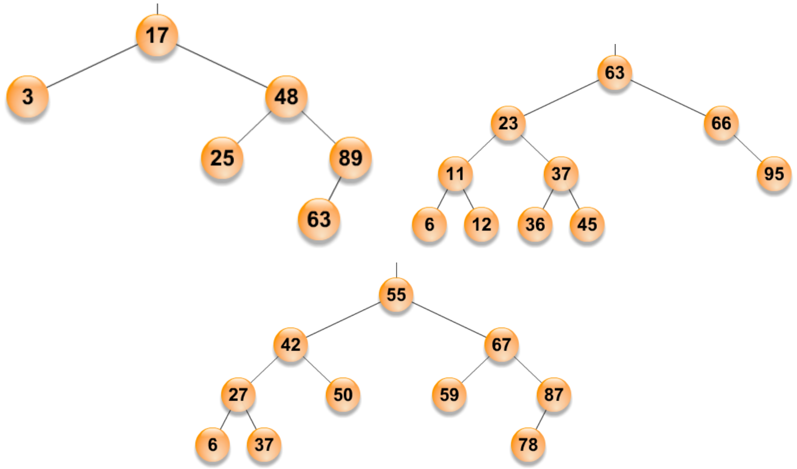


0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	17	3	48			25	89							63

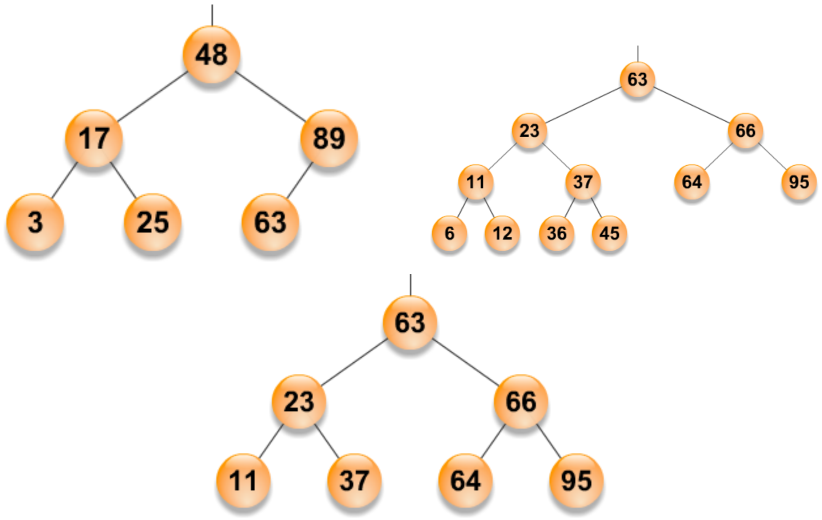
Complete Trees

- Only "missing" nodes in their bottom row (level set)
- Nodes in bottom row are as far left as possible

Not Complete (Why?)



Complete



- Complete trees don't waste space in arrays: no gaps
- Hard for general BSTs, easy for binary heaps. . .

PQ Ops with Binary Heaps

- Use an internal `T array[]` of queue contents
- Maintaint min-heap order in `array`

Define

Tree-like ops for `array[]`

```
root()     => 1
left(i)    => i*2
right(i)   => i*2 + 1
parent(i) => i / 2
```

`insert(T x)`

Ensure heap is a complete tree

- Insert at next `array[size]`
- Increment size
- Percolate new element up

`T findMin()`

Super easy

```
return array[root()];
```

`deleteMin()`

Ensure heap is a complete tree

- Decrement size
- Replace root with last data
- Percolate root down

Demos of Binary Heaps

Not allowed on exams, but good for studying

Min Heap from David Galles @ Univ SanFran

- Visualize both heap and array version
- All ops supported

Max Heap from Steven Halim

- Good visuals
- No array
- Slow to load

Operations for Heaps

```
// Binary Heap, 1-indexed
public class BinaryHeapPQ<T>{
  private T [] array;
  private int size;

  // Helpers
  static int root(){
    return 1;
  }
  static int left(int i){
    return i*2;
  }
  static int right(int i){
    return i*2+1;
  }
  static int parent(int i){
    return i / 2;
  }
```

```
// Insert a data
public void insert(T x){
  size++;
  ensureCapacity(size+1);
  array[size] = x;
  percolateUp(size);
}
// Remove the minimum element
public void deleteMin(){
  array[root()] = array[size];
  array[size] = null;
  size--;
  percolateDown(root());
}
```

Percolate Up/Down

Up

```
void percolateUp(int xdx){
  while(xdx!=root()){
    T x = array[xdx];
    T p = array[parent(xdx)];
    if(doCompare(x,p) < 0){
      array[xdx] = p;
      array[parent(xdx)] = x;
      xdx = parent(xdx);
    }
    else{ break; }
  }
}
```

Down

```
void percolateDown(int xdx){
  while(true){
    T x = array[xdx];
    int cdx = left(xdx);
    // Determine which child
    // if any to swap with
    if(cdx > size){ break; } // No left, bottom
    if(right(xdx) < size &&  // Right valid
       doCompare(array[right(xdx)], array[cdx]) < 0){
      cdx = right(xdx);      // Right smaller
    }
    T child = array[cdx];
    if(doCompare(child,x) < 0){ // child smaller
      array[cdx] = x;        // swap
      array[xdx] = child;
      xdx = cdx;             // reset index
    }
    else{ break; }
  }
}
```

PQ/Binary Heap Code

`BinaryHeapPQ.java`

- Code distribution today contains working heap
- `percolateUp()` and `percolateDown()` do most of the work
- Uses "root at index 1" convention

Text Book Binary Heap

- Weiss uses a different approach in percolate up/down
- Move a "hole" around rather than swapping
- Probably saves 1 comparison per loop iteration
- Have a look in `weiss/util/PriorityQueue.java`

Complexity of Binary Heap PQ methods?

```
T findMin();
void insert(T x); // x knows its priority
void deleteMin();
```

Give the complexity and justify for each

Height Again. . .

Efficiency of Binary Heap PQs

`findMin()`	clearly $O(1)$
`deleteMin()`	worst case height
`insert(x)`	worst case height

Height of a Complete Binary Tree wrt number of nodes N?

- Guesses?

Summary of Binary Heaps

Op	Worst Case	Avg Case
`findMin()`	$O(1)$	$O(1)$
`insert(x)`	$O(\log N)$	$O(1)$
`deleteMin()`	$O(\log N)$	$O(\log N)$

- Notice: No `get(x)` method or `remove(x)` methods
- These would involve searching the whole binary heap/priority queue if they did existed: $O(N)$